This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

Educ T 118.95.463

HARVARD UNIVERSITY

LIBRARY OF THE

Department of Education

COLLECTION OF TEXT-BOOKS
Contributed by the Publishers

TRANSFERRED
TO
HARVARD COLLEGE

3 2044 097 003 099

A
COMPLETE
ARITHMETIC,

WITH ORAL AND WRITTEN EXERCISES.

BY

GEO. W. HULL, M.A., PH.D.,
PROFESSOR OF MATHEMATICS IN THE FIRST PENNSYLVANIA STATE
NORMAL SCHOOL, MILLERSVILLE, PA.

BUTLER, SHELDON & COMPANY
PHILADELPHIA, NEW YORK, CHICAGO, BOSTON

Educ T 118.95.463

Harvard College Library
Dec. 20, 1918.
Transferred from
Education Library.

COPYRIGHT, 1895, BY E. H. BUTLER & CO.

HULL'S COM. ARITH.

17

PREFACE.

THE object of this work is to give a complete course in all those portions of Arithmetic which are required for the actual business of life, and at the same time to furnish the pupil with a clear and comprehensive knowledge of the science of numbers.

The principles, therefore, that have been followed in the preparation of this work are embodied in the following questions: 1st, *What are the actual demands of practical life?* 2d, *What does a logical development of the subject require?*

In the application of these principles it has been found necessary to omit some subjects usually found in Arithmetics; to greatly abridge and simplify the discussion of Denominate Numbers, Percentage, Proportion, etc.; and to place a number of other topics of local application, or of minor importance to the majority of pupils of the public schools of America, in a Supplement.

Another object which has been kept constantly in view through the entire work is the development of *thought-power*—to make thinkers rather than imitators. Greater importance is therefore attached to *illustrations*, *processes*, and *principles* than to abstract definitions, long solutions, and unnecessary rules.

GEO. W. HULL.

MILLERSVILLE, PA.,
April 1, 1895.

CONTENTS.

PAGE

PREFACE 3

SECTION I.

NUMERATION AND NOTATION. 7
- Arabic System of Notation . 7
- Roman Notation 13
- Numeration and Notation of U. S. Money 15

SECTION II.

FUNDAMENTAL PROCESSES . . 16
- Addition 16
- Subtraction 25
- Multiplication 32
- Division 43
- Use of the Parenthesis . . 52
- Arithmetical Analysis . . . 55

SECTION III.

FACTORS, DIVISORS, AND MULTIPLES 56
- Factoring 56
- Divisibility of Numbers . . 57
- Cancellation 60
- Greatest Common Divisor . 62
- Least Common Multiple . . 66

SECTION IV.

PAGE

COMMON FRACTIONS 72
- Definitions 72
- Notation and Numeration of Fractions 74
- Reduction of Fractions . . 74
- Addition of Fractions . . . 80
- Subtraction of Fractions . 81
- Multiplication of Fractions. 84
- Division of Fractions . . . 88
- Complex Fractions 92
- Fractional Parts of Numbers. 93
- Arithmetical Analysis . . 95

SECTION V.

DECIMAL FRACTIONS 101
- Definitions 101
- Numeration of Decimals . 103
- Notation of Decimals . . . 104
- Reduction of Decimals . . 106
- Addition of Decimals . . . 108
- Subtraction of Decimals . . 109
- Multiplication of Decimals. 110
- Division of Decimals . . . 112
- Application of Decimals . . 114
- Bills and Accounts 120

SECTION VI.

PAGE
DENOMINATE NUMBERS . . 124
Measures of Value 124
Measures of Weight . . . 126
Measures of Length . . . 127
Measures of Surface . . . 128
Measures of Volume . . . 129
Measures of Capacity . . . 130
Measures of Time 131
Angular Measure 132
Miscellaneous Tables . . . 133
Miscellaneous Measures . . 133
Reduction 134
Addition 142
Subtraction 143
Multiplication 145
Division 145
Longitude 146
Longitude and Time . . . 147

SECTION VII.

PRACTICAL MEASUREMENTS . 154
Measures of Surfaces . . . 154
Painting, Papering, Plastering, etc. 159
Measures of Volume . . . 161
Bins, Tanks, and Cisterns . 164
Boards and Timber 165
Masonry and Brickwork . 167

SECTION VIII.

PERCENTAGE 172
Applications of Percentage. 182
Commission 182
Taxes 185
Insurance 187
Interest 189
Promissory Notes . . . 199
Partial Payments . . . 202
PAGE
Annual Interest 205
Compound Interest . . . 207
Discount 209
Stocks and Bonds . . . 215

SECTION IX.

RATIO AND PROPORTION . . 222
Partnership 230
Equation of Payments . . 233
Averaging of Accounts . . 237

SECTION X.

INVOLUTION AND EVOLUTION 240
Involution 240
Squaring Numbers Geometrically 242
Cubing Numbers Geometrically 243
Evolution 244
Evolution by Factoring . . 246
Square Root 247
Of Decimals 249
Of Imperfect Squares . . 250
Of Fractions 251
Applications of Square Root 251
The Square 251
The Right Triangle . . 252
Similar Figures 254
Cube Root 255
Short Method 259
Of Decimals 260
Of Imperfect Cubes . . 261
Of Fractions 261
Applications of Cube Root. 262
The Cube 262
Similar Volumes 263

SECTION XI.

PAGE
MENSURATION 265
  Lines 265
  Angles 265
  Surfaces 266
  Area of Polygons 268
    Rectangle 268
    Parallelogram 269
    Triangle 270
    Trapezoid 271
    Circle 272
  Solids 276
    Prism 276
    Pyramid 279
    Cylinder 280
    Cone 282
    Frustums 283
    Sphere 285

SECTION XII.

PAGE
PROGRESSIONS 289
  Arithmetical Progression . 289
  Geometrical Progression . 291

SECTION XIII.

MISCELLANEOUS PROBLEMS . 295

SECTION XIV.

SUPPLEMENT 305
  Business Forms 305
  Exchange 306
    Domestic Exchange . . 309
    Foreign Exchange . . . 311
  Duties or Customs 313
  Annuities 314
  Building Associations . . . 317
  Metric System 323

ARITHMETIC.

SECTION I.

NUMERATION AND NOTATION.

1. IF we desire to know how many trees there are in a certain grove, we must count them; thus, *one*, *two*, *three*, *four*, *five*, *six*, *seven*, *eight*, *nine*, etc., until all have been counted.

These expressions, *one*, *two*, *three*, etc., are called **Whole Numbers**, or **Integers**.

2. Notation is the method of representing numbers by means of characters.

Numeration is the method of reading numbers when expressed by characters.

ARABIC SYSTEM OF NOTATION.

3. Ten characters, called **Figures**, are used to represent numbers. They are—

FIGURES,	1,	2,	3,	4,	5,	6,	7,	8,	9,	0,
Names,	one,	two,	three,	four,	five,	six,	seven,	eight,	nine,	zero.

These ten characters are also called **Digits**.

A figure standing alone expresses ones, or **Units**.

4. Numbers greater than *nine* are expressed by writing two or more of these figures side by side.

If we add one more to nine, we shall have ten. To express this, we write 1 in the second place and zero in the first place. Hence

A figure standing in the second place expresses **Tens.** Thus,

10 is ten *units* or 1 *ten*, called ten.
11 is 1 *ten* and 1 *unit*, or eleven.
12 is 1 *ten* and 2 *units*, or twelve.
13 is 1 *ten* and 3 *units*, or thirteen.
14 is 1 *ten* and 4 *units*, or fourteen.

And so on to 19, or nineteen.

20 is 2 *tens*, called twenty.
21 is 2 *tens* and 1 *unit*, or twenty-one.
22 is 2 *tens* and 2 *units*, or twenty-two.
23 is 2 *tens* and 3 *units*, or twenty-three.

And so on to 29, or twenty-nine.

In like manner we have 30, 40, 50, 60, 70, 80, 90, with the intervening numbers.

EXERCISES.

1. Write neatly in figures the numbers from ten to twenty; from thirty to forty; from fifty to sixty; from seventy to eighty; from thirty-four to fifty-six; from sixty-six to eighty-six.

How many tens and units in—

2.	*3.*	*4.*	*5.*	*6.*	*7.*	*8.*
14?	15?	25?	37?	40?	50?	99?
26?	32?	39?	59?	72?	79?	80?
35?	47?	54?	60?	88?	77?	21?

9. What is the largest number that can be expressed by two figures? What is the smallest?

Read the following numbers:

10.	*11.*	*12.*	*13.*	*14.*	*15.*	*16.*
13	16	26	39	41	51	99
25	33	40	60	73	80	81
39	49	55	61	89	78	22
20	30	50	70	90	95	85

Write in figures—

17.	*18.*	*19.*
Twenty-two;	Fifty-seven;	Sixty-four;
Eighty-nine;	Seventy;	Thirty-five;
Twenty-nine;	Sixty-eight;	Forty-one;
Thirty-three;	Ninety-nine;	Eighty-seven;
Forty-eight;	Forty-six;	Ninety-five;
Fifty-two.	Eighty-eight.	Fifty-four.

5. Hundreds.—If we add one more to ninety-nine, we shall have *ten tens,* or one hundred. To express this, we write 1 in the third place. Hence

A figure standing in the third place expresses **Hundreds.** Thus,

100 is 10 *tens,* called one hundred.
234 is 2 *hundreds,* 3 *tens,* and 4 *units,* or two hundred and thirty-four.
999 is 9 *hundreds,* 9 *tens,* and 9 *units,* or nine hundred and ninety-nine.

EXERCISES.

1. Write neatly in figures the numbers from one hundred to one hundred and twenty-five; from two hundred and twenty-five to two hundred and fifty; from three hundred and fifty to three hundred and seventy-five; from four hundred and seventy-five to five hundred.

How many hundreds, tens, and units in—

2.	*3.*	*4.*	*5.*	*6.*	*7.*
314?	541?	324?	587?	500?	749?
238?	652?	836?	809?	909?	927?

Read the following numbers:

8.	*9.*	*10.*	*11.*	*12.*	*13.*
100	500	900	450	890	789
200	600	120	560	912	406
300	700	230	670	846	709

6. Periods.—For convenience in reading and writing numbers the figures of a number are divided into periods of three places each. The first three places make the *first* or *units period*, the second three places make the *second* or *thousands period*, etc.

The first six periods are as follows:

NUMERATION TABLE.

Names of Periods,	Quadrillions.	Trillions.	Billions.	Millions.	Thousands.	Units.
Names of Places,	Hundred-quadrillions, Ten-quadrillions, Quadrillions,	Hundred-trillions, Ten-trillions, Trillions,	Hundred-billions, Ten-billions, Billions,	Hundred-millions, Ten-millions, Millions,	Hundred-thousands, Ten-thousands, Thousands,	Hundreds, Tens, Units.
Places,	8 8 8,	8 8 8,	8 8 8,	8 8 8,	8 8 8,	8 8 8
Periods,	6th.	5th.	4th.	3d.	2d.	1st.

The periods are separated from each other by commas.

EXERCISES ON THE TABLE.

1. Name the periods in order from units to quadrillions.

2. Name the places in order from units to quadrillions.

Name the following periods:

3.	*4.*	*5.*
1st period.	4th period.	6th period.
2d period.	2d period.	4th period.
3d period.	5th period.	3d period.

In what period is—

6.	*7.*	*8.*
Millions?	Billions?	Quadrillions?
Thousands?	Units?	Trillions?
Trillions?	Thousands?	Millions?

Read the following numbers:

9.	*10.*	*11.*	*12.*	*13.*
1,000	9,000	8,210	8,021	3,432
2,000	7,300	7,320	6,047	7,872
3,000	6,200	9,760	5,036	3,140

Write in figures—

14.	*15.*
5 thousand.	8 thousand 200.
7 thousand 314.	1 thousand and 11.
9 thousand and 20.	2 thousand 890.
6 thousand and 81.	3 thousand and 3.
4 thousand 780.	9 thousand and 9.

Read the following numbers:

16.	*17.*	*18.*	*19.*
10,000	60,214	100,000	625,324
20,000	70,809	200,000	704,897
30,000	80,029	300,000	820,614

Write in figures—

20.

32 thousand 316.
50 thousand and 16.
85 thousand and 8.
97 thousand 306.
90 thousand and 9.

21.

316 thousand 425.
102 thousand and 25.
400 thousand and 40.
526 thousand and 4.
700 thousand and 7.

Read the following numbers:

22.

1,000,000
2,000,000
30,000,000
40,000,000
500,000,000
600,000,000

23.

700,500,300
850,430,520
961,432,618
804,503,702
21,032,041
64,008,009

24.

401,420,401
45,045,045
100,100,100
60,060,060
4,004,004
164,164,164

25.

164,310,806
8,008,008
80,080,080
800,800,800
888,888,888
941,862,721

Write in figures—

26.

1 million 320 thousand 412.
2 million 145 thousand and 75.
8 million 65 thousand and 86.
28 million 4 thousand 608.
75 million 6 thousand 102.
92 million 84 thousand 189.

27.

126 million 216 thousand 812.
400 million 300 thousand 200.
520 million 62 thousand 182.
903 million 9 thousand and 9.
90 million 90 thousand and 90.
999 million 999 thousand 999.

Copy, point off, and read—

28. 1234.
29. 64876.
30. 103625.
31. 2186234.
32. 60450217.
33. 430041671.
34. 6102032416.
35. 80634262321.
36. 70010003.
37. 600001024.
38. 3000700271.
39. 20006024028.
40. 321027320300.
41. 3206087042107.
42. 20014206104027.
43. 416876341216162.

Write in figures—

44. Fifty-six thousand, four hundred.

45. Four million, two hundred and sixteen thousand, and one.

46. Forty million, forty thousand, and forty.

47. Six billion, six million, six thousand, and six.

48. Nine hundred and forty million, eight hundred and six thousand, four hundred and six.

49. Seven trillion, six billion, five million, four thousand, and three.

7. The table which has been given will enable us to read a number consisting of eighteen figures: the periods which follow them are—

Quintillions, Sextillions, Septillions, Octillions, Nonillions, Decillions, Undecillions, Duodecillions, etc.

ROMAN NOTATION.

8. In this system seven letters are used to express numbers. Thus,

LETTERS,	I,	V,	X,	L,	C,	D,	M,
Value,	1.	5.	10.	50.	100.	500.	1000.

Other numbers are expressed by writing two or more of these letters side by side.

(1) *When a letter is repeated its value is repeated;* thus, II represents 2; XXX, 30; CCCC, 400, etc.

(2) *When a letter is placed before one of greater value, their* **Difference** *is expressed;* thus, IV represents 4; XL, 40; XC, 90, etc.

(3) *When a letter is placed after one of greater value, their* **Sum** *is expressed;* thus, VI represents 6; XV, 15; LX, 60, etc.

(4) *A dash placed over a letter increases its value a thousand fold;* thus, $\overline{\text{V}}$ represents 5000; $\overline{\text{VII}}$, 7000; $\overline{\text{XI}}$, 11000, etc.

ROMAN TABLE.

Roman.	Arabic.	Roman.	Arabic.	Roman.	Arabic.
I	1	XVI	16	CCC	300
II	2	XVII	17	CCCC	400
III	3	XVIII	18	D	500
IV	4	XIX	19	DC	600
V	5	XX	20	DCC	700
VI	6	XXI	21	DCCC	800
VII	7	XXX	30	DCCCC	900
VIII	8	XL	40	M	1000
IX	9	L	50	MD	1500
X	10	LX	60	MM	2000
XI	11	LXX	70	MMM	3000
XII	12	LXXX	80	MMMM	4000
XIII	13	XC	90	$\overline{\mathrm{V}}$	5000
XIV	14	C	100	$\overline{\mathrm{VI}}$	6000
XV	15	CC	200	$\overline{\mathrm{VII}}$	7000

Express in Arabic notation—

1.	*2.*	*3.*	*4.*
XIX;	CIX;	XXXIX;	$\overline{\mathrm{CC}}$XC;
XXXIII;	CXI;	LXXXIX;	CXLIX;
XXIX;	XCI;	CLXXIX;	CLI;
XLV;	DCXC;	CCCCXLVII;	MD;
LXV.	DCCX.	$\overline{\mathrm{VIII}}$.	MMD.

Express in Roman notation—

1.	*2.*	*3.*	*4.*
15	56	100	587
36	40	1600	436
42	49	3600	789
87	63	5000	207
66	99	5010	999

Express 1895 in Roman notation.

NUMERATION AND NOTATION OF U. S. MONEY.

9. In the currency of the United States one hundred cents make a dollar.

The sign $, called the **Dollar Sign,** is written before the number. Thus, one dollar is written $1, or $1.00; and fifteen dollars and twenty-five cents is written $15.25. The period between the dollars and cents is called the *decimal point.*

Cents occupy the first two places at the right of the decimal point.

Read the following:

1. $8.42.
2. $12.64.
3. $28.04.
4. $38.23.
5. $125.05.
6. $362.14.
7. $400.06.
8. $920.10.
9. $3020.06.
10. $1214.86.
11. $9024.09.
12. $4321.12.

Write the following:

13. Five dollars. Fifty dollars. Five hundred dollars.

14. Four dollars and 25 cents.

15. One hundred and forty-seven dollars and thirty-seven cents.

16. Six hundred and twenty dollars and nine cents.

17. Three thousand and ten dollars and fifty cents.

DEFINITIONS.

10. A **Unit** is one of any kind.

A **Number** is a unit or a collection of units, or rather the "how many" of the collection.

Arithmetic treats of numbers and their uses.

11. There are three ways of expressing numbers:

1. By *words;* as, one, two, three, etc.
2. By *letters,* called the *Roman* method; as, I, II, III, etc.
3. By *figures,* called the *Arabic* method; as, 1, 2, 3, etc.

SECTION II.

FUNDAMENTAL PROCESSES.

ADDITION.

ORAL EXERCISES.

12.—*1.* A class has 4 boys and 9 girls in it; how many pupils in the class?

2. Horace is 5 years old; how old will he be in 8 years? In 9 years? In 6 years? In 5 years?

3. A farmer has 6 cows; how many will he have if he buys 5 more? 8 more? 7 more? 6 more? 9 more?

4. Arthur has 7 marbles; how many will he have if he buys 5 more? 6 more? 9 more? 8 more? 7 more?

5. Ruth has 8 cents; how many will she have if May gives her 6 more? 5 more? 9 more? 8 more? 7 more?

6. A teacher has 9 pupils in arithmetic; if 9 more are added, how many will there be in the class? If 8 more? If 7 more? If 6 more? If 5 more? If 4 more?

7. How many are—

8 and 5?
7 and 6?
5 and 9?
4 and 8?
9 and 9?
8 and 9?
7 and 8?
8 and 8?
6 and 9?
9 and 7?
6 and 8?
6 and 7?
4 and 7?
8 and 7?
9 and 8?
7 and 7?

---

8. Count by 2's from 2 to 30. From 30 to 60. From 1 to 31. From 31 to 61.

9. Count by 3's from 3 to 45. From 1 to 46. From 2 to 47.

10. Count by 4's from 4 to 48. From 1 to 49. From 2 to 50. From 3 to 51.

11. Count by 5's from 5 to 100. From 1 to 51. From 2 to 52. From 3 to 53. From 4 to 54.

12. In the same way count by 6's, 7's, 8's, 9's, 10's, to 50.

13. To each of the following numbers,

8, 6, 7, 9, 5, 4, 3, 5, 9, 7, 8,

add 2, 5, 4, 3, 9, 8, 7, 6, naming the results only; thus, 10, 8, 9, etc.

Name the results only—

14. $3 + 3 + 3 =$
15. $4 + 4 + 4 =$
16. $5 + 5 + 5 =$
17. $6 + 6 + 6 =$
18. $7 + 7 + 7 =$
19. $4 + 3 + 2 + 1 =$
20. $5 + 6 + 7 + 8 =$

21. $3 + 4 + 7 + 8 =$
22. $6 + 2 + 3 + 7 =$
23. $6 + 2 + 5 + 4 =$
24. $6 + 4 + 3 + 2 =$
25. $8 + 5 + 4 + 3 =$
26. $9 + 2 + 3 + 6 =$
27. $7 + 8 + 3 + 7 =$

28.	*29.*	*30.*	*31.*	*32.*	*33.*
3	4	9	4	9	7
6	8	6	5	8	3
8	7	7	6	7	8
5	6	3	7	6	2
6	2	2	8	5	1

34.	*35.*	*36.*	*37.*	*38.*	*39.*
4	9	1	4	7	8
8	8	2	7	6	9
5	7	3	6	4	7
3	6	4	3	8	8
2	5	5	2	3	9
6	4	6	1	2	7

40. Add 7 to 18, 15, 16, 29, 24, 15, 13, 12, 21, 30. Add 8; 4; 6; 8; 9; 5.

41. Add 6 to 36, 42, 48, 24, 35, 52, 49, 78, 63, 88. Add 4; 5; 3; 7; 8; 9.

Add—

42.	*43.*	*44.*	*45.*	*46.*	*47.*
3	2	8	5	4	4
4	1	7	4	3	8
8	6	6	9	7	7
7	5	8	9	8	9
6	4	7	4	9	3
5	3	6	5	6	4
4	7	8	5	4	8
3	8	7	4	3	6
5	6	6	9	2	7

48–71. Add rapidly by columns and by lines—

$6 + 4 + 3 + 4 + 7 + 8 =$
$3 + 7 + 8 + 5 + 4 + 7 =$
$4 + 8 + 6 + 9 + 4 + 7 =$
$2 + 3 + 8 + 6 + 3 + 5 =$
$4 + 8 + 7 + 5 + 6 + 3 =$
$2 + 7 + 6 + 7 + 7 + 7 =$

$2 + 3 + 4 + 5 + 6 + 7 =$
$4 + 5 + 6 + 7 + 8 + 9 =$
$5 + 6 + 7 + 8 + 9 + 4 =$
$6 + 7 + 8 + 9 + 5 + 4 =$
$7 + 9 + 8 + 7 + 6 + 2 =$
$4 + 8 + 7 + 6 + 3 + 2 =$

DEFINITIONS.

13. Addition is the process of finding a number which shall contain as many units as the numbers added.

The **Sum** is the result obtained by adding.

The **Sign of Addition** is +, and is read *plus.* It indicates that the numbers between which it is placed are to be added.

14. The **Sign of Equality** is =, and is read *equals.* Thus, $5 + 3 = 8$ is read 5 *plus* 3 *equals* 8.

WRITTEN EXERCISES.

15.—*1.* What is the sum of 65 and 87?

PROCESS.

```
  65
  87
 152, Ans.
```

For convenience we write the numbers with units under units and tens under tens, and begin at the right to add.

The sum of 7 units and 5 units is 12 units, which equal 1 *ten* and 2 *units;* we write the 2 units under the column of units, and add the 1 ten to the column of tens. The sum of 1 ten, 8 tens, and 6 tens is 15 tens, which equal 1 *hundred* and 5 tens; we write the 5 tens under the column of tens, and place the 1 hundred on the left in the place of hundreds.

2. What is the sum of 455, 789, and 347?

PROCESS.

```
  455
  789
  347
 1591, Ans.
```

In practice we name *results* only in adding; thus, we say 7, 16, 21; write 1 and carry 2.

Then 2, 6, 14, 19; write 9 and carry 1.

Then 1, 4, 11, 15; write 15.

PROOF.—Begin at the top and add downward; if the two results agree, the work is probably correct.

Find the sum of—

3.	*4.*	*5.*	*6.*	*7.*	*8.*	*9.*	*10.*	*11.*
57	46	68	69	57	79	88	49	37
64	75	64	75	75	81	92	86	93
36	21	50	45	48	33	26	21	44

12.	*13.*	*14.*	*15.*	*16.*	*17.*	*18.*	*19.*	*20.*
32	33	56	63	21	32	63	63	98
18	14	23	87	87	18	45	47	87
62	86	45	76	65	92	95	89	64

21.	*22.*	*23.*	*24.*	*25.*	*26.*	*27.*	*28.*
$387	$382	$678	$564	$8.65	$1.65	$4.78	$2.82
496	146	407	736	4.89	3.78	3.92	5.62
387	205	562	785	3.72	4.67	8.71	4.44

29–49. Add each group by columns and by lines—

26 + 82 + 62 =
32 + 16 + 59 =
14 + 32 + 80 =
16 + 93 + 76 =

321 + 621 + 261 =
468 + 873 + 782 =
562 + 410 + 341 =
324 + 602 + 562 =

36 + 18 + 21 =
88 + 27 + 46 =
25 + 49 + 15 =
19 + 86 + 76 =

50.	*51.*	*52.*	*53.*	*54.*	*55.*	*56.*	*57.*
672	918	864	444	999	362	428	429
456	362	365	333	888	157	327	598
654	548	627	666	434	687	165	876
547	678	507	555	216	416	982	597
862	732	828	888	555	728	681	672

58–74. Add each group by columns and by lines—

216 + 321 + 432 =
347 + 342 + 162 =
821 + 156 + 786 =
218 + 342 + 432 =
572 + 321 + 548 =

382 + 436 + 532 + 812 =
567 + 782 + 687 + 416 =
621 + 584 + 409 + 512 =
149 + 327 + 207 + 362 =
826 + 126 + 341 + 178 =

75. What is the sum of 364 + 582 + 649 + 874?

76. What is the sum of 5628 + 6785 + 2648 + 5000?

77. What is the sum of 6784 + 4329 + 6543 + 8962 + 3456?

78. What is the sum of 1567 + 8650 + 1075 + 4865 + 6372?

79. What is the sum of 6532 + 6457 + 3754 + 6372 + 8547?

80. What is the sum of 8632 + 1678 + 6754 + 6750 + 9762?

81. What is the sum of 8672 + 5401 + 7890 + 386 + 9021?

82. What is the sum of 37 + 375 + 3754 + 37546 + 64 + 645 + 4573 + 57373?

83–102. Add each group by columns and by lines—

341 + 467 + 508 + 632 =
263 + 821 + 700 + 216 =
567 + 463 + 436 + 348 =
873 + 562 + 321 + 516 =
142 + 702 + 786 + 922 =
378 + 433 + 105 + 483 =

421 + 322 + 234 + 621 =
812 + 444 + 423 + 578 =
287 + 555 + 342 + 760 =
359 + 666 + 167 + 420 =
614 + 777 + 761 + 706 =
786 + 888 + 504 + 933 =

103. What is the sum of 80965 + 6789 + 1940 + 2071 + 8900 + 6886 + 904 + 17895 + 7907 + 999 + 90899.

PRACTICAL PROBLEMS.

1. A farmer bought three horses: the first cost $275, the second $360, and the third $416; what did they all cost?

2. The distance from Harrisburg to Hagerstown is 74 miles, from Hagerstown to Roanoke 240 miles, from Roanoke to Bristol 150 miles, from Bristol to Chattanooga 242 miles; how far from Harrisburg to Chattanooga by this route?

3. A bought 7590 pounds of pea coal, 3765 pounds of nut coal, 6834 pounds of stove coal, and 2505 pounds of bituminous coal; how much coal did he purchase?

4. A dealer in eggs packed 1080 eggs in a barrel, nine hundred in a second, 1008 in a third, and eleven remained; how many eggs had he altogether?

5. How much money did four harvest-hands earn if Mr. Jones earned $23.16, Mr. Piper $22.02, Mr. Herr $30.30, and Mr. Binkley $14.40?

6. A florist sold eleven pinks for $1.10, forty roses for $5, thirteen lilies for $3.25, and two hundred geraniums for $10; how many flowers did he sell, and how much money did he receive for them?

7. A hunter shot 21 pheasants, one hundred and twenty-two pigeons, 300 rabbits, three wild turkeys, 33 wild ducks, and 10 snipe in a season; how many animals did he shoot?

8. Harold bought a wagon for $25, a sleigh for $10.50, a lap robe for $1.75, and a whip for $1.25; what was his bill?

9. A lady bought an extension-table for $12, a stove for $17.50, an oil-cloth for $2.25, bed-room suite for $95, a rocking-chair for $11.15, some dishes for $45, carpet for $37.50, a lamp for $3.75, and a piano for $375; what was the entire bill?

10. A owes his grocer $7.95, his butcher $14.59, his tailor $16.78, his baker $6.37, his hired help $28: he also owes a coal-bill of $25.25, a wood-bill of $3.38, and school-tax of $2.12; how much does he owe?

11. In 1890 in New York City there were 312,706 families, in Chicago 220,320, in Philadelphia 205,135, in Brooklyn 170,970, in St. Louis 91,756, in Boston 86,716, in Baltimore 86,654, in Cincinnati 63,530, in Cleveland 53,052, and in San Francisco 52,535; how many families in these ten cities?

12. The superintendent of a factory received $1800 a year, the engineer $800, the fireman $600, the book-keeper $750, the fuel cost $1500, the raw material $111,000, and the pay-roll of the other employés amounted to $50,000; what were the expenses of the factory for the year?

13. Four men, A, B, C, and D, entered into partnership: A put into the business $1650, B put in $75 more than A, C put in $150 more than B, and D put in as much as A and C; what was the amount of money invested?

14. B paid $650 for a lot, on which he built a house for $5785, the fence around the lot cost $167, the pavement cost $243, and the plumbing cost $315. He sold the property and cleared $1200; what did he receive for it?

15. The number of immigrants coming into the United States in 1882 was 788,992; in 1883, 603,322; in 1884, 518,592; in 1885, 395,346; in 1886, 334,203; in 1887, 490,109; in 1888, 546,889; in 1889, 444,427; in 1890, 455,302; in 1891, 560,319; and in 1892, 623,084. How many immigrants arrived in the eleven years?

16. What was the whole amount of money in circulation in the United States on Jan. 1, 1894, if there were $508,602,811 in gold, $57,867,589 in standard silver dollars, $65,854,740 in subsidiary silver coins, $77,412,179 in gold certificates, $32,954,650 in silver certificates, $151,960,267 in silver Treasury notes, $302,541,814 in U. S. notes, $39,045,000 in currency certificates, and $196,181,216 in national bank-notes?

17. The following problem in addition was given in the Civil Service examination of the United States in 1892:

$79,654,321,908.35
47,776,013,703.30
92,773,331,673.25
7,774,910,336.15
44,297,794,329.37
6,105,733,266.59
232,173.63
8,859,367,397.45
42,223,001,764.86
63,337,476,074.03
2,335,602,047.90
293,827,764,501.77

18.	*19.*	*20.*	*21.*	*22.*
123	456	789	1234	3344
123	456	789	5678	4433
123	456	789	9012	5566
123	456	789	3456	6655
123	456	789	7890	7788
123	456	789	1234	8877
123	456	789	5678	9900
123	456	789	9012	2299
123	456	789	4567	4567
123	456	789	4001	5005

23.	*24.*	*25.*	*26.*
999	573	9087	5768
888	375	8709	7568
777	753	799	6578
666	537	7980	8567
555	375	8970	5678
444	735	9857	6875
333	648	5798	7856
222	486	7589	8675
111	864	8597	6758
999	648	958	6875
888	468	7895	5867
777	846	5978	7685
666	357	9587	6587

27. Add the following numbers by lines, and place the sums in the column provided for them; then add the column of sums to find the total:

$8176.23	$143.16	$9432.75	$7006.02	. . . . .
103.42	60.25	710.10	604.23	. . . . .
41.02	621.37	45.86	9.25	. . . . .
8.05	1210.62	1234.08	1260.32	. . . . .
900.46	602.89	672.69	432.80	. . . . .
167.80	3211.40	6321.75	4321.08	. . . . .
8321.45	902.47	43.76	98.42	. . . . .
3216.82	1122.23	8628.43	6219.80	. . . . .
432.75	704.50	347.24	100.75	. . . . .
96.84	1200.25	6007.80	3210.40	. . . . .
			Total,	. . . . .

Prove the problem by adding by columns.

28.

86	123	4361	76543	. . . . . .
78234	4216	184	42	. . . . . .
576	47	54320	6210	. . . . . .
8120	98765	42	803	. . . . . .
67843	9872	3476	804	. . . . . .
3028	4988	9888	8888	. . . . . .
98706	14329	23186	487	. . . . . .
9643	8704	3216	50476	. . . . . .
702	32806	439	8102	. . . . . .
70304	4027	98432	47862	. . . . . .
			Total,	. . . . . .

SUBTRACTION.

ORAL EXERCISES.

16.—*1.* A boy has 13 marbles; how many will he have left if he gives away 5? If 6? If 8? If 9? If 7? If 4?

2. Ross has 14 papers; how many will he have left if he sells 6? If 4? If 9? If 8? If 5? If 7?

3. Lewis had 15 cents, and bought a ball for 6 cents; how many cents had he left? How much would he have had left if the ball had cost 7 cents? 9 cents? 8 cents?

4. May paid 16 cents for ribbon; what did she pay for a collar if it cost 9 cents less? If 7 cents less? If 8 cents less?

5. What number must you add to 5 to make 12? 11? 13? 14?

6. What number must you add to 9 to make 13? 14? 18? 17? 16? 15?

7. What number must you add to 8 to make 16? 18? 17? 14? 13? 15?

8. Begin at 30 and count backward by 2's to 0. By 3's. By 4's. By 5's. By 6's. By 7's. By 8's. By 9's.

9. Begin at 29 and count backward by 2's, 3's, 4's, 5's, 6's, 7's, 8's, 9's.

10. Begin at 28 and count backward by 3's, 4's, 5's, 6's, 7's, 8's, 9's.

11. Begin at 100 and count backward by 10's.

12.	*13.*	*14.*	*15.*	*16.*
16 − 8 =	14 − 5 =	12 − 6 =	17 − 9 =	18 − 9 =
16 − 9 =	14 − 6 =	12 − 5 =	17 − 8 =	15 − 7 =
16 − 7 =	14 − 8 =	12 − 9 =	17 − 7 =	15 − 8 =
16 − 6 =	14 − 9 =	12 − 4 =	17 − 4 =	15 − 9 =
13 − 6 =	13 − 7 =	13 − 9 =	13 − 8 =	15 − 6 =
13 − 5 =	13 − 3 =	14 − 7 =	12 − 3 =	18 − 8 =

Name the results only in 17 to 24. Thus, 38, 35, 31, etc.—

17. 40 − 2 − 3 − 4 − 5 − 6 − 7 − 8.
18. 39 − 2 − 2 − 2 − 4 − 4 − 4 − 4.
19. 38 − 4 − 5 − 6 − 7 − 8 − 4 − 3.
20. 37 − 2 − 4 − 5 − 3 − 2 − 6 − 7.
21. 46 − 4 − 5 − 6 − 7 − 8 − 9 − 7.
22. 50 − 8 − 7 − 7 − 6 − 6 − 6 − 5.
23. 54 − 9 − 8 − 7 − 6 − 9 − 5 − 4.
24. 60 − 8 − 9 − 8 − 7 − 9 − 8 − 7.

Subtract—

25.	*26.*	*27.*	*28.*	*29.*	*30.*	*31.*
9	10	11	12	13	14	15
5	8	9	7	6	4	7

32.	*33.*	*34.*	*35.*	*36.*	*37.*	*38.*
15	14	12	16	13	14	15
5	5	5	5	5	6	6

39.	*40.*	*41.*	*42.*	*43.*	*44.*	*45.*
13	12	16	13	17	16	18
6	6	6	7	7	7	9

DEFINITIONS.

17. **Subtraction** is the process of taking a less number from a greater.

The **Minuend** is the number from which we subtract.

The **Subtrahend** is the number to be subtracted.

The **Difference** is the result obtained by subtracting.

18. The **Sign of Subtraction** is —. It is read *minus*. It shows that the number after it is to be subtracted from the number before it.

WRITTEN EXERCISES.

CASE I.

19. To subtract when no term of the subtrahend is greater than the corresponding term of the minuend.

1. Subtract 342 from 689.

PROCESS.

Minuend, 689
Subtrahend, 342
Difference, 347, *Ans.*

For convenience we write the subtrahend under the minuend, with units under units, tens under tens, etc.

2 units from 9 units leave 7 units, which we write under the column of units.

4 tens from 8 tens leave 4 tens, which we write under the column of tens.

3 hundred from 6 hundred leave 3 hundred, which we write under the column of hundreds.

Hence the difference is 347.

342
347
689

PROOF.—342, the subtrahend, plus 347, the difference, equals 689, the minuend; hence the result is correct.

2.	*3.*	*4.*	*5.*	*6.*	*7.*
485	675	923	485	976	758
321	234	412	232	323	134

8.	*9.*	*10.*	*11.*	*12.*
9387	9876	8947	9986	9476
7164	2155	7616	8574	3126

13.	*14.*	*15.*
$9875.43	$68451.67	$59782.16
2153.41	12131.25	17351.14

16. Subtract 34216 from 69718.
17. Subtract 48754 from 98977.
18. Subtract 26427 from 66859.
19. Subtract 12345 from 72345.

PRACTICAL PROBLEMS.

1. A man bought a horse for $225, and sold him for $385; what was his gain?

2. A farm was bought for $8650, and sold for $7240; what was the loss?

3. An army went into battle with 5767 men, and came out with only 1422 men; how many were missing?

4. A's income for a year is $9874; how much does he save if his expenses are $4370?

5. A grain-dealer, having 6780 bushels of grain, sold 5260; how many bushels had he remaining?

6. My house cost $7854, and my barn $2324; what is the difference of their costs?

7. A farm was bought for $9860, and sold for $8740; what was the loss?

8. A bought some Western land for $7659, and sold it for $9789; what was his gain?

CASE II.

20. To subtract when a term in the subtrahend is greater than the corresponding term in the minuend.

1. Subtract 26 from 83.

Process.
Minuend, 83
Subtrahend, 26
Difference, 57, *Ans.*

We cannot take 6 units from 3 units, so we add 10 units to 3 units, making 13 units; 6 units from 13 units leave 7 units, which we write under the units column.

Since we have added 10 units to the minuend, we must add 10 units or 1 ten to the subtrahend; 2 tens and 1 ten make 3 tens; 3 tens from 8 tens leave 5 tens, which we write under the tens column; hence the result is 57.

2. Subtract 2367 from 5146.

Process.
5146
2367
2779, *Ans.*

In practice we say, 7 from 16 leaves 9; 7 from 14 leaves 7; 4 from 11 leaves 7; 3 from 5 leaves 2.

3. Subtract 3162 from 5000.

Process.
5000
3162
1838, *Ans.*

Say, 2 from 10 leaves 8; 7 from 10 leaves 3; 2 from 10 leaves 8; 4 from 5 leaves 1.

Proof.—3162 + 1838 = 5000, the minuend; hence the result is correct.

4.	*5.*	*6.*	*7.*	*8.*
643	596	421	863	686
218	389	245	125	429

9.	*10.*	*11.*	*12.*	*13.*
572	643	605	520	400
385	275	427	234	135

14.	*15.*	*16.*	*17.*	*18.*
$281.16	$864.28	$572.38	$810.70	$562.41
175.08	265.37	212.22	128.20	318.75

19.	*20.*	*21.*	*22.*
50000	80000	60000	70000
12861	23712	28654	25672

23–33. Subtract 478 from each of the following numbers: 541, 926, 604, 718, 1126, 1001, 800, 675, 808, 1000, 1621.

34–42. Subtract each of the following numbers from 1216: 462, 185, 862, 564, 162, 904, 628, 436, 1082.

43–48. Find the difference between 892 and each of the following numbers: 1214, 1604, 2000, 1122, 4216, 1616.

Find the difference between—

49. 81214 and 53467.
50. 934210 and 473508.
51. 720307 and 226418.
52. 938402 and 128103.
53. 630026 and 246219.
54. 751382 and 618592.
55. 821621 and 418299.
56. 987654 and 456789.

57.	*58.*	*59.*	*60.*
8,216,214	9,160,218	4,216,820	8,162,640
2,475,032	432,612	2,485,318	1,162,873

61.	*62.*	*63.*	*64.*
$64,210,000	$100,260,604	$4,216,818.75	$8,217,900
5,821,684	65,128,412	2,890,569.08	6,989,289

65–72. Subtract 268 from each of the numbers: 319, 454, 660, 400, 521, 899, 470, 1001.

73–81. Subtract 4675 from each of the numbers: 5555, 6271, 7001, 10304, 12064, 140021, 5000, 8372, 8008.

82–90. Subtract from 7214 each of the numbers: 486, 1275, 6572, 4009, 5021, 4307, 5000, 6215, 3456.

What is the value—

91. Of 10324 + 3756 -- 12186?

92. Of 12621 − 8367 + 9162?

93. Of 14546 − 9787 + 1486?

94. Of 10101 − 9012 + 1001?

95. Of 121461 − 90687 + 67217?

96. Of $127.60 − $42.25 + $11.16 − $60.25?

PRACTICAL PROBLEMS.

1. A merchant deposited $3251 in a bank, at another time $6005, then drew out $4786; how much remained in bank?

2. What number added to 3262 will equal 6122?

3. How much more does 18263 differ from 31126 than from 9212?

4. At an election Mr. Smith received 45216 votes, and his opponent received 23580 votes; what was Mr. Smith's majority?

5. A firm's income is $20160, and its expenses are $12328; what is the gain?

6. A Western cattle-dealer had 9382 cattle, and bought 3621 more, 132 died, and he sold 8416; how many remained?

7. A contractor received $50000 for building a library, and he expended upon it $42,382.75; what was his profit?

8. A house was bought for $3325, $125 were paid for repairs; it was afterward sold for $3750; what was the gain?

9. A teacher's salary is $1500 a year: he pays $375.25 for board, $85.60 for clothing, and $215.15 for other expenses; how much does he save?

10. A farmer receives $415 for his wheat, $216 for his corn, and $167 for his potatoes: he paid $221 rent, $75 for labor, and $163 for other expenses; how much did he save?

11. A paid $8000 for his farm, $7365 for a new house, and $762 for a barn, and then sold them for $18500; what was his gain?

12. The cost of tuition of the public schools of Pennsylvania for 1892 was $7,766,657.16, and the cost of fuel for the same year was $3,301,159.51; what was the difference between the cost of tuition and the cost of fuel?

13. Philadelphia paid for teachers' salaries during the year 1892, $1,738,637.25, for school buildings $574,845.31, and for fuel, etc. $351,126.05; how much more did the city pay for teachers' salaries than for buildings and fuel?

14. The population of New York in 1880 was 5,082,871, and of Pennsylvania 4,292,891. In 1890 the population of New York was 5,997,853, and of Pennsylvania 5,258,014; how much more did Pennsylvania gain in ten years than New York?

15. The receipts and expenditures of the United States government for the last six months of 1893 were as follows:

	Receipts.	Expenditures.
July,	$30,905,776.19	$39,675,888.60
Aug.,	23,890,885.30	33,305,228.48
Sept.,	24,582,756.10	25,478,010.17
Oct.,	24,553,394.97	29,588,892.34
Nov.,	23,079,400.81	31,302,026.41
Dec.,	22,312,027.00	30,058,260.51

What was the deficit for six months?

MULTIPLICATION.

ORAL EXERCISES.

21.—*1.* What will 6 books cost at 5 cents apiece? At 9 cts.? At 7 cts.? At 6 cts.? At 8 cts.?

2. What will 7 pounds of sugar cost at 5 cents a pound? At 6 cts.? At 8 cts.? At 9 cts.? At 7 cts.? At 10 cts.?

3. What will 8 sheep cost at $7 apiece? At $9? At $8? At $10? At $11? At $12?

4. There are 8 quarts in 1 peck; how many quarts in 7 pecks? 9 pecks? 11 pecks? 12 pecks? 8 pecks? 10 pecks?

5. There are 9 square feet in 1 square yard; how many square feet in 5 square yards? 6 sq. yd.? 8 sq. yd.? 7 sq. yd.? 10 sq. yd.? 12 sq. yd.? 11 sq. yd.?

6. How many are 6 times 5? 8? 9? 10? 7? 6? 11? 12? 4? 50? 80? 100? 70? 60? 110? 120? 40? 90? 30?

7. How many are 7 times 4? 5? 3? 12? 11? 10? 9? 8? 7? 6? 20? 120? 110? 100? 90? 80? 70? 60? 200?

8. How many are 8 times 6? 7? 9? 10? 11? 12? 8? 60? 70? 90? 100? 110? 120? 80? 600? 700?

9. How many are 9 times 6? 7? 9? 10? 11? 12? 8? 60? 70? 90? 100? 110? 120? 80? 600? 700?

10. Multiply 6000, 5000, 8000, 9000, 7000, 11000, 12000, by 9; 8; 5; 6; 7.

ADDITION.

121
121
121
363

The sum of 121 taken three times is the same as 3 times 121.

MULTIPLICATION.

121, Multiplicand.
3, Multiplier.
363, Product.

DEFINITIONS.

22. Multiplication is the process of taking one number as many times as there are units in another.

The **Multiplicand** is the number to be multiplied or taken.

The **Multiplier** is the number by which we multiply.

The **Product** is the result obtained by multiplying.

NOTE.—The multiplicand and multiplier are factors of the product.

23. The **Sign of Multiplication** is ×, and is read *multiplied by,* or *times.* When placed between two numbers it indicates that one is to be multiplied by the other.

MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

1. What is the product of 8 × 9?

To find by the above table the product of any two numbers from 1 to 12 inclusive, look in the first or left-hand vertical line for either of the numbers, then follow out its horizontal line until the column is reached which is headed by the other number; the product of the two numbers will be found at the intersection of the two lines. In this way we find that the product of 8 × 9 is 72.

2. Find in the same manner the product of 8 × 7. 7 × 8. 9 × 12. 12 × 9. 6 × 7. 12 × 12.

WRITTEN EXERCISES.

CASE I.

24. When the multiplier is one figure.

1. Multiply 396 by 7.

PROCESS.
Multiplicand, 396
Multiplier, 7
Product, 2772, *Ans.*

We write the multiplier under the units figure of the multiplicand, and begin at the right to multiply.

7 times 6 units are 42 units, or 4 tens and 2 units. We write the 2 units under the units column and add the 4 tens to the next product.

7 times 9 tens are 63 tens, plus the 4 tens are 67 tens, or 6 hundreds and 7 tens. We write the 7 tens under the tens column and add the 6 hundreds to the next product.

7 times 3 hundreds are 21 hundreds, plus the 6 hundreds are 27 hundreds, or 2 thousands and 7 hundreds, which we write in their proper places.

2. Multiply 956 by 8.

PROCESS.
956
8
7648, *Ans.*

In practice we say, 8 times 6 are 48; write 8 and carry 4. 8 times 5 are 40, and 4 are 44; write 4 and carry 4. 8 times 9 are 72, and 4 are 76; write 76.

3.	*4.*	*5.*	*6.*	*7.*	*8.*
23	45	61	123	1234	3216
3	4	5	5	4	3

9.	*10.*	*11.*	*12.*	*13.*	*14.*
$4.23	$5.32	$6.14	$5.32	$1.62	$51.46
6	7	8	9	6	5

15–122. Multiply each of the following numbers by 4. By 5. By 6. By 7. By 8. By 9:

2136	8216	4316	6172	6050	4216
3142	5432	8709	5006	7089	7863
6142	1618	5492	3120	5142	4567

Multiply—

123. 246 by 5.
124. 589 by 5.
125. 456 by 6.
126. 789 by 6.
127. 562 by 7.
128. 678 by 7.
129. 357 by 8.
130. 368 by 9.
131. 2406 by 5.
132. 3045 by 6.
133. 4210 by 7.
134. 1216 by 8.
135. 24321 by 7.
136. 216312 by 8.
137. 3162172 by 9.
138. 40027621 by 5.
139. 123456789 by 9.
140. 987654321 by 9.

PRACTICAL PROBLEMS.

1. At $165.10 an acre, what will 9 acres of land cost?

2. If sound moves 1120 feet in a second, how far will it move in 9 seconds?

3. Light moves about 186,330 miles per second; how far will it move in 8 seconds?

4. There are 5280 feet in a mile; how many feet in 7 miles?

5. A farmer bought 6 cows at an average price of $21.75 a head; what did he pay for them?

6. Mr. Bitner sold 9 acres of land at $315.25 an acre; what did he receive for the land?

7. A firm sells on an average $3216.80 worth of goods in a month for 8 months; how much is sold in 8 months?

8. The distance to the moon is 240,000 miles; what is 7 times this distance?

9. The diameter of the earth is about 7925 miles; what is 8 times this distance?

10. In 1 mile there are 1760 yards; how many yards are there in 9 miles?

11. What will 9 yards of broadcloth cost at the rate of $4.75 per yard?

CASE II.

25. When the multiplier consists of two or more figures.

1. Multiply 856 by 48.

Process.
856
48
6848
3424
41088, *Ans.*

We write the multiplier under the multiplicand, with units under units, tens under tens, etc.

We must multiply 856 by 8 units and 4 tens. 8 times 856 equals 6848, and 4 tens times 856 equals 3424 tens. Adding these two partial products, we have 41088.

2. Multiply 428 by 234.

Process.
428
234
1712
1284
856
100152, *Ans.*

The pupil should observe that the first figure of each partial product is placed in the column in which the multiplier is found which produced it.

234
428
1872
468
936
100152

Proof.—Multiply the multiplier by the multiplicand; if the results agree, the work is probably correct.

3.	*4.*	*5.*	*6.*
216	372	736	546
14	15	22	25

7.	*8.*	*9.*	*10.*
$12.25	$16.75	$20.22	$31.82
34	42	51	63

11.	*12.*	*13.*	*14.*
332	562	375	782
32	42	61	52

Multiply—

15. 271 by 18.
16. 273 by 24.
17. 304 by 42.
18. 423 by 26.
19. 561 by 62.
20. 624 by 34.
21. 732 by 43.
22. 816 by 35.
23. 902 by 63.
24. 482 by 81.
25. 567 by 72.
26. 518 by 85.
27. 666 by 75.
28. 809 by 49.
29. 632 by 61.
30. 729 by 92.

31–174. Multiply each of the following numbers by 16. By 56. By 84. By 92. By 123. By 234:

216	418	618	432	421	639	234	489
302	316	908	567	482	415	705	622
416	524	862	893	921	789	981	930

175.	*176.*	*177.*
3467	3467	3467
74	704	7004
13868	13868	13868
24269	24269	24269
256558	2440768	24282868

178. Multiply 5984 by 72.
179. Multiply 6375 by 45.
180. Multiply 1234 by 54.
181. Multiply 4133 by 111.
182. Multiply 6234 by 202.
183. Multiply 2416 by 209.
184. Multiply 8206 by 312.
185. Multiply 1834 by 413.
186. Multiply 2004 by 562.
187. Multiply 1346 by 345.
188. Multiply 3167 by 621.
189. Multiply 1321 by 3121.
190. Multiply 3021 by 2132.
191. Multiply 6331 by 4215.
192. Multiply 5023 by 3215.
193. Multiply 1623 by 4021.
194. Multiply 3024 by 3004.
195. Multiply 3546 by 4203.
196. Multiply 9218 by 8219.
197. Multiply 8273 by 6571.
198. Multiply 6204 by 3125.
199. Multiply 71232 by 4182.
200. Multiply 67893 by 4321.
201. Multiply 30605 by 4216.

PRACTICAL PROBLEMS.

1. What will a farm of 102 acres cost at $162 an acre?

2. What will 24 horses cost at an average price of $180 a head?

3. What will 225 shares of railroad stock cost at $127 a share?

4. What will 415 bushels of timothy-seed cost at $4.05 a bushel?

5. If a barrel of coal oil contains 53 gallons, how many gallons in 518 barrels?

6. An army eats on an average 3216 pounds of bread in one day; how much will it require for 365 days?

7. What will 325 barrels of sugar cost, each containing 205 pounds, at 6 cents a pound?

8. A merchant bought 327 boxes of oranges at $5.25 a box; how much did they cost?

9. An army is composed of 325 regiments, each regiment containing 1087 men; how many men in the army?

10. A imported 428 bags of coffee, each bag containing 176 pounds, at 28 cents a pound; what did it cost?

11. A father earns $135.71 a month, and his son earns $76.10 a month; how much will they both earn in 52 months at the same rates?

12. A freight train has 35 cars, each car weighs 20168 pounds, and its freight 56820 pounds; what is the weight of the entire train?

13. There are 24 hours in one day, and 365 days in one year; how many hours in 18 years?

14. A and B start from the same place and travel in opposite directions, A at the rate of 57 miles a day, and B at the rate of 61 miles a day; how far apart will they be in 115 days?

15. When wheat is selling at $1.10 a bushel and corn at $.47 a bushel, what is the difference in the cost of 5245 bushels of each?

16. A grocer bought 4239 pounds of butter in one month at an average price of 32 cents a pound, and during the same time he bought 5826 dozen of eggs at an average price of 24 cents a dozen; what did he pay for both?

17. The receipts of the United States government for December, 1893, were $22,312,027.00, and the expenses for the same month were $30,058,260.51. At this rate, what would be the deficit for the 12 months of 1894?

18. If the daily circulation of a paper is 216,344, what would be the circulation for 313 days?

19. The United States contains about 2,999,154 square miles, and in 1890 there were about 21 inhabitants to each square mile; what was the entire population?

20. In 1893 there were 25963 teachers in the public schools of Pennsylvania, with an average salary of $326.13 a year; how much did they all receive?

21. Great Britain and Ireland have an area of 118949 square miles, and in 1890 they contained a population of 319 to the square mile; what was their entire population?

22. A railroad is 847 miles long and cost $52,168 per mile; what was the total cost?

23. In 1890 the national debt of the United States amounted to $15 for each individual, and the population was 62,982,234; what was the entire debt?

24–29. Multiply 34567 by 23. By 45. By 127. By 202. By 3002. By 4023.

30–35. Multiply 30206 by 86. By 107. By 309. By 4002. By 504. By 1007.

36–40. Multiply 123456789 by 9. By 18. By 27. By 36. By 45.

41–45. Multiply 987654321 by 9. By 18. By 27. By 36. By 45.

46–50. Multiply 10090009 by 9. By 999. By 9009. By 101. By 306.

CASE III.

26. When there are ciphers at the right of either or both factors.

1. Multiply 2, 3, 4, 5, 6 by 10.

2. How may these products be obtained without multiplying?

3. Multiply 6, 7, 8, 9, 12 by 100.

4. How may these products be obtained without multiplying?

5. Multiply 146 by 2400; also, 460 by 3200.

PROCESS.

```
  146            460
   2400          3200
  584             92
 292            138
 350400         1472000
```

24 times 146 equals 3504, and 2400 times 146 is 100 times as great, or 350400.

32 times 46 equals 1472, and 32 times 460 is 10 times as much, or 14720; 3200 times 460 is 100 times 14720, or 1472000.

6. Multiply 430 by 300.

7. Multiply 260 by 3400.

8. Multiply 5020 by 3200.

9. Multiply 600700 by 6000.

10. Multiply 200800 by 3004.

PRACTICAL PROBLEMS.

1. There are 5280 feet in a mile; how many feet in 200 miles?

2. There are 7000 Troy grains in 1 pound avoirdupois; how many grains in 120 pounds?

3. There are 4840 square yards in 1 acre; how many square yards in 640 acres?

4. How many pounds in 460 tons, if there are 2000 pounds in a ton?

5. At $250 an acre, what will 120 acres of land cost?

6. How many feet around the earth, if it is 25000 miles and each mile contains 5280 feet?

7. How many square rods in 150 acres, if there are 160 square rods in 1 acre?

MISCELLANEOUS PROBLEMS.

1. A drover bought 42 cows at $21 each, 63 sheep at $12 each, and 15 horses at $150 each, and sold them all for $4000; how much did he gain?

2. A merchant sold 324 barrels of apples at $4.75 a barrel, and gained $162 on the transaction; what did the apples cost him?

3. In 1894 there were 88 Senators in the United States Senate, each receiving $5000 a year, and 356 members of the House of Representatives, each receiving the same salary as a Senator; what sum of money will pay the salaries of both houses?

4. There are 60 seconds in 1 minute, 60 minutes in 1 hour, 24 hours in 1 day; how many seconds in a day?

5. A train of 35 cars is loaded with apples, each car containing 128 barrels, and each barrel containing 3 bushels; how many bushels of apples in the entire cargo?

6. A train of 42 cars is loaded with flour, each car containing 105 barrels, and each barrel weighing 196 pounds; what is the entire weight of the cargo?

7. A contracted to build 25 miles of railroad for 2 millions of dollars: the road cost him $44,500 per mile; how much did he make by the contract?

8. The multiplicand is 526 and the multiplier 73; how much more do I increase the product by adding 62 to the multiplier than I would by adding 62 to the multiplicand?

DIVISION.

ORAL EXERCISES.

27.—*1.* At 5 cents a quart, how many quarts of milk can be bought for 25 cts.? 30 cts.? 35 cts.? 40 cts.? 45 cts.? 50 cts.? 55 cts.? 60 cts.?

2. At $6 a ton, how many tons of coal can be bought for $30? $48? $60? $72? $54? $66? $42? $36?

3. At $7 each, how many sheep can be bought for $49? $63? $42? $84? $77? $56?

4. At 8 cents a yard, how many yards of calico can be bought for 40 cents? 56 cts.? 64 cts.? 48 cts.? 72 cts.? 96 cts.? 88 cts.?

5. When berries are selling at 9 cents a quart, how many quarts can you buy for 27 cents? 45 cts.? 54 cts.? 63 cts.? 108 cts.? 99 cts.? 81 cts.? 72 cts.?

How many—

6. 5's in 25? 45? 60? 55? 30? 40? 35? 50?

NOTE.—Give the results only; thus, 5, 9, 12, etc.

7. 6's in 24? 48? 60? 54? 36? 72? 66? 42?
8. 7's in 42? 56? 84? 77? 70? 35? 63? 49?
9. 8's in 56? 40? 32? 96? 80? 88? 64? 72?
10. 9's in 27? 72? 63? 90? 81? 99? 108? 54?
11. 10's in 60? 40? 50? 70? 80? 90? 100? 120?

How many—

12. 4's in—	*13.* 5's in—	*14.* 6's in—	*15.* 7's in—	*16.* 8's in—
160?	250?	240?	420?	320?
240?	350?	360?	490?	560?
320?	300?	420?	560?	630?
3600?	4500?	4800?	6300?	8800?
2800?	5500?	6600?	8400?	9600?
4400?	6000?	7200?	7700?	7200?

If an apple is divided into *two equal* parts, one of these parts is called *one-half;* if into *three equal* parts, *one-third;* and so on.

$\frac{1}{2}$ expresses one-half, or one of the two equal parts of anything.

$\frac{1}{3}$ expresses one-third, or one of the three equal parts of anything.

$\frac{2}{3}$ expresses two-thirds, or two of the three equal parts of anything.

17. Read $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$, $\frac{1}{10}$, $\frac{1}{11}$.

18. Read $\frac{2}{3}$, $\frac{2}{4}$, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{4}{5}$, $\frac{3}{5}$, $\frac{7}{8}$, $\frac{2}{6}$, $\frac{3}{6}$, $\frac{4}{6}$.

19. What is $\frac{1}{2}$ of 4? 8? 12? 16? 18? 24? 30? 36?

20. What is $\frac{1}{3}$ of 9? 24? 18? 21? 27? 30? 36? 33?

21. What is $\frac{1}{4}$ of 8? 16? 20? 24? 32? 72? 60? 48?

22. What is $\frac{2}{3}$ of 6? 9? 15? 21? 30? 36? 60? 72?

DEFINITIONS.

28. **Division** is the process of finding how many times one number is contained in another.

The **Dividend** is the number to be divided.

The **Divisor** is the number by which we divide.

The **Quotient** is the number of times the divisor is contained in the dividend.

The **Remainder** is a part of the dividend which is sometimes left after dividing.

29. The **Sign of Division** is ÷, and is read *divided by.* When placed between two numbers it indicates that the number before it is to be divided by the number after it. Thus, $24 \div 4 = 6$, is read 24 *divided by* 4 *equals* 6.

Division is also expressed by writing the dividend above and the divisor below a short horizontal line. Thus, $24 \div 4 = 6$, may be written $\frac{24}{4} = 6$.

WRITTEN EXERCISES.

CASE I.

30. When the divisor is one figure.

1. Divide 765 by 5.

PROCESS.

Divisor, 5)765, Dividend.
Quotient, 153, *Ans.*

We write the divisor at the left of the dividend, with a) between them.

5 is contained in 7 hundreds 1 hundred times, with a remainder of 2 hundreds, or 20 tens. We write the 1 under the hundreds figure.

20 tens and 6 tens are 26 tens; 5 is contained in 26 tens 5 tens times, with a remainder of 1 ten, or 10 units. We write the 5 under the tens figure.

10 units and 5 units are 15 units; 5 is contained in 15 units 3 units times. We write the 3 under the units figure.

The required result is 153.

This process is called **Short Division.**

2. Divide 4573 by 7.

PROCESS.

Divisor, 7)4573, Dividend.
Quotient, 653—2 Rem.

In practice we say, 7 into 45 six times, with 3 to carry; 7 into 37 five times, with 2 to carry; 7 into 21 three times, and 2 remaining.

Quotient, 653
Divisor, 7
4571
2, Rem.
Dividend, 4573

PROOF.—Multiply the quotient by the divisor, and add the remainder to the product; if the result is equal to the dividend, the division is correct.

3.	*4.*	*5.*	*6.*	*7.*
2)576	3)681	4)1284	5)1875	5)1722

8.	*9.*	*10.*	*11.*	*12.*
7)$2268	8)$3424	9)$6831	6)$42.66	7)$140.28

Divide—

13. 4756 by 4.
14. 8295 by 5.
15. 4026 by 6.
16. 9248 by 8.
17. 3024 by 7.
18. 4104 by 9.
19. 1872 by 9.
20. 4321 by 8.
21. 8621 by 9.
22. 3075 by 7.
23. 4060 by 8.
24. 9876 by 6.
25. 4238 by 5.
26. 8970 by 9.

27. 486 ÷ 3 =
28. 572 ÷ 4 =
29. 671 ÷ 5 =
30. 846 ÷ 6 =
31. 786 ÷ 7 =
32. 980 ÷ 8 =
33. 1286 ÷ 9 =
34. 3206 ÷ 8 =
35. 4620 ÷ 7 =
36. 3621 ÷ 6 =
37. 4005 ÷ 5 =
38. 8908 ÷ 4 =

39. 60256 ÷ 3 =
40. 82167 ÷ 4 =
41. 80086 ÷ 5 =
42. 120626 ÷ 6 =
43. 420861 ÷ 7 =
44. 999999 ÷ 8 =
45. 865242 ÷ 7 =
46. 728156 ÷ 9 =
47. 964800 ÷ 6 =
48. 34566 ÷ 6 =
49. 85645 ÷ 7 =
50. 65512 ÷ 8 =
51. 92736 ÷ 9 =
52. 402030 ÷ 9 =
53. 1621522 ÷ 7 =
54. 8654751 ÷ 9 =
55. 9007648 ÷ 8 =
56. 4215672 ÷ 8 =

PRACTICAL PROBLEMS.

1. There are 3 feet in a yard; how many yards in 627 feet?

2. There are 7 days in a week; how many weeks in 365 days?

3. There are 8 quarts in a peck; how many pecks in 9680 quarts?

4. At $6 a ton how many tons of coal can be bought for $1872?

5. A man earns $8 a week; how long must he labor to earn $2048?

6. How many sheep at $7 a head can be bought for $1372?

7. A father divided $19,875 equally among his 6 sons; how much did each receive?

8. If sound travels 10,080 feet in 9 seconds, how far will it travel in 1 second?

9. If light travels 1,490,640 miles in 8 seconds, how far will it travel in 1 second?

10. There are 12 months in 1 year; how many years in 4804 months?

CASE II.

31. When the divisor is two or more figures.

1. Divide 26649 by 63.

PROCESS.

```
   |423, Ans.
63)26649
   252
   ---
    144
    126
    ---
     189
     189
     ---
```

As 63 is not contained in 2 nor in 26, we take three figures, 266, for the first partial dividend. 63 is contained in 266 four times, with a remainder; we write the 4 over the last figure of 266 as the first figure of the quotient; multiplying 63 by 4, the product is 252, which we write under 266; subtracting 252 from 266, the remainder is 14; bringing down the next dividend figure, 4, we have 144, the next partial dividend. 63 is contained in 144 two times, with a remainder; 2 times 63 is 126, which subtracted from 144 leaves 18; bringing down 9, the last dividend figure, we have 189, the last partial dividend.

63 is contained in 189 three times; 3 times 63 is 189; subtracting, there is no remainder.

Hence the required result is 423.

This process is called **Long Division.**

Divide—

2. 451 by 11.
3. 528 by 12.
4. 624 by 13.
5. 728 by 14.
6. 855 by 15.
7. 976 by 16.

8. 884 by 17.
9. 702 by 18.
10. 798 by 19.
11. 714 by 21.
12. 770 by 22.
13. 828 by 23.

14. 984 by 24.
15. 675 by 25.
16. 988 by 26.
17. 864 by 27.
18. 868 by 28.
19. 957 by 29.

20. 1612 ÷ 31 =
21. 1536 ÷ 32 =
22. 1485 ÷ 33 =
23. 1768 ÷ 34 =
24. 1225 ÷ 35 =
25. 2196 ÷ 36 =
26. 2664 ÷ 37 =
27. 1504 ÷ 47 =
28. 5289 ÷ 43 =
29. 9464 ÷ 52 =
30. 5612 ÷ 61 =
31. 6336 ÷ 72 =
32. 4557 ÷ 21 =
33. 3264 ÷ 24 =

34–35. Divide 50061 by 123; also 730458 by 243.

PROCESS.

$$\begin{array}{r} \underline{|407} \\ 123)50061 \\ \underline{492} \\ 861 \\ \underline{861} \end{array}$$

PROCESS.

$$\begin{array}{r} \underline{|3006} \\ 243)730458 \\ \underline{729} \\ 1458 \\ \underline{1458} \end{array}$$

NOTES.—1. If a remainder with the next figure of the dividend annexed does not contain the divisor, place a cipher in the quotient and annex another figure of the dividend.

2. If the product is greater than the partial dividend, the figure in the quotient is too large.

3. If the remainder is greater than the divisor, the figure in the quotient is too small.

4. To find the figure of the quotient approximately, divide the left-hand figure of the dividend by the left-hand figure of the divisor.

Divide—

36. 6615 by 63.
37. 5304 by 26.
38. 8456 by 28.
39. 8910 by 22.
40. 6630 by 78.
41. 3690 by 82.
42. 7812 by 21.
43. 9591 by 23.
44. 7946 by 29.
45. 4991 by 23.
46. 5612 by 92.
47. 4824 by 72.
48. 6076 by 98.
49. 13392 by 62.
50. 28569 by 89.
51. 11340 by 36.
52. 85699 by 43.
53. 16536 by 53.
54. 29786 by 53.
55. 19581 by 61.

56. 403326 ÷ 126 =
57. 562520 ÷ 245 =
58. 798657 ÷ 246 =
59. 866775 ÷ 195 =
60. 620365 ÷ 265 =
61. 982260 ÷ 321 =
62. 236250 ÷ 375 =

63. 356534 by 89 =
64. 472012 by 312 =
65. 647680 by 506 =
66. 232696 by 512 =
67. 242808 by 604 =
68. 531288 by 628 =
69. 632186 by 321 =

70. Divide 522776 by 843.
71. Divide 1056566 by 326.
72. Divide 3113815 by 365.
73. Divide 7845682 by 8462.
74. Divide 8426327 by 3784.
75. Divide 10365051 by 3021.
76. Divide 17590800 by 3425.
77. Divide $83146235 by 4216.
78. Divide $9654321.68 by 11521.
79. Divide $80745.28 by 6328.

PRACTICAL PROBLEMS.

1. How many barrels, each containing 196 pounds, can be filled from 9408 pounds of flour?

2. If a barrel of oil contains 51 gallons, how many barrels of the same size can be filled from 6273 gallons?

3. A farm of 84 acres was sold for $17976; what was the average price per acre?

4. There are 231 cubic inches in a gallon; how many gallons will a tank hold that contains 139755 cubic inches?

5. There are 32 quarts in 1 bushel; how many bushels in 18432 quarts?

6. There are 144 square inches in 1 square foot; how many square feet in 54432 square inches?

7. At an average price of $82 an acre, how many acres of land can be bought for $25,912?

8. The driving-wheel of a locomotive is 15 feet in circumference; how many revolutions will it make in going from Philadelphia to New York, a distance of 90 miles, there being 5280 feet in a mile?

9. The product of three numbers is 61318: one of the numbers is 23, and another 43; what is the third?

10. The salary of the President of the United States is $50,000 a year; how much does he receive per day?

11. There are 1728 cubic inches in a cubic foot; how many cubic feet in 404,352 cubic inches?

12. How many horses at $125 each can be exchanged for 375 cows at an average price of $22 a head?

13. 123 hogsheads contain 7749 gallons; how many gallons in a hogshead?

14. The distance to the moon is about 240,000 miles; how long would it take to travel that distance at the rate of 75 miles an hour?

CASE III.

32. When there are ciphers at the right of the divisor.

1. Divide 428 by 10.

1st Process.	2d Process.
|42, Quotient. 10)428 40 28 20 8, Rem.	1|0)42|8 Quotient, 42—8 Rem.

By comparing these processes it is seen that *we can divide by 10 by cutting off one figure from the right of the dividend; the figure cut off will be the remainder, and the part remaining will be the quotient.*

Similarly we can divide by 100, 1000, etc.

2–37. Divide each of the following numbers by 10. By 100. By 1000:

5216 2167 5062 4216 4216 4321
8621 8162 7540 8156 5432 2000

38. Divide 8547 by 3000.

PROCESS.

|2, Quotient.
3000)8547
6000
2547, Rem.

3|000)8|547
Quotient, 2—2547, Rem.

It will be observed that the result here is the same as if 8 were divided by 3 and the remainder prefixed to the figures cut off from the dividend.

39. 562 ÷ 20 =
40. 846 ÷ 30 =
41. 575 ÷ 40 =
42. 680 ÷ 50 =
43. 789 ÷ 60 =
44. 862 ÷ 70 =
45. 963 ÷ 80 =
46. 985 ÷ 90 =
47. 4625 ÷ 200 =
48. 6200 ÷ 300 =
49. 9200 ÷ 400 =
50. 8730 ÷ 500 =
51. 762 ÷ 600 =
52. 941 ÷ 700 =
53. 984 ÷ 800 =
54. 962 ÷ 900 =
55. 4216 ÷ 2000 =
56. 8645 ÷ 3000 =
57. 9321 ÷ 4000 =
58. 9026 ÷ 5000 =
59. 8285 ÷ 6000 =
60. 9005 ÷ 7000 =
61. 9260 ÷ 8000 =
62. 9462 ÷ 9000 =

63. Divide 4547 by 2100.

PROCESS.

|2, Quotient.
2100)4547
4200
347, Rem.

|2, Quotient.
21|00)45|47
42
347, Rem.

It will be observed that the result here is the same as if 45 were divided by 21 and the remainder prefixed to the figures cut off from the dividend.

64. 86547 ÷ 2200 =
65. 48514 ÷ 3500 =
66. 67584 ÷ 4200 =
67. 78654 ÷ 3300 =
68. 57432 ÷ 5300 =
69. 843000 ÷ 240 =
70. 400785 ÷ 9100 =
71. 374321 ÷ 12300 =
72. 295848 ÷ 20400 =
73. 830927 ÷ 13400 =
74. 657337 ÷ 21200 =
75. 433326 ÷ 27000 =

PRACTICAL PROBLEMS.

1. There are 10 cents in a dime; how many dimes in 8765 cents?

2. There are 80 chains in a mile; how many miles are there in 4208 chains?

3. There are 160 perches in an acre; how many acres in 4212 perches?

4. If a box will hold 200 oranges, how many boxes will be required to ship 86400 oranges?

5. There are 2000 pounds in a ton; how many tons in 863125 pounds?

6. There are 7000 Troy grains in a pound avoirdupois; how many pounds in 118125 grains?

7. There are 5760 grains in a pound Troy; how many pounds in 691200 grains?

8. How many miles in 432960 feet, if there are 5280 feet in 1 mile?

9. There are 3600 seconds in an hour; how many hours in 12950 seconds?

USE OF THE PARENTHESIS.

33. The **Parenthesis** ( ) is used to show that the numbers included within it are subjected to the same operation.

Thus, (9 − 2 + 4) × 6 indicates that 9 − 2 + 4, or 11, is to be multiplied by 6.

The Vinculum (———) is sometimes used instead of the parenthesis or in connection with it. Thus, $\overline{9-2+4}\times 6$ may be used instead of $(9-2+4)\times 6$.

WRITTEN EXERCISES.

1. What is the value of $(18+2-4)\times 7$?

18 + 2 − 4 equals 16, and 16 multiplied by 7 equals 112, *Ans.*

Find the value—

2. Of $(28+17-20)\times 9$.
3. Of $(55-21+81)\times 8$.
4. Of $(75-32+83)\div 3$.
5. Of $(236+810-42)\times 15$.
6. Of $(325-218+610)\times 52$.
7. Of $(482-192)-(84-16)+9$.
8. Of $(650-210)\div(720-680)$.
9. Of $(420-\overline{270-30})\div(426-420)$.
10. Of $(628-\overline{475-325})+(982-\overline{620-82})$.

MISCELLANEOUS PROBLEMS

On the Four Fundamental Processes.

1. The minuend is 900000 and the subtrahend is 212345; what is the difference?

2. The minuend is 300300 and the difference is 186214; what is the subtrahend?

3. The subtrahend is 102043 and the difference is 8624; what is the minuend?

4. The product of two numbers is 456104, and one of the numbers is 3124; what is the other?

5. The multiplier is 2006 and the multiplicand is 2060; what is the product?

6. The product is 13392 and the multiplier is 62; what is the multiplicand?

7. The product is 19581 and the multiplicand is 321; what is the multiplier?

8. The divisor is 326 and the quotient is 3241; what is the dividend?

9. The divisor is 843, the quotient is 620, and the remainder 116; what is the dividend?

10. What number divided by 462 will give a quotient of 104 and a remainder of 78?

11. Of what number is 512 both the divisor and the quotient?

12. What number is that from which if 80806 be taken the remainder will be twice as much as it is when 100008 is subtracted from it?

13. A's house cost $7825, which was $4218 less than the cost of the farm; what was the cost of both?

14. B owns three houses: the first cost $3212, the second twice as much as the first, and the third as much as the first and second together; what did they all cost?

15. A drover has $1050; how many cows can he buy at $30 each? $25 each? $35 each?

16. There were 24,012 public schools in Pennsylvania in 1893, with 994,407 pupils; what was the average number of pupils in each school?

17. If 12 horses and 13 cows cost $2082, and the average price of a horse is $128, what is the average price of a cow?

18. A boy traveled 215 miles in a week on his bicycle; three times as far, lacking 117 miles, the next week; how far did he travel in both weeks?

19. A miller bought 475 barrels of flour for $2850, and sold 228 barrels for $1710 and the remainder for cost; what did he gain?

20. A miller bought 240 barrels of flour for $1680; if he sells 80 barrels for cost, at what price per barrel must he sell the remainder to gain $320?

ARITHMETICAL ANALYSIS.

34. In the solution of problems we frequently compare the parts of a problem through their relation to 1. This process is called **Arithmetical Analysis.**

WRITTEN EXERCISES.

1. If 8 apples cost 48 cents, what will 5 apples cost?

PROCESS.

8 apples = 48 cts.
1 apple = 6 "
5 apples = 30 "

If 8 apples cost 48 cents, 1 apple will cost $\frac{1}{8}$ of 48 cents, or 6 cents, and 5 apples will cost 5 times 6 cents, or 30 cents.

2. If 15 plows cost $180, what will 11 plows cost?

3. If 23 wagons cost $575, what will 40 wagons cost?

4. What will 85 bushels of corn cost, if 34 bushels cost $24.48?

5. If 25 men can do a piece of work in 42 days, how long will it take 15 men to do it?

6. If 8 men can do a piece of work in 145 days, how long will it take 29 men to do the same work?

7. If 16 men earn $560 a week, how much will 57 men earn in the same time and at the same rate?

8. If 312 acres of land cost $26208, what will 208 acres cost?

9. What will 125 horses cost, at the rate of 23 horses for $3450?

10. A quantity of provisions lasts an army of 2500 men 72 days; how long would it last 18000 men?

11. A paid $27144 for a farm, at the rate of 15 acres for $3510; how many acres did he buy?

12. If 8 acres of land cost $656, what will 35 acres cost at $4 more an acre?

13. If 16 horses cost $1952, what will 22 horses cost at $6 less a head?

SECTION III.

FACTORS, DIVISORS, AND MULTIPLES.

FACTORING.

ORAL EXERCISES.

1. WHAT two numbers will exactly divide 6? 8? 14? 15? 21? 35? 45?

35. A number that exactly divides another number is a **Factor** of that number.

One (1) and the number itself are not regarded as factors.

2. What are the factors of 12? 16? 20? 30? 32? 40? 49? 50? 63? 72?

36. A number that has factors is a **Composite Number.**

3. Name the composite numbers between 0 and 20. Between 20 and 40. Between 40 and 60. Between 60 and 80. Between 80 and 100.

4. Name all the numbers from 0 to 20 that have no factors.

37. A number that has no factors is a **Prime Number.**

5. Name the prime numbers from 20 to 50. From 50 to 100.

6. What numbers between 1 and 21 are exactly divisible by 2? Which are not?

38. A number that is exactly divisible by 2 is an **Even Number.** A number that is not exactly divisible by 2 is an **Odd Number.**

7. What prime numbers will exactly divide 35? 55? 33? 39? 49? 121? 72? 81? 100? 56? 77?

39. The prime numbers which exactly divide another number are its **Prime Factors.**

8. What are the prime factors of 24? 28? 36? 42? 45? 52? 54? 64? 70? 72? 80? 81? 85? 87? 91? 93? 94? 96? 98? 99? 100? 102?

40. Factoring is the process of finding the factors of a number.

What numbers between 30 and 100—

9. Have 2 for a factor? What is the units figure in each of these numbers?

10. Have 3 for a factor? Divide the sum of the digits of each of these numbers by 3 and give the remainder.

11. Have 5 for a factor? What is the units figure in each of these numbers?

12. Have 9 for a factor? Divide the sum of the digits of each of these numbers by 9 and give the remainder.

DIVISIBILITY OF NUMBERS.

41. Since an exact divisor and a factor are the same, the following principles will be found of service in factoring numbers:

A number is exactly divisible—

By 2 *if its right-hand figure is* 2, 4, 6, 8, or 0.
By 3 *if the sum of the digits is divisible by* 3.
By 4 *if the number formed by its last two digits is divisible by* 4.
By 5 *if its right-hand figure is* 5 *or* 0.
By 6 *if it is divisible by* 2 *and* 3.
By 8 *if the number formed by its last three digits is divisible by* 8.
By 9 *if the sum of its digits is divisible by* 9.
By 10 *if the right-hand figure is* 0.

13–21. Tell by inspection which of the following numbers are exactly divisible by 2. By 3. By 4. By 5. By 6. By 7. By 8. By 9. By 10:

134 1062 1424 18172 271272
260 1086 1632 62388 411480
412 1296 1881 10101 720864
564 1359 2345 28960 428424

22. Give five numbers above 100, all of which are exactly divisible by both 3 and 9.

23. Give ten numbers above 100, all of which are exactly divisible by both 5 and 10.

24. Give five numbers above 1000, all of which are exactly divisible by 2, 3, and 6.

25–28. Change one figure in each of the following numbers to make them exactly divisible by 2. By 3. By 5. By 9:

133 317 421 329 4321 3029 1427
211 409 527 803 2437 5123 8129

The following table gives a list of the prime numbers from 1 to 971, inclusive, for convenience of reference:

1	47	113	197	281	379	463	571	659	761	863
2	53	127	199	283	383	467	577	661	769	877
3	59	131	211	293	389	479	587	673	773	881
5	61	137	223	307	397	487	593	677	787	883
7	67	139	227	311	401	491	599	683	797	887
11	71	149	229	313	409	499	601	691	809	907
13	73	151	233	317	419	503	607	701	811	911
17	79	157	239	331	421	509	613	709	821	919
19	83	163	241	337	431	521	617	719	823	929
23	89	167	251	347	433	523	619	727	827	937
29	97	173	257	349	439	541	631	733	829	941
31	101	179	263	353	443	547	641	739	839	947
37	103	181	269	359	449	557	643	743	853	953
41	107	191	271	367	457	563	647	751	857	967
43	109	193	277	373	461	569	653	757	859	971

WRITTEN EXERCISES.

42.—*1.* Find the prime factors of 210.

PROCESS.	
2)210	210 is divisible by 2, since the units figure is 0.
5)105	Dividing 210 by 2, the quotient is 105.
3)21	105 is divisible by 5, since the units figure is 5.
7	Dividing 105 by 5, the quotient is 21.
	21 is divisible by 3. Dividing 21 by 3, the quotient is 7, a prime number.

Hence the prime factors of 210 are 2, 5, 3, and 7.

From this problem we have the following

RULE.

Divide the number by any of its prime factors; do the same with each quotient until the quotient is a prime number.

The several divisors and the last quotient are the prime factors required.

2. Find the prime factors of 114. Of 426.

PROCESS.	PROCESS.
2)114	2)426
3)57	3)213
19	71

Hence the prime factors of 114 are 2, 3, and 19; and of 426 are 2, 3, and 71.

Find the prime factors of—

3. 105.
4. 231.
5. 429.
6. 550.
7. 770.
8. 910.
9. 1225.
10. 2310.
11. 7007.
12. 1155.
13. 1050.
14. 1485.
15. 616.
16. 1040.
17. 1260.
18. 1728.
19. 1890.
20. 1584.
21. 1800.
22. 2079.
23. 2430.
24. 3645.
25. 5929.
26. 6125.
27. 6300.
28. 7290.
29. 8320.
30. 12705.
31. 15125.
32. 27625.
33. 34375.
34. 38148.

CANCELLATION.

ORAL EXERCISES.

43. Divide—

1. $\frac{9 \text{ times five}}{3 \text{ times five}}$.

2. $\frac{9 \text{ times six}}{3 \text{ times six}}$.

3. $\frac{9 \text{ times nine}}{3 \text{ times nine}}$.

4. $\frac{9 \times 17}{3 \times 17}$.

5. $\frac{12 \times 57}{3 \times 57}$.

6. $\frac{18 \times 117}{3 \times 117}$.

7. $\frac{15 \times 85}{3 \times 85}$.

8. $\frac{21 \times 97}{3 \times 97}$.

9. $\frac{36 \times 207}{3 \times 207}$.

10. What number may be omitted from both dividend and divisor in the first problem without changing the quotient? In the 2d? In the 3d? In the 4th? In the 5th? In the 6th? In the 7th? In the 8th? In the 9th?

Find the quotient in the following problems by inspection, and tell what *factors* may be omitted:

11. $\frac{16 \times 47}{8 \times 47}$.

12. $\frac{21 \times 108}{7 \times 108}$.

13. $\frac{35 \times 62}{7 \times 62}$.

14. $\frac{23 \times 9}{9 \times 23}$.

15. $\frac{6 \times 7 \times 8}{2 \times 7 \times 8}$.

16. $\frac{24 \times 6 \times 9}{8 \times 6 \times 9}$.

17. $\frac{42 \times 8 \times 75}{6 \times 8 \times 75}$.

18. $\frac{25 \times 7 \times 6}{5 \times 7 \times 6}$.

19. $\frac{54 \times 11 \times 13}{9 \times 11 \times 13}$.

20. $\frac{12 \times 6 \times 7}{12 \times 2 \times 7}$.

21. $\frac{8 \times 9 \times 4}{8 \times 9 \times 2}$.

22. $\frac{14 \times 15 \times 16}{7 \times 15 \times 16}$.

44. It is therefore evident that *rejecting equal factors from both dividend and divisor does not change the quotient.*

45. This process of shortening operations in division by rejecting equal factors from both divisor and dividend is called **Cancellation.**

WRITTEN EXERCISES.

1. Divide 70 × 33 by 35 × 3.

PROCESS.

$$\frac{70 \times 33}{35 \times 3} = \frac{2 \times \cancel{5} \times \cancel{7} \times \cancel{3} \times 11}{\cancel{5} \times \cancel{7} \times \cancel{3}} = 22, \textit{ Ans.}$$

Resolving the numbers into their prime factors, we see that 5, 7, and 3 are found in both dividend and divisor, and may be rejected or cancelled. We have remaining in the dividend 2 × 11, or 22, which is the required quotient.

2. Divide 15 × 18 × 40 × 125 by 5 × 36 × 20 × 75.

In practice it is customary to cancel without factoring; thus,

$$\frac{\overset{\cancel{3}}{\cancel{15}} \times \cancel{18} \times \overset{\cancel{2}}{\cancel{40}} \times \overset{5}{\cancel{125}}}{\cancel{5} \times \underset{\cancel{2}}{\cancel{36}} \times \cancel{20} \times \underset{\cancel{3}}{\cancel{75}}} = 5, \textit{ Ans.}$$

Perform by cancellation—

3. $\frac{33 \times 45 \times 75}{5 \times 81 \times 55} = 5.$

4. $\frac{84 \times 72 \times 6}{63 \times 12 \times 12} = 4.$

5. $\frac{72 \times 45 \times 140}{18 \times 24 \times 35} = 30.$

6. $\frac{60 \times 40 \times 70}{80 \times 30 \times 7} = 10.$

7. 11 × 39 × 14 × 96 ÷ 44 × 18 × 26 × 14.

8. 2 × 4 × 8 × 13 × 7 × 16 ÷ 26 × 14 × 8.

9. 125 × 60 × 24 × 42 ÷ 25 × 120 × 36 × 5.

10. 28 × 56 × 400 ÷ 112 × 280.

11. A farmer exchanged 15 barrels of apples, each containing 3 bushels, at 80 cents a bushel, for 8 pieces of cloth, each containing 30 yards; what was the price of the cloth per yard?

12. A merchant exchanged 20 boxes of tea, each containing 56 pounds, at 60 cents a pound, for 28 boxes of coffee, each containing 100 pounds; what was the price of the coffee?

13. How many dozen of eggs, worth 14 cents a dozen, must be given for 18 pounds of sugar, at 7 cents a pound?

14. How many pounds of butter, at 24 cents a pound, will pay for 45 yards of dress goods, at 32 cents a yard?

15. How many boxes of tea, each containing 24 pounds, at 75 cents a pound, must be given for 145 bags of wheat, each bag containing 2 bushels, at 180 cents a bushel?

16. A man exchanged 25 loads of lime, each load containing 15 bushels, at 22 cents a bushel, for 11 loads of oats, each containing 10 bags, with 3 bushels in a bag; what was the price of the oats per bushel?

GREATEST COMMON DIVISOR.

ORAL EXERCISES.

What numbers will exactly divide—

1. 8? 12? 16?
2. 9? 15? 32?
3. 45? 51? 75?
4. 57? 81? 100?

46. A **Divisor** of a number is a number that exactly divides it.

What numbers will exactly divide both—

5. 27 and 45?
6. 32 and 48?
7. 24 and 56?
8. 45 and 60?
9. 50 and 75?
10. 60 and 90?

47. A **Common Divisor** of two or more numbers is a divisor of each of them.

What is the *greatest* number that will exactly divide both—

11. 9 and 12?
12. 12 and 18?
13. 21 and 24?
14. 24 and 36?
15. 40 and 56?
16. 60 and 72?
17. 50 and 75?
18. 72 and 96?

48. The **Greatest Common Divisor** (G. C. D.) of two or more numbers is the greatest number that is a divisor of each of them.

What factors of each of the following numbers are found in their greatest common divisors:

19. Of 12 and 18?
20. Of 36 and 42?
21. Of 28 and 42?
22. Of 72 and 96?
23. Of 75 and 100?
24. Of 72 and 144?

49. Principle 1.—*The greatest common divisor of two or more numbers is the product of all their common prime factors.*

WRITTEN EXERCISES.

CASE I.

50. When the numbers can be readily factored.

1. Find the greatest common divisor of 210 and 330.

Process.

2	210 – 330
3	105 – 165
5	35 – 55
	7 – 11

G. C. D. = 2 × 3 × 5 = 30, *Ans.*

To find the common prime factors we divide by any prime number that will divide *all* of the numbers. We thus find that 2, 3, and 5 are the only common prime factors; and their product, 2 × 3 × 5 = 30, is the greatest common divisor (Prin. Art. 49).

2. Find the greatest common divisor of 105, 210, and 315; also, of 385, 770, and 1125.

Process.

3	105 – 210 – 315
5	35 – 70 – 105
7	7 – 14 – 21
	1 – 2 – 3

G. C. D. = 3 × 5 × 7 = 105.

5	385 – 770 – 1155
7	77 – 154 – 231
11	11 – 22 – 33
	1 – 2 – 3

G. C. D. = 5 × 7 × 11 = 385.

RULE.

Divide by any common prime factor of all the numbers; divide the quotients in the same manner, and thus continue until the quotients have no common factor.

Take the product of all the divisors; the result will be the greatest common divisor.

What is the greatest common divisor—

3. Of 20, 30, and 40?
4. Of 36, 48, and 60?
5. Of 30, 45, and 60?
6. Of 44, 66, and 88?
7. Of 48, 72, and 96?
8. Of 150, 210, and 330?
9. Of 175, 210, and 245?
10. Of 280, 386, and 392?
11. Of 252, 336, and 420?
12. Of 315, 420, and 525?
13. Of 210, 420, and 630?
14. Of 468, 780, and 1092?

Find the greatest common divisor of—

15. 462 and 546.
16. 608 and 736.
17. 720 and 756.
18. 390 and 330.
19. 672 and 744.
20. 704 and 832.
21. 720 and 792.
22. 385 and 525.
23. 616 and 728.
24. 945 and 1125.
25. 1056 and 1152.
26. 1050 and 1155.
27. 840 and 1080.
28. 1200 and 1440.
29. 1260 and 1470.

30. A has three farms, whose areas are 30, 45, and 75 acres, respectively; find the size of the largest fields, all of the same size, into which the farms can be divided.

31. What is the greatest length of board that can be used without cutting in fencing a triangular field whose sides are 80, 112, and 144 feet, respectively?

32. There are three piles of apples, containing 280, 315, and 350 bushels, respectively; what is the size of the largest bin that will contain each pile an exact number of times?

33. I wish to divide three tracts of land, containing 320, 640, and 1008 acres, respectively, into fields of the same size having the largest number of acres possible in each; what will be the size of each field, and how many can be made?

CASE II.

51. When the numbers cannot be readily factored.

Principle 2.—*A common divisor of two or more numbers is a divisor of their sum, and also of their difference.*

1. Find the G. C. D. of 91 and 117.

Process.

91)117(1
91
26)91(3
78
G. C. D. = 13)26(2
26

We divide 117 by 91, and the divisor 91 by the remainder 26, and finally the divisor 26 by the remainder 13, and have no remainder; then 13 is the G. C. D.

91 and 117 are each a number of times the G. C. D.; hence their difference, or 26, is a number of times the G. C. D. (Prin. 2).

Now, since 91 and 26 are each a number of times the G. C. D. sought, the question is reduced to finding the G. C. D. of 91 and 26.

In like manner we can reduce it to finding the G. C. D. of 26 and 13; but this is 13. Hence the G. C. D. of 91 and 117 is 13.

2. Find the G. C. D. of 115 and 161; also, of 217 and 341.

1st Form.

115)161(1
115
46)115(2
92
23)46(2
46

G. C. D. = 23.

2d Form.

115	161	1
92	115	2
23	46	2
	46	

1st Form.

217)341(1
217
124)217(1
124
93)124(1
93
31)93(3
93

G. C. D. = 31.

2d Form.

217	341	1
124	217	1
93	124	1
93	93	3
	31	

RULE.

Divide the greater number by the less, the divisor by the remainder, and thus continue until there is no remainder. The last divisor will be the greatest common divisor.

Find the greatest common divisor—

3. Of 92 and 115.
4. Of 186 and 217.
5. Of 205 and 246.
6. Of 329 and 423.
7. Of 424 and 583.
8. Of 488 and 671.
9. Of 536 and 737.
10. Of 415 and 664.
11. Of 721 and 1329.
12. Of 961 and 1953.
13. Of 1218 and 2233.
14. Of 560, 672, and 784.

SUGGESTION.—Find the G. C. D. of the first two numbers, then of that divisor and the third number.

15. Of 861, 984, and 1230.
16. Of 705, 987, and 1551.

Find the G. C. D. of—

17. 549 and 671.
18. 697 and 820.
19. 909 and 1414.
20. 923 and 1065.
21. 927 and 1339.
22. 533 and 1271.
23. 873 and 1067.
24. 1220 and 2013.
25. 1617 and 2058.
26. 1917 and 2556.
27. 2001 and 3247.
28. 2052 and 2565.
29. 3596 and 3768.
30. 3608 and 3731.
31. 5312 and 10043.

LEAST COMMON MULTIPLE.

ORAL EXERCISES.

What number less than 100 is one or more times—

1. 3? 5? 6?
2. 4? 8? 12?
3. 10? 9? 15?
4. 16? 20? 25?
5. 30? 40? 45?
6. 34? 36? 31?

52. A **Multiple** of a number is one or more times that number.

What number less than 100 is one or more times—

7. 6 and 12?
8. 8 and 10?
9. 9 and 12?
10. 10 and 15?
11. 9 and 12?
12. 15 and 18?
13. 16 and 20?
14. 18 and 24?

53. A **Common Multiple** of two or more numbers is a number that is a multiple of each of them.

What is the least number that will contain—

15. 12 and 18?
16. 15 and 20?
17. 20 and 30?
18. 21 and 63?
19. 20 and 25?
20. 30 and 40?

54. The **Least Common Multiple** (L. C. M.) of two or more numbers is the least number that is a multiple of each of them.

55. Principle.—*The least common multiple of two or more numbers must contain all the prime factors of each number and no others.*

WRITTEN EXERCISES.

CASE I.

56. When the numbers can be readily factored.

1. Find the least common multiple of 25, 35, and 56.

PROCESS.

$$\begin{array}{r|l} 5 & 25 - 35 - 56 \\ \hline 7 & 5 - 7 - 56 \\ \hline & 5 - 1 - 8 \end{array}$$

L. C. M. = 5 × 7 × 5 × 8 = 1400.

We divide by any number that will divide at least two of the numbers.

Dividing 25 and 35 by 5, the second line is 5, 7, and 56.

Dividing 7 and 56 by 7, the third line is 5, 1, and 8.

Since 5 and 8 have no common divisors, the L. C. M. is the product of the divisors 5 and 7 and the numbers in the last line, 5 and 8.

Hence the L. C. M. = 5 × 7 × 5 × 8 = 1400.

2. Find the L. C. M. of 24, 60, and 105.

PROCESS.

$$\begin{array}{r|l}
2 & 24 - 60 - 105 \\ \hline
2 & 12 - 30 - 105 \\ \hline
3 & 6 - 15 - 105 \\ \hline
5 & 2 - 5 - 35 \\ \hline
 & 2 - 1 - 7
\end{array}$$

$$\text{L. C. M.} = 2 \times 2 \times 3 \times 5 \times 2 \times 7 = 840.$$

RULE.

Divide the numbers by any number that will divide two or more of them, write the quotient and undivided numbers beneath, and thus continue until the numbers in the last line are prime to each other.

The product of the divisors and the numbers in the last line will be the least common multiple.

Find the L. C. M.—

3. Of 8, 12, and 20.
4. Of 15, 18, and 20.
5. Of 6, 8, 10, and 12.
6. Of 15, 16, 18, and 20.
7. Of 6, 7, 8, 9, and 10.
8. Of 12, 18, 22, and 24.
9. Of 36, 48, 84, and 126.
10. Of 22, 33, 44, 55, and 66.
11. Of 64, 84, 96, and 216.
12. Of 65, 78, 104, and 130.

Find the L. C. M. of—

13. 8, 11, 22, and 30.
14. 120, 180, 200, and 240.
15. 140, 160, 280, and 320.
16. 98, 126, 140, and 168.
17. 96, 128, and 160.
18. 108, 180, and 252.
19. 150, 200, and 250.
20. 180, 200, and 240.

21. What is the smallest sum of money with which a drover can purchase cows at $32 each, mules at $88 each, or horses at $121 each?

22. What is the least number of acres that can be exactly divided into fields of 12 acres, 15 acres, or 16 acres each?

23. What is the least sum of money with which I can purchase an exact number of yards of cloth at 50 cts., $1.20, or $2.16 a yard?

24. What is the least quantity of water that can be exactly measured by vessels which hold either 9, 32, 48, or 63 gallons each?

25. Three trolley-cars make round trips in 40, 60, and 90 minutes, respectively; if they start out at the same time, after how many minutes will all three be again at their respective starting-points at the same time?

CASE II.

57. When the numbers are not readily factored.

1. Find the L. C. M. of 115 and 161.

Since the common factors are not readily seen by inspection, their product may be found by finding the G. C. D.

PROCESS.

115)161(1
115
46)115(2
92
G. C. D. = 23)46(2
46

$$\begin{array}{r|l} 23 & 115 - 161 \\ \hline & 5 \; - \; 7 \end{array}$$

L. C. M. = $23 \times 5 \times 7$, by Case I.
= $115 \times \frac{161}{23} = 805$.

The L. C. M. as found by the first case is $23 \times 5 \times 7$. But $23 \times 5 = 115$, and $7 = \frac{161}{23}$; hence the L. C. M. $= 115 \times \frac{161}{23} = 805$.

RULE.

Find the greatest common divisor of the two numbers.

Divide either number by the greatest common divisor, and multiply the quotient by the other number.

2. Find the L. C. M. of 217 and 341.

PROCESS.

GREATEST COMMON DIVISOR.

217)341(1
217
124)217(1
124
93)124(1
93
G. C. D. = 31)93(3
93

QUOTIENT.

31)217(7
217

LEAST COMMON MULTIPLE.

341
7
2387, *Ans.*

Find the L. C. M.—

3. Of 261 and 319.
4. Of 527 and 589.
5. Of 861 and 943.
6. Of 731 and 817.
7. Of 703 and 814.
8. Of 529 and 713.
9. Of 459 and 629.
10. Of 671 and 915.

Find the L. C. M. of—

11. 609 and 957.
12. 710 and 1065.
13. 856 and 963.
14. 1081 and 1175.
15. 1111 and 1313.
16. 1435 and 1681.
17. 1403 and 1817.
18. 1491 and 1917.
19. 2205 and 3465.
20. 6649 and 7957.

MISCELLANEOUS PROBLEMS.

1. Find the prime factors of 44100.

2. A and B agreed to purchase cows at a uniform price per head, provided they could invest all their money: A had $253, and B had $299; what was the average price of a cow, and how many cows did each buy?

3. A can dig 9 rods of ditch, B 12 rods, C 15 rods, and D 16 rods, in a day; what is the least number of rods that would furnish an exact number of days' labor to each man working alone, and how long would it take him to dig it?

4. Find the G. C. D. of 1829 and 2419.

5. Find the difference between the L. C. M. of 1, 3, 5, 7, 9, and the L. C. M. of 2, 4, 6, 8, 10.

6. Divide 14 × 36 × 49 × 24 × 25 × 112 by 75 × 7 × 16 × 9 × 7, and add 364 to the quotient.

7. A farmer has bought a triangular piece of land whose sides are 357, 391, and 425 feet, respectively; what is the length of the longest boards that can be used to fence it without cutting them?

8. What is the least sum of money with which I can purchase exact numbers of sheep at $6, cows at $22, or horses at $78 a head?

9. How much does the L. C. M. of 1751 and 2369 exceed their G. C. D.?

10. Divide 213 × 84 × 190 × 264 by 30 × 56 × 36, and add 161 to the quotient.

11. What is the largest-sized vessel that will exactly measure the contents of each of four cans, holding, respectively, 28, 36, 52, and 76 quarts? How many times would the measure have to be filled?

12. Four men start at the same time and place to travel around a circular island 360 miles in circumference: A goes 45 miles, B 36 miles, C 30 miles, and D 24 miles a day; in how many days will they all be again together at the starting-point, and how often will each have gone round?

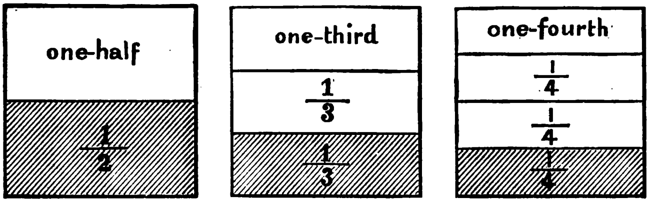
13. A manufacturer has a quantity of wire in lengths of 105, 120, 135, and 165 feet, respectively, which he wishes to cut without waste into the longest possible equal lengths; what must be the length of each of these?

14. A school building contains 175 pupils in the high school department, 420 in the grammar school department, and 595 in the primary department; into how many classes can each department be subdivided so as to have the greatest possible equal number in all the classes?

SECTION IV.

COMMON FRACTIONS.

DEFINITIONS.



58. DRAW three square inches. Divide the first into two *equal* parts. What is one of these parts called? Divide the second into three *equal* parts. What is one of these parts called? What are two of them called? Divide the third into four *equal* parts. What is one of these parts called? What are two of them called? Three of them? Four of them?

These expressions, *one-half, one-third, two-thirds,* etc., are called *Fractions.*

59. A **Fraction** is therefore *one* or *more* of the equal parts of a unit.

60. A fraction is expressed by writing one number above another with a short horizontal line between them. Thus,

One-half is written $\frac{1}{2}$.
One-third is written $\frac{1}{3}$.
Three-fourths is written $\frac{3}{4}$.

When a fraction is thus expressed it is called a **Common Fraction.**

What does the number below the line show?

What does the number above the line show?

The **Denominator** is the number below the line. The **Numerator** is the number above the line.

The numerator and the denominator are called the **Terms** of a fraction.

61. How many thirds are there in anything? How many fourths? How many fifths?

Is $\frac{3}{5}$ greater or less than a unit? $\frac{7}{8}$? $\frac{9}{10}$?

A **Proper Fraction** is one whose value is less than a unit.

62. Is $\frac{5}{4}$ greater or less than a unit? $\frac{7}{5}$? $\frac{9}{9}$? $\frac{12}{11}$?

An **Improper Fraction** is one whose value is equal to or greater than a unit.

63. The sum of 6 and $\frac{2}{3}$, or $6 + \frac{2}{3}$, is written $6\frac{2}{3}$. What kind of number is 6? What kind of number is $\frac{2}{3}$? What kind of number is $6\frac{2}{3}$?

A **Mixed Number** is the sum of a whole number and a fraction.

64. What is 1 divided by 3? By 4? By 5? By 6? By 7? By 9? By 12?

The **Reciprocal** of a number is 1 divided by that number.

65. The fraction $\frac{5}{6}$ is the result of either of two processes: first, of taking $\frac{5}{6}$ of 1, or, second, of taking $\frac{1}{6}$ of 5. Hence a fraction may be regarded as an expression of *division,* in which the numerator is the dividend and the denominator is the divisor.

It is evident that an *integer* can be expressed in the form of a fraction by writing 1 for its denominator. Thus, 5 is the same as $\frac{5}{1}$.

NOTATION AND NUMERATION OF FRACTIONS.

66. Read the following fractions:

1. $\frac{2}{3}$.
2. $\frac{11}{12}$.
3. $\frac{20}{9}$.
4. $\frac{17}{18}$.
5. $\frac{10}{100}$.
6. $\frac{28}{60}$.
7. $\frac{45}{70}$.
8. $\frac{27}{111}$.
9. $6\frac{2}{3}$.
10. $7\frac{5}{8}$.
11. $4\frac{11}{12}$.
12. $9\frac{7}{20}$.
13. $\frac{851}{4001}$.
14. $\frac{8}{100}$.
15. $\frac{116}{1000}$.
16. $\frac{9}{1296}$.
17. $\frac{361}{8162}$.
18. $\frac{4002}{18261}$.
19. $4\frac{210}{216}$.
20. $19\frac{18}{19}$.

Express with figures—

21. Five-eighths.
22. Nine-tenths.
23. Six-twentieths.
24. Five and two-thirds.
25. Sixty-seventieths.
26. Eighty-one-ninetieths.
27. 212-tenths.
28. 1001-millionths.
29. Nine and five-sixths.
30. One and twelve-twentieths.
31. Seventy-five-eightieths.
32. 225-ninetieths.

What kind of fractions are the following?—

33. $\frac{2}{3}$.
34. $\frac{8}{5}$.
35. $6\frac{2}{3}$.
36. $\frac{9}{9}$.
37. $1\frac{2}{3}$.
38. $\frac{8}{10}$.
39. $\frac{1}{21}$.
40. $\frac{921}{81}$.
41. $6\frac{7}{9}$.
42. $\frac{14}{14}$.
43. $\frac{1}{500}$.
44. $\frac{6}{1}$.

REDUCTION OF FRACTIONS.

CASE I.

67. To reduce an improper fraction to a whole or a mixed number.

ORAL EXERCISES.

1. How many ones in 2 halves? 4 halves?
2. How many ones in $\frac{3}{3}$? $\frac{6}{3}$? $\frac{15}{3}$? $\frac{18}{3}$? $\frac{20}{3}$?
3. How many ones in $\frac{4}{4}$? $\frac{8}{4}$? $\frac{10}{4}$? $\frac{21}{4}$? $\frac{30}{4}$?
4. How many ones in $\frac{5}{5}$? $\frac{10}{5}$? $\frac{18}{5}$? $\frac{27}{5}$? $\frac{45}{5}$?
5. How many ones in $\frac{6}{6}$? $\frac{12}{6}$? $\frac{30}{6}$? $\frac{48}{6}$? $\frac{55}{6}$?

WRITTEN EXERCISES.

6. Reduce $\frac{72}{8}$ to a whole number.

Since a fraction is an expression of division (ART. 65),

$$\frac{72}{8} = 72 \div 8 = 9, \textit{ Ans.}$$

7. Reduce $\frac{539}{34}$ to a mixed number.

PROCESS.

```
34)539(15 29/34, Ans.
   34
   ---
   199
   170
   ---
    29
```

RULE.

Divide the numerator by the denominator.

Reduce to whole or mixed numbers—

8. $\frac{211}{8}$.
9. $\frac{312}{11}$.
10. $\frac{805}{12}$.
11. $\frac{389}{16}$.
12. $\frac{807}{20}$.
13. $\frac{929}{32}$.
14. $\frac{1878}{88}$.
15. $\frac{2285}{57}$.
16. $\frac{3217}{84}$.
17. $\frac{4168}{96}$.
18. $\frac{4862}{73}$.
19. $\frac{9092}{91}$.
20. $\frac{1049}{22}$.
21. $\frac{1216}{31}$.
22. $\frac{1560}{33}$.
23. $\frac{1682}{35}$.
24. $\frac{2047}{42}$.
25. $\frac{2167}{43}$.
26. $\frac{2865}{44}$.
27. $\frac{3187}{45}$.
28. $\frac{4028}{92}$.
29. $\frac{7877}{128}$.
30. $\frac{6285}{131}$.
31. $\frac{8361}{241}$.
32. $\frac{6788}{74}$.
33. $\frac{7816}{75}$.
34. $\frac{6821}{111}$.
35. $\frac{4025}{126}$.
36. $\frac{6060}{101}$.
37. $\frac{8296}{315}$.
38. $\frac{10000}{412}$.
39. $\frac{12416}{245}$.

CASE II.

68. To reduce a whole or a mixed number to an improper fraction.

ORAL EXERCISES.

1. How many halves in 1 apple? In 4 apples?
2. How many thirds in 1? In 3? In 6? In 8?
3. How many fourths in 1? In 8? In 10? In 12?
4. How many thirds in $7\frac{2}{3}$? In $8\frac{1}{3}$? In $12\frac{2}{3}$? In $20\frac{1}{3}$?
5. How many fifths in $6\frac{1}{5}$? In $8\frac{2}{5}$? In $10\frac{3}{5}$? In $12\frac{4}{5}$?

WRITTEN EXERCISES.

6. Reduce $24\frac{2}{5}$ to an improper fraction.

PROCESS.

$$\begin{array}{r} 24\frac{2}{5} \\ 5 \\ \hline 122 \text{ fifths} = \frac{122}{5}. \end{array}$$

In 1 there are 5 fifths, and in 24 there are 24 × 5 fifths, which are 120 fifths; 120 fifths + 2 fifths are 122 fifths, or $\frac{122}{5}$.

RULE.

Multiply the whole number by the denominator of the fraction, to the product add the numerator, and write the result over the denominator.

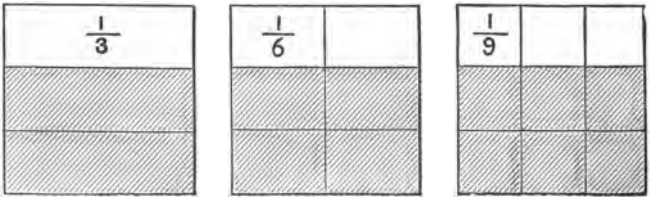
Reduce to improper fractions—

7. $62\frac{8}{21}$.
8. $85\frac{7}{25}$.
9. $100\frac{7}{15}$.
10. $30\frac{7}{8}$.
11. $41\frac{29}{37}$.
12. $62\frac{31}{32}$.
13. $75\frac{41}{72}$.
14. $106\frac{6}{17}$.
15. $516\frac{4}{5}$.

16. $67\frac{3}{4}$.
17. $75\frac{5}{9}$.
18. $62\frac{9}{10}$.
19. $73\frac{8}{11}$.
20. $29\frac{4}{31}$.
21. $84\frac{6}{25}$.
22. $102\frac{12}{25}$.
23. $205\frac{19}{31}$.
24. $315\frac{12}{53}$.
25. $400\frac{7}{10}$.
26. $101\frac{100}{101}$.
27. $16\frac{128}{116}$.
28. $27\frac{216}{129}$.
29. $103\frac{12}{47}$.
30. $122\frac{7}{10}$.
31. $212\frac{8}{9}$.
32. $702\frac{10}{11}$.
33. $615\frac{3}{5}$.
34. $122\frac{1}{15}$.
35. $241\frac{4}{21}$.

CASE III.

69. **To reduce a fraction to its lowest terms.**



From this illustration it is seen that $\frac{2}{3} = \frac{4}{6} = \frac{6}{9}$.

ORAL EXERCISES.

1. How many sixths in $\frac{2}{3}$? How many ninths?

2. How may $\frac{4}{6}$ and $\frac{6}{9}$ be obtained from $\frac{2}{3}$?

3. How may $\frac{2}{3}$ be obtained from $\frac{4}{6}$ or $\frac{6}{9}$?

4. What, then, may be done to the terms of a fraction without changing its value?

70. Principle.—*Multiplying or dividing both terms of a fraction by the same number does not change its value.*

5. How many thirds in $\frac{3}{9}$? In $\frac{8}{12}$? In $\frac{10}{15}$?

6. How many fifths in $\frac{5}{25}$? In $\frac{12}{30}$? In $\frac{21}{35}$?

WRITTEN EXERCISES.

7. Reduce $\frac{24}{30}$ to its lowest terms.

PROCESS.

$$6 \left| \frac{24}{30} \right. = \frac{4}{5}, \textit{ Ans.}$$

Since dividing both terms of a fraction by the same number does not change its value, we divide both terms of $\frac{24}{30}$ by 6, and have $\frac{4}{5}$. Hence $\frac{24}{30}$ equals $\frac{4}{5}$.

8. Reduce $\frac{232}{261}$ to its lowest terms.

When it is not readily seen what number will divide both terms of the fraction, it is best to find their G. C. D.

PROCESS.

$$\begin{array}{r} 232)261(1 \\ \underline{232} \\ \text{G. C. D.} = 29)232(8 \\ \underline{232} \end{array}$$

$$29 \left| \frac{232}{261} \right. = \frac{8}{9}, \textit{ Ans.}$$

By the method of ART. 51 we find the G. C. D. to be 29. Dividing both terms of the fraction by 29, we have $\frac{8}{9}$, the required result.

RULE.

Divide both terms of the fraction by the same number, and continue to divide until the terms have no common divisor.

Or, divide both terms by their greatest common divisor.

Reduce to their lowest terms—

9. $\frac{40}{48}$, $\frac{35}{45}$.
10. $\frac{22}{99}$, $\frac{36}{81}$.
11. $\frac{112}{119}$, $\frac{128}{132}$.
12. $\frac{144}{156}$, $\frac{104}{156}$.
13. $\frac{108}{396}$, $\frac{48}{176}$.
14. $\frac{161}{207}$, $\frac{261}{319}$.
15. $\frac{155}{279}$, $\frac{205}{287}$.
16. $\frac{333}{407}$, $\frac{164}{205}$.
17. $\frac{212}{371}$, $\frac{301}{473}$.
18. $\frac{282}{329}$, $\frac{427}{488}$.
19. $\frac{306}{561}$, $\frac{289}{442}$.
20. $\frac{300}{825}$, $\frac{672}{784}$.

21. $\frac{182}{273}$.
22. $\frac{204}{306}$.
23. $\frac{202}{303}$.
24. $\frac{435}{522}$.
25. $\frac{357}{408}$.
26. $\frac{819}{936}$.
27. $\frac{291}{388}$.
28. $\frac{415}{913}$.
29. $\frac{305}{1037}$.
30. $\frac{850}{935}$.
31. $\frac{690}{805}$.
32. $\frac{282}{1551}$.
33. $\frac{1177}{1284}$.
34. $\frac{1026}{2565}$.
35. $\frac{5184}{6912}$.
36. $\frac{3168}{5148}$.

CASE IV.

71. To reduce fractions to their least common denominator.

ORAL EXERCISES.

1. How many fourths in $\frac{1}{2}$?

In 1 there are 4 fourths, and in $\frac{1}{2}$ there are $\frac{1}{2}$ of 4 fourths, which are 2 fourths, or $\frac{2}{4}$.

2. How many sixths in $\frac{1}{2}$? In $\frac{1}{3}$? In $\frac{2}{3}$?
3. How many twelfths in $\frac{1}{2}$? In $\frac{1}{3}$? In $\frac{1}{4}$? In $\frac{1}{6}$?
4. To what denominator can $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{6}$ be reduced?

Fractions have a **Common Denominator** when they have the same number for a denominator.

The **Least Common Denominator** (L. C. D.) of several fractions is the least denominator to which all may be reduced.

5. Reduce $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$ to twelfths.
6. Reduce $\frac{1}{3}$, $\frac{1}{5}$, $\frac{2}{3}$, $\frac{2}{5}$, $\frac{3}{5}$ to fifteenths.

Reduce to their least common denominator—

7. $\frac{2}{3}$ and $\frac{5}{6}$.
8. $\frac{3}{4}$ and $\frac{7}{8}$.
9. $\frac{5}{6}$ and $\frac{7}{12}$.
10. $\frac{2}{5}$ and $\frac{7}{10}$.
11. $\frac{3}{4}$ and $\frac{5}{6}$.
12. $\frac{7}{8}$ and $\frac{15}{16}$.
13. $\frac{2}{3}$ and $\frac{7}{8}$.
14. $\frac{5}{6}$ and $\frac{3}{5}$.
15. $\frac{7}{8}$ and $\frac{2}{5}$.

72. Principle.—*The least common denominator of several fractions is the least common multiple of their denominators.*

WRITTEN EXERCISES.

16. Reduce $\frac{2}{5}$, $\frac{3}{10}$, and $\frac{7}{15}$ to their common denominator.

PROCESS.

$$\begin{array}{r|l} 5 & 5 - 10 - 15 \\ \hline & 1 - 2 - 3 \end{array}$$

L. C. M. $= 5 \times 2 \times 3 = 30.$

$$\frac{2}{5} = \frac{2 \times 6}{5 \times 6} = \frac{12}{30},$$

$$\frac{3}{10} = \frac{3 \times 3}{10 \times 3} = \frac{9}{30},$$

$$\frac{7}{15} = \frac{7 \times 2}{15 \times 2} = \frac{14}{30}.$$

By ART. 55 the L. C. M. of 5, 10, and 15 is 30.

Dividing 30 by 5, the denominator of $\frac{2}{5}$, we find we must multiply 5 by 6 to produce 30. Multiplying both terms of $\frac{2}{5}$ by 6, we have $\frac{2}{5} = \frac{12}{30}$ (Prin., ART. 70).

Dividing 30 by 10, the denominator of $\frac{3}{10}$, we find we must multiply 10 by 3 to produce 30. Multiplying both terms of $\frac{3}{10}$ by 3, we have $\frac{3}{10} = \frac{9}{30}$, etc.

RULE.

Find the L. C. M. of the given denominators.

Divide this by each denominator and multiply both terms of the fraction by the quotient.

NOTE.—Reduce mixed numbers to improper fractions, and all to their lowest terms before finding the least common denominator.

Reduce to their least common denominator—

17. $\frac{2}{3}$, $\frac{7}{8}$, and $\frac{5}{12}$.
18. $\frac{3}{4}$, $\frac{7}{8}$, and $\frac{9}{10}$.
19. $\frac{3}{7}$, $\frac{5}{14}$, and $\frac{7}{10}$.
20. $\frac{5}{14}$, $\frac{13}{21}$, and $\frac{6}{28}$.
21. $2\frac{1}{3}$, $\frac{12}{18}$, and $\frac{24}{30}$.
22. $5\frac{1}{3}$, $\frac{15}{18}$, and $\frac{17}{72}$.
23. $1\frac{1}{3}$, $\frac{3}{7}$, $\frac{9}{14}$, and $\frac{5}{49}$.
24. $\frac{5}{6}$, $\frac{7}{8}$, $\frac{9}{10}$, and $\frac{11}{12}$.
25. $\frac{6}{17}$, $\frac{7}{34}$, $\frac{8}{68}$, and $\frac{9}{136}$.
26. $\frac{3}{5}$, $\frac{7}{15}$, $\frac{3}{14}$, and $\frac{9}{70}$.
27. $\frac{5}{18}$, $\frac{13}{26}$, and $\frac{7}{39}$.
28. $\frac{3}{5}$, $\frac{8}{15}$, and $\frac{7}{35}$.
29. $\frac{9}{10}$, $\frac{11}{12}$, and $\frac{13}{14}$.
30. $\frac{3}{5}$, $\frac{7}{9}$, and $\frac{5}{21}$.
31. $\frac{7}{12}$, $\frac{11}{16}$, and $\frac{19}{36}$.
32. $\frac{2}{5}$, $\frac{7}{20}$, and $\frac{5}{32}$.
33. $\frac{3}{10}$, $\frac{13}{14}$, and $\frac{9}{35}$.
34. $\frac{11}{12}$, $\frac{15}{28}$, and $\frac{17}{42}$.

ADDITION OF FRACTIONS.

ORAL EXERCISES.

73.—*1.* Find the sum of 2 thirds and 5 thirds.

2. Find the sum of 2 sixths and 3 sixths. $\frac{2}{8}$ and $\frac{5}{8}$.

3. Find the sum of $\frac{3}{8}$, $\frac{4}{8}$, and $\frac{7}{8}$. $\frac{2}{9}$, $\frac{5}{9}$, and $\frac{4}{9}$.

4. Find the sum of $\frac{1}{2}$ and $\frac{1}{3}$.

SUGGESTION.—$\frac{1}{2} = \frac{3}{6}$, and $\frac{1}{3} = \frac{2}{6}$; $\frac{3}{6}$ and $\frac{2}{6}$ are $\frac{5}{6}$.

Find the sum of—

5. $\frac{1}{2}$ and $\frac{1}{4}$.
6. $\frac{1}{3}$ and $\frac{1}{4}$.
7. $\frac{2}{3}$ and $\frac{1}{4}$.
8. $\frac{2}{3}$ and $\frac{3}{4}$.
9. $\frac{1}{3}$ and $\frac{1}{6}$.
10. $\frac{2}{3}$ and $\frac{1}{6}$.
11. $\frac{1}{3}$ and $\frac{5}{6}$.
12. $\frac{2}{3}$ and $\frac{5}{6}$.
13. $\frac{1}{4}$ and $\frac{1}{8}$.
14. $\frac{3}{4}$ and $\frac{5}{8}$.
15. $\frac{2}{4}$ and $\frac{7}{8}$.
16. $\frac{5}{4}$ and $\frac{9}{8}$.
17. $\frac{1}{5}$ and $\frac{1}{10}$.
18. $\frac{2}{5}$ and $\frac{3}{10}$.
19. $\frac{3}{5}$ and $\frac{2}{15}$.
20. $\frac{4}{5}$ and $\frac{7}{20}$.

74. Principle.—*To add fractions they must be reduced to a common denominator.*

WRITTEN EXERCISES.

21. Find the sum of $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{7}{8}$.

PROCESS.—$\frac{2}{3} + \frac{3}{4} + \frac{7}{8} = \frac{16}{24} + \frac{18}{24} + \frac{21}{24} = \frac{55}{24} = 2\frac{7}{24}$, *Ans.*

The L. C. M. of 3, 4, and 8 is 24.

Reducing the fractions to 24ths, we have $\frac{2}{3} = \frac{16}{24}$, $\frac{3}{4} = \frac{18}{24}$, and $\frac{7}{8} = \frac{21}{24}$. Since the fractions now have a common denominator, we add their numerators: 16 *twenty-fourths* + 18 *twenty-fourths* + 21 *twenty-fourths*, equal 55 *twenty-fourths*, or $\frac{55}{24}$, or $2\frac{7}{24}$.

RULE.

Reduce the fractions to a common denominator, then add their numerators, and write the sum over the common denominator.

NOTE.—Always reduce the result to its lowest terms.

Find the sum—

22. Of $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{5}{6}$.
23. Of $\frac{4}{5}$, $\frac{5}{6}$, and $\frac{7}{8}$.
24. Of $\frac{5}{7}$, $\frac{8}{56}$, and $\frac{6}{35}$.
25. Of $\frac{6}{7}$, $\frac{7}{8}$, and $\frac{5}{14}$.

Find the sum—

26. Of $\frac{2}{3}$, $\frac{5}{7}$, and $\frac{6}{8}$.
27. Of $\frac{2}{5}$, $\frac{3}{8}$, and $\frac{7}{12}$.
28. Of $\frac{3}{4}$, $\frac{4}{7}$, and $\frac{7}{9}$.
● *29.* Of $\frac{2}{3}$, $\frac{12}{18}$, and $\frac{9}{24}$.
30. Of $\frac{4}{5}$, $\frac{6}{8}$, and $\frac{9}{12}$.
31. Of $\frac{4}{7}$, $\frac{3}{4}$, and $\frac{11}{14}$.
32. Of $\frac{4}{9}$, $\frac{7}{10}$, $\frac{5}{8}$, and $\frac{11}{18}$.
33. Of $\frac{6}{7}$, $\frac{8}{21}$, $\frac{9}{14}$, and $\frac{2}{3}$.
34. Of $\frac{7}{9}$, $\frac{1}{3}$, $\frac{9}{15}$, and $\frac{7}{27}$.
35. Of $\frac{2}{3}$, $\frac{12}{18}$, $\frac{9}{14}$, and $\frac{5}{21}$.
36. Of $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{5}{6}$.
37. Of $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{7}$, $\frac{7}{12}$, and $\frac{5}{24}$.
38. Of $\frac{9}{24}$, $\frac{7}{15}$, $\frac{10}{21}$, $\frac{18}{35}$, and $\frac{5}{7}$.
39. Of $\frac{2}{3}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{5}{10}$.

40. Find the sum of $13\frac{2}{3}$ and $15\frac{3}{4}$.

PROCESS.		
$13\frac{2}{3}$	$\frac{8}{12}$	The sum of $\frac{2}{3}$ and $\frac{3}{4}$ is $1\frac{5}{12}$.
$15\frac{3}{4}$	$\frac{9}{12}$	The sum of 13 and 15 is 28.
$29\frac{5}{12}$, *Ans.*		The sum of 28 and $1\frac{5}{12}$ is $29\frac{5}{12}$.

41. Find the sum of $8\frac{1}{2}$, $10\frac{5}{6}$, and $14\frac{3}{4}$.
42. Find the sum of $102\frac{5}{6}$, $204\frac{7}{8}$, and $90\frac{5}{12}$.
43. Find the sum of $22\frac{1}{3}$, $34\frac{3}{4}$, $42\frac{5}{7}$, and $60\frac{1}{2}$.
44. Find the sum of $62\frac{1}{2}$, $87\frac{1}{2}$, $75\frac{3}{4}$, and $112\frac{1}{2}$.
45. Find the sum of $67\frac{3}{11}$, $4\frac{9}{33}$, $600\frac{2}{3}$, and $50\frac{2}{22}$.

SUBTRACTION OF FRACTIONS.

ORAL EXERCISES.

75.—*1.* Subtract 2 thirds from 5 thirds.
2. Subtract 2 sevenths from 6 sevenths.
3. Subtract $\frac{3}{5}$ from $\frac{8}{5}$. $\frac{9}{11}$ from $\frac{13}{11}$. $\frac{8}{13}$ from $\frac{10}{13}$.
4. Subtract $\frac{1}{3}$ from $\frac{1}{2}$.

SUGGESTION.—$\frac{1}{3} = \frac{2}{6}$, and $\frac{1}{2} = \frac{3}{6}$; $\frac{2}{6}$ from $\frac{3}{6}$ leaves $\frac{1}{6}$.

Subtract—

5. $\frac{1}{4}$ from $\frac{1}{2}$.
6. $\frac{1}{4}$ from $\frac{1}{3}$.
7. $\frac{1}{2}$ from $\frac{2}{3}$.
8. $\frac{2}{3}$ from $\frac{3}{4}$.
9. $\frac{1}{6}$ from $\frac{1}{3}$.
10. $\frac{1}{2}$ from $\frac{5}{6}$.
11. $\frac{2}{3}$ from $\frac{5}{6}$.
12. $\frac{1}{3}$ from $\frac{7}{6}$.
13. $\frac{1}{8}$ from $\frac{1}{4}$.
14. $\frac{3}{8}$ from $\frac{3}{4}$.
15. $\frac{1}{2}$ from $\frac{5}{8}$.
16. $\frac{3}{4}$ from $\frac{7}{8}$.
17. $\frac{1}{10}$ from $\frac{1}{5}$.
18. $\frac{3}{10}$ from $\frac{2}{5}$.
19. $\frac{3}{5}$ from $\frac{14}{15}$.
20. $\frac{4}{5}$ from $\frac{18}{20}$.

76. Principle.—*To subtract fractions they must be reduced to a common denominator.*

WRITTEN EXERCISES.

21. Subtract $\frac{3}{10}$ from $\frac{7}{8}$.

PROCESS.

$\frac{7}{8} - \frac{3}{10} = \frac{35}{40} - \frac{12}{40} = \frac{23}{40}$, *Ans.*

The L. C. M. of 8 and 10 is 40. Reducing the fractions to 40ths, we have $\frac{7}{8} = \frac{35}{40}$, and $\frac{3}{10} = \frac{12}{40}$. Since the fractions now have a common denominator, we subtract their numerators: 35 fortieths − 12 fortieths equals 23 fortieths, or $\frac{23}{40}$.

RULE.

Reduce the fractions to a common denominator, subtract their numerators, and write the result over the common denominator.

NOTE.—Always reduce the result to its lowest terms.

Subtract—

22. $\frac{4}{7}$ from $\frac{5}{6}$.
23. $\frac{2}{5}$ from $\frac{7}{10}$.
24. $\frac{5}{11}$ from $\frac{6}{18}$.
25. $\frac{5}{18}$ from $\frac{8}{21}$.
26. $\frac{10}{11}$ from $\frac{11}{12}$.
27. $\frac{13}{14}$ from $\frac{20}{21}$.
28. $\frac{18}{19}$ from $\frac{40}{41}$.
29. $\frac{5}{6}$ from $\frac{7}{8}$.
30. $\frac{1}{6}$ from $\frac{8}{15}$.
31. $\frac{2}{9}$ from $\frac{22}{21}$.
32. $\frac{5}{14}$ from $\frac{22}{35}$.
33. $\frac{28}{39}$ from $\frac{7}{6}$.
34. $\frac{2}{45}$ from $\frac{11}{20}$.
35. $\frac{2}{21}$ from $\frac{7}{12}$.
36. $\frac{7}{15}$ from $\frac{11}{14}$.
37. $\frac{11}{15}$ from $\frac{18}{19}$.
38. $\frac{1}{11}$ from $\frac{11}{12}$.
39. $\frac{11}{12}$ from $\frac{12}{13}$.

40. Subtract $8\frac{5}{6}$ from $21\frac{2}{3}$.

PROCESS.

$$\begin{array}{ll} 21\frac{2}{3} & 1\frac{2}{3} = \frac{10}{6} \\ \underline{8\frac{5}{6}} & \underline{\phantom{1\frac{2}{3} = }\frac{5}{6}} \\ 12\frac{5}{6}, \textit{Ans.} & \end{array}$$

We cannot subtract $\frac{5}{6}$ from $\frac{2}{3}$; we take 1 from 21, which added to $\frac{2}{3}$ equal $1\frac{2}{3}$, or $\frac{10}{6}$; $\frac{5}{6}$ from $\frac{10}{6}$ leaves $\frac{5}{6}$, and 8 from 20 leaves 12. Hence the required result is $12\frac{5}{6}$.

What is the value of—

41. $6\frac{1}{2} - \frac{1}{2}$?
42. $10\frac{2}{3} - 5\frac{1}{2}$?
43. $18\frac{7}{8} - 10\frac{1}{3}$?
44. $25\frac{9}{10} - 12\frac{2}{3}$?
45. $32\frac{13}{14} - 20\frac{5}{6}$?
46. $60\frac{1}{3} - 14\frac{2}{3}$?
47. $70\frac{1}{2} - 20\frac{5}{8}$?
48. $81\frac{2}{3} - 25\frac{7}{8}$?
49. $96\frac{1}{2} - 32\frac{5}{14}$?
50. $62\frac{1}{2} - 33\frac{5}{6}$?
51. $118\frac{7}{8} - 28\frac{8}{9}$?
52. $500\frac{2}{3} - 125\frac{9}{10}$?

PRACTICAL PROBLEMS

In Addition and Subtraction of Fractions.

1. A bought $16\frac{2}{3}$ tons of coal, and B bought $8\frac{5}{6}$ tons; how much did they both buy, and how much more did A buy than B?

2. The first of three farms contains $119\frac{3}{8}$ acres, the second $97\frac{5}{6}$ acres, and the third $216\frac{5}{9}$ acres; how many acres in the three farms?

3. A teacher's salary per month is $\$135\frac{7}{10}$, and his expenses average $\$51\frac{7}{8}$; how much does he save per month?

4. My agent charged me $\$110\frac{1}{2}$ for traveling expenses and $\$104\frac{3}{4}$ for boarding; his sales amounted in cash to $1000; how much should he return to me?

5. A merchant sold $5000 worth of goods in four months: the first month he sold $\$970\frac{1}{4}$ worth, the second $\$1002\frac{1}{2}$ worth, and the third $\$1260\frac{7}{17}$ worth; how much did he sell the fourth month?

6. A farmer raised 800 bushels of wheat, corn, and rye; how much wheat did he raise, if there were $108\frac{5}{7}$ bushels of rye and $257\frac{9}{10}$ bushels of corn?

7. A miller had $250\frac{1}{2}$ bushels of wheat: after buying $316\frac{7}{9}$ bushels he sold $426\frac{7}{8}$ bushels; how much less had he than he had at first?

8. A had $\$2045\frac{1}{4}$, then lost $\$120\frac{3}{4}$, then gained $\$316\frac{1}{2}$, after which he lost $\$528\frac{3}{4}$; how much remained?

9. The sides of a field are $42\frac{1}{8}$ rods, $63\frac{5}{6}$ rods, $70\frac{5}{12}$ rods, $53\frac{2}{7}$ rods, and $37\frac{1}{2}$ rods, respectively; what is the distance around the field?

10. A man went to a store with $500: he paid $\$75\frac{1}{2}$ for a reaper, $\$34\frac{3}{4}$ for a stove, and $\$125\frac{1}{4}$ for a wagon; how much money had he remaining?

11. If I pay my grocer $\$18\frac{3}{4}$, my coal-dealer $\$27\frac{1}{2}$, and my tailor $\$22\frac{1}{2}$, how much will I have left out of four twenty-dollar bills?

MULTIPLICATION OF FRACTIONS.

CASE I.

77. To multiply a fraction by an integer.

ORAL EXERCISES.

1. What is $\frac{1}{7} + \frac{1}{7} + \frac{1}{7}$? What, then, is $3 \times \frac{1}{7}$?

2. What is $\frac{2}{7} + \frac{2}{7} + \frac{2}{7}$? What, then, is $3 \times \frac{2}{7}$?

3. What is $\frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8}$? What, then, is $4 \times \frac{3}{8}$?

4. How may we multiply a fraction by an integer?

5. What is $\frac{5}{6} + \frac{5}{6} + \frac{5}{6}$ expressed in its lowest terms? *Ans.*, $\frac{5}{2}$. What, then, is $3 \times \frac{5}{6}$? How may we obtain this result from $\frac{5}{6}$?

6. In what other way, then, besides multiplying the numerator, may we multiply a fraction by an integer?

78. Principle.—*Multiplying the numerator or dividing the denominator of a fraction by any number, multiplies the value of the fraction by that number.*

What is—

7. $3 \times \frac{2}{7}$?
8. $4 \times \frac{1}{9}$?
9. $5 \times \frac{2}{11}$?
10. $6 \times \frac{3}{19}$?
11. $7 \times \frac{5}{43}$?
12. $4 \times \frac{7}{8}$?
13. $8 \times \frac{11}{24}$?
14. $9 \times \frac{14}{27}$?
15. $10 \times \frac{16}{40}$?
16. $12 \times \frac{19}{60}$?
17. $9 \times \frac{2}{5}$?
18. $9 \times \frac{13}{36}$?
19. $8 \times \frac{2}{7}$?
20. $8 \times \frac{7}{64}$?
21. $11 \times \frac{3}{121}$?

WRITTEN EXERCISES.

22. Multiply $\frac{8}{225}$ by 23.

Multiplying the numerator, we have $\frac{8}{225} \times 23 = \frac{184}{225}$, *Ans.*

23. Multiply $\frac{8}{225}$ by 25.

Dividing the denominator, we have $\frac{8}{225} \times 25 = \frac{8}{9}$, *Ans.*

24. Multiply $16\frac{2}{3}$ by 7.

PROCESS.

$16\frac{2}{3}$ Multiplying 16 by 7, we have 112.
$\underline{7}$ Multiplying $\frac{2}{3}$ by 7, we have $4\frac{2}{3}$.
112 Adding 112 and $4\frac{2}{3}$, we have $116\frac{2}{3}$.
$\underline{4\frac{2}{3}}$
$116\frac{2}{3}$, *Ans.*

NOTE.—Multiplying an integer by a fraction is the same as multiplying a fraction by an integer. Thus, $7 \times \frac{2}{3} = \frac{2}{3} \times 7$.

Cancel all common factors.

RULE.

Multiply the numerator or divide the denominator by the integer.

Multiply—

25. $\frac{25}{63}$ by 21.
26. $\frac{45}{88}$ by 33.
27. $\frac{60}{144}$ by 12.
28. $\frac{57}{58}$ by 11.
29. 24 by $\frac{28}{72}$.
30. 27 by $\frac{220}{1107}$.
31. $6\frac{5}{6}$ by 12.
32. $42\frac{2}{3}$ by 9.
33. $27\frac{15}{16}$ by 8.
34. $40\frac{3}{5}$ by 20.
35. $102\frac{5}{7}$ by 6.
36. $100\frac{3}{5}$ by 12.

37. 8 by $\frac{43}{80}$.
38. $\frac{5}{9}$ by 11.
39. 12 by $\frac{33}{16}$.
40. $\frac{15}{64}$ by 72.
41. 105 by $\frac{7}{85}$.
42. 35 by $15\frac{5}{7}$.
43. $16\frac{7}{9}$ by 81.
44. 151 by $4\frac{8}{15}$.
45. $16\frac{2}{3}$ by 64.
46. $20\frac{5}{7}$ by 48.
47. 156 by $3\frac{11}{18}$.
48. $4\frac{7}{8}$ by 140.
49. 184 by $\frac{25}{32}$.
50. 196 by $\frac{15}{28}$.
51. $5\frac{70}{81}$ by 729.

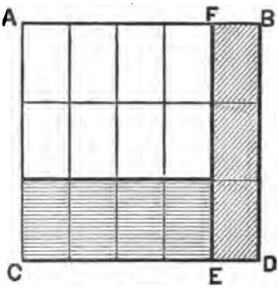
52. $16\frac{2}{5}$ by 6.
53. $33\frac{1}{3}$ by 15.
54. $102\frac{1}{2}$ by 17.
55. 28 by $72\frac{5}{14}$.
56. $81\frac{3}{64}$ by 40.
57. 23 by $11\frac{5}{46}$.
58. 62 by $101\frac{2}{31}$.
59. 17 by $17\frac{1}{2}$.
60. $18\frac{2}{9}$ by 14.
61. 132 by $17\frac{1}{2}$.
62. $65\frac{1}{2}$ by 13.
63. $11\frac{5}{6}$ by 22.
64. $65\frac{2}{11}$ by 3.
65. $18\frac{5}{9}$ by 9.
66. $17\frac{8}{21}$ by 21.

CASE II.

79. To multiply a fraction by a fraction.

1. Multiply $\frac{4}{5}$ by $\frac{2}{3}$.

To multiply $\frac{4}{5}$ by $\frac{2}{3}$ is to find $\frac{2}{3}$ of $\frac{4}{5}$. Let the square represent the unit divided into 15 equal parts. Then A F E C will represent $\frac{4}{5}$ of the unit, or 12 parts. $\frac{2}{3}$ of these $\frac{4}{5}$, or 12 parts, are 8 parts. One of these parts is $\frac{1}{15}$ of the unit, and 8 parts are $\frac{8}{15}$ of the unit. Hence $\frac{4}{5} \times \frac{2}{3}$ is $\frac{8}{15}$.



From this we derive the following

RULE.

Multiply the numerators together for the numerator of the product, and the denominators for the denominator.

The word "of" between two fractions is equivalent to the *sign of multiplication.* Thus, $\frac{2}{3}$ of $\frac{4}{5} = \frac{4}{5} \times \frac{2}{3}$. Such expressions are called *Compound Fractions.*

WRITTEN EXERCISES.

2. Multiply $\frac{3}{4}$ by $\frac{5}{7}$.

PROCESS.—$\frac{3}{4} \times \frac{5}{7} = \frac{15}{28}$, *Ans.*

3. Find the value of $\frac{3}{4} \times \frac{9}{15} \times \frac{11}{12} \times 1\frac{2}{3}$.

$$\text{PROCESS.—}\frac{\cancel{3}}{4} \times \frac{\overset{3}{\cancel{9}}}{\underset{\cancel{5}}{\cancel{15}}} \times \frac{11}{\underset{4}{\cancel{12}}} \times \frac{\cancel{5}}{\cancel{3}} = \frac{11}{16}, \textit{Ans.}$$

4. Multiply $3\frac{3}{7}$ by $1\frac{5}{11}$.

PROCESS.—$3\frac{3}{7} \times 1\frac{5}{11} = \frac{24}{7} \times \frac{16}{11} \times \frac{384}{77} = 4\frac{76}{77}$, *Ans.*

Reduce mixed numbers to improper fractions. Cancel all common factors.

Find the value of—

5. $\frac{8}{9} \times \frac{12}{20}$.
6. $\frac{15}{17} \times \frac{11}{27}$.
7. $\frac{11}{25} \times \frac{35}{44}$.
8. $\frac{21}{36} \times \frac{45}{49}$.
9. $\frac{15}{32} \times \frac{55}{56}$.
10. $\frac{20}{81} \times \frac{63}{50}$.
11. $\frac{11}{48} \times \frac{80}{99}$.
12. $\frac{2}{3} \times \frac{4}{5} \times \frac{9}{8}$.
13. $\frac{2}{3} \times \frac{18}{35} \times \frac{49}{60}$.
14. $\frac{7}{8} \times \frac{32}{126} \times \frac{65}{87}$.
15. $\frac{5}{9} \times \frac{6}{7} \times \frac{11}{12}$.
16. $\frac{81}{91} \times \frac{26}{27} \times \frac{133}{126}$.
17. $\frac{14}{15} \times \frac{49}{50} \times \frac{75}{76}$.
18. $\frac{48}{55} \times \frac{27}{85} \times \frac{102}{96}$.

19. $7\frac{7}{8} \times 5\frac{5}{8}$.
20. $5\frac{3}{4} \times 2\frac{2}{23}$.
21. $20\frac{1}{4} \times 5\frac{7}{9}$.
22. $12\frac{1}{2} \times 7\frac{2}{3}$.
23. $16\frac{2}{3} \times 5\frac{2}{7}$.
24. $62\frac{1}{2} \times 4\frac{4}{25}$.
25. $\frac{3}{4} \times \frac{5}{6} \times \frac{6}{7} \times \frac{7}{8}$.
26. $\frac{20}{121} \times \frac{33}{80} \times \frac{44}{96}$.
27. $\frac{24}{35} \times \frac{7}{15} \times \frac{15}{18} \times \frac{5}{12}$.
28. $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{6}{7}$ of $\frac{5}{8}$.
29. $\frac{15}{16}$ of $\frac{32}{35}$ of $\frac{14}{25}$.
30. $\frac{8}{9}$ of $\frac{72}{88}$ of $\frac{115}{120}$.
31. $\frac{24}{25}$ of $\frac{125}{144}$ of $\frac{168}{185}$.
32. $\frac{18}{24} \times \frac{26}{45} \times \frac{96}{112}$.

33. Multiply $\frac{4}{5}$ of $\frac{7}{12}$ by $\frac{8}{35}$ of $\frac{25}{32}$.
34. Multiply $4\frac{4}{5}$ of $\frac{9}{15}$ of $\frac{11}{12}$ by $6\frac{2}{3}$.
35. Multiply $\frac{24}{35}$ by $\frac{25}{16}$ of $\frac{16}{50}$ of $\frac{49}{75}$ of $\frac{57}{19}$.
36. Multiply $\frac{45}{51}$ of $\frac{17}{40}$ of $\frac{9}{25}$ by $\frac{10}{11}$ of $\frac{4}{25}$.
37. Multiply $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{6}{7}$ by $\frac{8}{9}$ of $\frac{10}{71}$ of $\frac{12}{13}$.

What is the cost of—

38. 8 tons of coal at \$$5\frac{3}{4}$ a ton?
39. $5\frac{1}{4}$ cords of wood at \$$6\frac{3}{4}$ a cord?
40. $\frac{2}{3}$ of a ton of hay at \$$5\frac{1}{4}$ a ton?
41. $6\frac{7}{9}$ barrels of flour at \$$4\frac{5}{6}$ a barrel?
42. $75\frac{1}{2}$ pounds of sugar at $6\frac{1}{3}$ cents a pound?
43. 20 cows at an average price of \$$31\frac{3}{4}$ a head?
44. 24 horses at an average price of \$$175\frac{5}{8}$ a head?
45. $21\frac{2}{3}$ yards of cloth at \$8 a yard?
46. 32 dozen of eggs at $28\frac{1}{2}$ cents a dozen?
47. 18 stoves at \$$34\frac{2}{5}$ apiece?
48. $16\frac{1}{2}$ tons of coal at \$$5\frac{1}{2}$ a ton?

49. There are $24\frac{3}{4}$ cubic feet in a perch of stone; how many cubic feet in $5\frac{3}{11}$ perches?

50. A sold $\frac{5}{6}$ of $150\frac{5}{8}$ cords of wood at $\$10\frac{2}{3}$ a cord; what did he receive for it?

51. B owns $\frac{7}{8}$ of a farm containing $112\frac{3}{4}$ acres; what is the value of B's share when land is selling at $\$91\frac{1}{2}$ an acre?

52. A miller bought $236\frac{5}{8}$ bushels of grain at 90 cents a bushel: $\frac{1}{3}$ of it he sold at $1.10 a bushel, $\frac{1}{5}$ of it at 95 cents a bushel, and the remainder at cost; how much did he gain?

DIVISION OF FRACTIONS.

CASE I.

80. To divide a fraction by an integer.

ORAL EXERCISES.

1. If 4 times $\frac{2}{3}$ is $\frac{8}{3}$, what is $\frac{8}{3}$ divided by 4?
2. If 5 times $\frac{3}{7}$ is $\frac{15}{7}$, what is $\frac{15}{7}$ divided by 5?
3. How, then, may we divide a fraction by an integer?
4. If 5 times $\frac{2}{15}$ is $\frac{2}{3}$, what is $\frac{2}{3}$ divided by 5?
5. If 6 times $\frac{5}{18}$ is $\frac{5}{3}$, what is $\frac{5}{3}$ divided by 6?
6. In what other way besides dividing the numerator may we divide a fraction by an integer?

81. Principle.—*Dividing the numerator or multiplying the denominator of a fraction by any number divides the value of the fraction by that number.*

What is—

7. $\frac{16}{7} \div 4$?
8. $\frac{22}{9} \div 11$?
9. $\frac{36}{7} \div 6$?
10. $\frac{72}{11} \div 8$?
11. $\frac{96}{13} \div 12$?
12. $\frac{108}{17} \div 12$?
13. $\frac{132}{15} \div 12$?
14. $\frac{5}{7} \div 4$?
15. $\frac{3}{5} \div 7$?
16. $\frac{2}{3} \div 9$?
17. $\frac{7}{9} \div 5$?
18. $\frac{8}{3} \div 7$?
19. $\frac{3}{4} \div 7$?
20. $\frac{5}{11} \div 3$?
21. $\frac{8}{7} \div 9$?

WRITTEN EXERCISES.

22. Divide $\frac{125}{7}$ by 25.

Dividing the numerator, we have

$$\tfrac{125}{7} \div 25 = \tfrac{5}{7}, \textit{ Ans.}$$

23. Divide $\frac{125}{7}$ by 26.

Multiplying the denominator, we have

$$\tfrac{125}{7} \div 26 = \tfrac{125}{182}, \textit{ Ans.}$$

24. Divide $586\frac{2}{3}$ by 7.

Process.

$7\underline{)586\frac{2}{3}}$

$83\frac{17}{21}$, *Ans.*

Dividing $586\frac{2}{3}$ by 7, we have 83 as a quotient, and $5\frac{2}{3}$, or $\frac{17}{3}$, remaining.

Dividing $\frac{17}{3}$ by 7, we have $\frac{17}{21}$. Hence the required result is $83\frac{17}{21}$.

RULE.

Divide the numerator or multiply the denominator.

Divide—

25. $\frac{16}{25}$ by 8.
26. $\frac{35}{49}$ by 5.
27. $\frac{121}{135}$ by 11.
28. $\frac{196}{201}$ by 14.
29. $\frac{216}{45}$ by 27.
30. $\frac{17}{19}$ by 11.
31. $25\frac{2}{3}$ by 6.
32. $87\frac{1}{2}$ by 12.
33. $42\frac{5}{7}$ by 18.
34. $53\frac{1}{3}$ by 17.
35. $37\frac{1}{2}$ by 15.
36. $62\frac{1}{2}$ by 25.
37. $\frac{18}{25}$ by 12.
38. $\frac{221}{17}$ by 21.
39. $\frac{27}{19}$ by 23.
40. $\frac{343}{75}$ by 35.
41. $\frac{160}{125}$ by 128.
42. $\frac{325}{67}$ by 300.
43. $\frac{184}{10}$ by 207.
44. $\frac{279}{101}$ by 341.
45. $862\frac{3}{5}$ by 5.
46. $936\frac{3}{8}$ by 7.
47. $1001\frac{5}{7}$ by 11.
48. $3854\frac{2}{3}$ by 12.
49. $2168\frac{4}{7}$ by 30.
50. $4285\frac{5}{13}$ by 45.
51. $2045\frac{7}{11}$ by 15.

52. Seven miners divided equally $141\frac{19}{20}$ ounces of gold; what was each man's share?

53. If a year of $365\frac{1}{4}$ days were divided into 12 months of equal length, what would be the length of each month?

CASE II.

82. To divide an integer or a fraction by a fraction.

WRITTEN EXERCISES.

1. Divide 2 by $\frac{5}{7}$.

PROCESS.—$2 \div \frac{5}{7} = 2 \times \frac{7}{5} = \frac{14}{5}$, or $2\frac{4}{5}$, *Ans.*

2 divided by 1 is 2; hence 2 divided by $\frac{1}{7}$ is 7 times 2, and 2 divided by $\frac{5}{7}$ is $\frac{1}{5}$ of 7 times 2, which is $\frac{7}{5}$ times 2, or $\frac{14}{5}$, or $2\frac{4}{5}$.

We may obtain the same result by *multiplying* 2 by $\frac{7}{5}$, which is the divisor inverted.

2. Divide $\frac{2}{3}$ by $\frac{5}{7}$.

PROCESS.—$\frac{2}{3} \div \frac{5}{7} = \frac{2}{3} \times \frac{7}{5} = \frac{14}{15}$, *Ans.*

$\frac{2}{3}$ divided by 1 is $\frac{2}{3}$; hence $\frac{2}{3}$ divided by $\frac{1}{7}$ is 7 times $\frac{2}{3}$, and $\frac{2}{3}$ divided by $\frac{5}{7}$ is $\frac{1}{5}$ of 7 times $\frac{2}{3}$, or $\frac{7}{5}$ times $\frac{2}{3}$, or $\frac{14}{15}$.

We may obtain the same result by *multiplying* $\frac{2}{3}$ by $\frac{7}{5}$, which is the divisor inverted; hence the

RULE.

Invert the divisor and proceed as in multiplication.

3. Divide $\frac{361}{512}$ by $\frac{19}{24}$.

PROCESS.—$\frac{361}{512} \div \frac{19}{24} = \frac{\overset{19}{\cancel{361}}}{\underset{64}{\cancel{512}}} \times \frac{\overset{3}{\cancel{24}}}{\cancel{19}} = \frac{57}{64}$, *Ans.*

4. Divide $18\frac{3}{4}$ by $9\frac{3}{8}$.

PROCESS.—$18\frac{3}{4} \div 9\frac{3}{8} = \frac{\cancel{75}}{\cancel{4}} \times \frac{\overset{2}{\cancel{8}}}{\cancel{75}} = 2$, *Ans.*

Divide—

5. 16 by $\frac{8}{9}$.
6. 220 by $\frac{22}{9}$.
7. 52 by $\frac{3}{13}$.
8. 87 by $\frac{14}{15}$.
9. 231 by $\frac{77}{80}$.
10. 216 by $2\frac{7}{10}$.
11. 606 by $\frac{8}{9}$.
12. 441 by $\frac{21}{22}$.
13. 660 by $4\frac{5}{7}$.

Divide—

14. $\frac{4}{11}$ by $\frac{5}{8}$.
15. $\frac{8}{13}$ by $\frac{4}{15}$.
16. $\frac{32}{63}$ by $\frac{28}{81}$.
17. $\frac{42}{72}$ by $\frac{18}{27}$.
18. $\frac{96}{121}$ by $\frac{27}{88}$.
19. $\frac{125}{147}$ by $\frac{110}{84}$.
20. $\frac{91}{101}$ by $\frac{21}{37}$.
21. $\frac{85}{121}$ by $\frac{17}{10}$.
22. $\frac{105}{144}$ by $\frac{245}{432}$.
23. 154 by $13\frac{4}{9}$.
24. 102 by $21\frac{6}{7}$.
25. 301 by $\frac{21}{113}$.
26. 1234 by $\frac{9}{10}$.
27. 205 by $18\frac{2}{9}$.
28. $\frac{324}{516}$ by $\frac{216}{624}$.
29. $\frac{196}{222}$ by $\frac{56}{370}$.
30. $\frac{169}{196}$ by $\frac{91}{140}$.
31. $32\frac{5}{6}$ by $18\frac{2}{3}$.
32. $125\frac{1}{2}$ by $62\frac{2}{5}$.
33. $35\frac{5}{9}$ by $12\frac{1}{27}$.
34. $108\frac{4}{7}$ by $14\frac{10}{21}$.
35. $2\frac{1}{81}$ by $8\frac{1}{3}$.
36. $21\frac{3}{11}$ by $5\frac{4}{9}$.
37. $172\frac{1}{2}$ by $8\frac{2}{3}$.

38. Divide $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{7}{8}$ by $\frac{17}{6}$ of $\frac{9}{21}$ of $\frac{42}{85}$.
39. Divide $\frac{3}{4}$ of $\frac{7}{9}$ of $\frac{16}{35}$ by $\frac{4}{5}$ of $\frac{9}{25}$ of $\frac{50}{66}$.
40. Divide $\frac{2}{3}$ of $\frac{25}{27}$ of $\frac{54}{75}$ by $\frac{7}{9}$ of $\frac{42}{1}$ of $\frac{81}{84}$.
41. Divide $\frac{12}{35}$ of $\frac{54}{27}$ of $\frac{105}{216}$ by $\frac{11}{45}$ of $\frac{105}{729}$ of $\frac{135}{99}$.

Required the price per lb., yd., etc.—

42. If $22\frac{1}{2}$ lb. of sugar cost \$1.40$\frac{5}{8}$.
43. If $75\frac{2}{3}$ yd. of cloth cost \$454.
44. If 102 sheep cost \$1183$\frac{1}{5}$.
45. If 218 tons of coal cost \$1471$\frac{1}{2}$.
46. If $47\frac{3}{4}$ lb. of flour cost \$1.52$\frac{4}{5}$.
47. If $16\frac{2}{3}$ barrels of flour cost \$70.
48. If $19\frac{5}{6}$ tons of hay cost \$136$\frac{17}{48}$.
49. If $25\frac{3}{4}$ bu. of apples cost \$41$\frac{1}{5}$.
50. If $113\frac{1}{3}$ yd. of calico cost \$5.44.
51. If $75\frac{4}{5}$ lb. of beef cost \$3.31$\frac{5}{8}$.
52. If $31\frac{5}{6}$ cords of wood cost \$191.
53. If 60 oranges cost \$1$\frac{1}{2}$.

54. What is the value of $\frac{2}{18}$ of $\frac{5}{12}$ of $\frac{54}{75}$ divided by $\frac{11}{12} \times \frac{13}{44} \times \frac{108}{130}$?

55. Divide $\frac{14}{25} \times \frac{16}{35} \times \frac{150}{144}$ by $\frac{8}{9} \times 5\frac{2}{5}$.

56. Divide $1\frac{4}{13} \times \frac{13}{17} \times 6$ by 6.

57. If $37\frac{1}{2}$ ounces of silver cost \$31$\frac{1}{4}$, what will 1 ounce cost? What will 150 ounces cost?

58. A earns $\$3\frac{1}{2}$ a day, and his expenses are \$11 a week; how many weeks must he labor to pay for 10 acres of land at \$30 an acre?

59. B sowed $63\frac{1}{3}$ bushels of oats on a field, averaging $3\frac{1}{3}$ bushels to the acre; what was the field worth, at $\$80\frac{4}{5}$ an acre?

60. How many times will a vessel whose capacity is $79\frac{3}{4}$ gallons hold the contents of each of two cans whose capacities are $5\frac{2}{3}$ and $8\frac{5}{6}$ gallons, respectively? and how many times will it hold the sum of their contents?

COMPLEX FRACTIONS.

83. A **Complex Fraction** is one whose numerator or denominator, or both, are fractional.

Thus, $\frac{\frac{2}{3}}{4}$, $\frac{4}{\frac{2}{3}}$, $\frac{\frac{2}{3}}{\frac{5}{7}}$, are complex fractions.

A complex fraction may be regarded as a problem in division of fractions, and solved by ART. 82.

1. Reduce $\frac{\frac{5}{8}}{\frac{11}{12}}$ to a simple fraction.

$\frac{\frac{5}{8}}{\frac{11}{12}}$ is the same as $\frac{5}{8} \div \frac{11}{12}$, which equals $\frac{5}{\underset{2}{\cancel{8}}} \times \frac{\overset{3}{\cancel{12}}}{11} = \frac{15}{22}$, *Ans.*

Another method of reducing a complex fraction to a simple one is to *multiply both numerator and denominator by the L. C. M. of the denominators.*

Thus, in the problem above, the L. C. M. of 8 and 12 is 24. Multiplying both terms of the fraction by 24, we have

$$\frac{\frac{5}{8}}{\frac{11}{12}} = \frac{\frac{5}{8} \times 24}{\frac{11}{12} \times 24} = \frac{15}{22}, \textit{ Ans.}$$

2. Reduce $\dfrac{\frac{2}{3} \text{ of } \frac{5}{7}}{\frac{3}{4} \text{ of } \frac{10}{21}}$ to a simple fraction.

PROCESS.

$$\frac{\frac{2}{3} \text{ of } \frac{5}{7}}{\frac{3}{4} \text{ of } \frac{10}{21}} = \frac{\frac{10}{21}}{\frac{5}{14}} = \frac{\frac{10}{21} \times 42}{\frac{5}{14} \times 42} = \frac{20}{15} = 1\tfrac{1}{3}, \textit{ Ans.}$$

Reduce—

3. $\dfrac{\frac{4}{15}}{\frac{7}{20}}$, $\dfrac{\frac{20}{33}}{\frac{14}{55}}$.

4. $\dfrac{\frac{112}{215}}{\frac{32}{86}}$, $\dfrac{\frac{45}{64}}{\frac{90}{96}}$.

5. $\dfrac{\frac{3}{4} \text{ of } \frac{12}{15}}{\frac{7}{8} \text{ of } \frac{16}{35}}$.

6. $\dfrac{\frac{2}{3} \text{ of } \frac{4}{5} \text{ of } \frac{18}{40}}{\frac{5}{6} \text{ of } \frac{9}{15} \text{ of } \frac{17}{30}}$.

7. $\dfrac{\frac{6}{16} \text{ of } \frac{11}{40} \text{ of } \frac{80}{88}}{\frac{8}{11} \text{ of } \frac{8}{13} \text{ of } \frac{44}{108}}$.

8. $\dfrac{\frac{14}{15} \times \frac{9}{32}}{\frac{15}{17} \times \frac{3}{16}}$.

9. $\dfrac{\frac{11}{15} \times \frac{75}{64}}{\frac{22}{40} \times \frac{15}{32}}$.

10. $\dfrac{\frac{16}{21} \times \frac{105}{9}}{\frac{18}{25} \times \frac{42}{45}}$.

11. $\dfrac{15\frac{2}{3}}{18\frac{5}{6}}$, $\dfrac{14\frac{1}{4}}{28\frac{1}{6}}$.

12. $\dfrac{\frac{2}{3} + \frac{5}{7}}{\frac{4}{24} + \frac{11}{42}}$.

13. $\dfrac{\frac{1}{2} + \frac{2}{3}}{\frac{3}{4} + \frac{5}{6}}$.

14. $\dfrac{\frac{9}{8} - \frac{5}{6}}{\frac{13}{12} - \frac{3}{4}}$.

FRACTIONAL PARTS OF NUMBERS.

CASE I.

84. To find what fractional part one number is of another.

1. What part of 24 is 8?

1 is $\frac{1}{24}$ of 24, and 8 is 8 times $\frac{1}{24}$, or $\frac{1}{3}$ of 24. Therefore 8 is $\frac{1}{3}$ of 24. From this solution we derive the following

RULE.

Divide the second number by the first.

What part—

2. Of 54 is 9?
3. Of 75 is 15?
4. Of 84 is 77?
5. Of 100 is 20?
6. Of 100 is 75?
7. Of 100 is 40?
8. Of 168 is 147?
9. Of 132 is 24?

10. What part of 6 is $\frac{3}{4}$?

1 is $\frac{1}{6}$ of 6, and $\frac{3}{4}$ is $\frac{3}{4}$ of $\frac{1}{6}$, or $\frac{1}{8}$ of 6. Therefore $\frac{3}{4}$ is $\frac{1}{8}$ of 6.

What part—

11. Of 8 is $\frac{3}{4}$?
12. Of 9 is $\frac{2}{3}$?
13. Of 12 is $\frac{6}{7}$?
14. Of 100 is $16\frac{2}{3}$?
15. Of 100 is $62\frac{1}{2}$?
16. Of 100 is $83\frac{1}{3}$?
17. Of 125 is $12\frac{1}{2}$?
18. Of 80 is $\frac{5}{12}$?

19. What part of $\frac{2}{3}$ is $\frac{5}{7}$?

$\frac{1}{3}$ is $\frac{1}{2}$ of $\frac{2}{3}$, and $\frac{3}{3}$ or 1 is 3 times $\frac{1}{2}$, or $\frac{3}{2}$ of $\frac{2}{3}$; and $\frac{5}{7}$ is $\frac{5}{7}$ times $\frac{3}{2}$, or $\frac{15}{14}$ of $\frac{2}{3}$. Therefore $\frac{5}{7}$ is $\frac{15}{14}$ of $\frac{2}{3}$.

What part—

20. Of $\frac{4}{7}$ is $\frac{5}{14}$?
21. Of $\frac{11}{12}$ is $\frac{7}{24}$?
22. Of $\frac{25}{48}$ is $\frac{5}{16}$?
23. Of $\frac{3}{16}$ is $\frac{3}{14}$?
24. Of $4\frac{1}{2}$ is $3\frac{3}{4}$?
25. Of $12\frac{4}{5}$ is 8?
26. Of $37\frac{1}{2}$ is $2\frac{1}{2}$?
27. Of $40\frac{2}{5}$ is $10\frac{1}{5}$?

CASE II.

85. To find a number when a fractional part is given.

1. 12 is $\frac{1}{6}$ of what number?

If $\frac{1}{6}$ of the required number is 12, 6 sixths, or the number, is 6 times 12, or 72.

From this solution we derive the following

RULE.

Divide the first number by the second.

2. 120 is $\frac{1}{5}$ of what number?
3. 150 is $\frac{1}{8}$ of what number?
4. 62 is $\frac{1}{12}$ of what number?
5. $12\frac{1}{2}$ is $\frac{1}{8}$ of what number?
6. $37\frac{1}{2}$ is $\frac{1}{12}$ of what number?
7. $\frac{5}{6}$ is $\frac{1}{20}$ of what number?
8. 60 is $\frac{2}{3}$ of what number?
9. 120 is $\frac{4}{5}$ of what number?
10. 17 is $\frac{8}{9}$ of what number?
11. 101 is $\frac{7}{8}$ of what number?
12. $\frac{4}{5}$ is $\frac{11}{12}$ of what number?
13. $4\frac{1}{2}$ is $\frac{2}{3}$ of what number?
14. $18\frac{3}{5}$ is $\frac{31}{32}$ of what number?
15. $66\frac{2}{3}$ is $\frac{2}{3}$ of what number?
16. $62\frac{1}{2}$ is $\frac{5}{8}$ of what number?
17. $87\frac{1}{2}$ is $\frac{7}{8}$ of what number?

ARITHMETICAL ANALYSIS.

WRITTEN EXERCISES.

86.—*1.* If 9 oranges cost $22\frac{1}{2}$ cents, what will 13 oranges cost?

PROCESS.
9 cost $22\frac{1}{2}$ cents,
1 cost $2\frac{1}{2}$ cents,
13 cost $13 \times 2\frac{1}{2} = 32\frac{1}{2}$ cents.

If 9 oranges cost $22\frac{1}{2}$ cents, 1 orange will cost $\frac{1}{9}$ of $22\frac{1}{2}$ cents, or $2\frac{1}{2}$ cents; and 13 oranges will cost 13 times $2\frac{1}{2}$ cents, or $32\frac{1}{2}$ cents.

2. If 7 bushels of apples cost $\$4\frac{2}{3}$, what will 11 bushels cost?

3. If 11 yards of calico cost $48\frac{2}{5}$ cents, what will 33 yards cost?

4. What will 15 hats cost, if 12 hats cost $\$40\frac{4}{5}$?

5. If 16 cows cost $\$320\frac{1}{5}$, what will 24 cows cost at the same rate?

6. What will 9 barrels of flour cost, if 18 barrels cost $\$113\frac{1}{7}$?

7. What will 100 pigs cost, if 42 pigs cost \$$136\frac{1}{2}$?

8. If $\frac{4}{7}$ of a barrel of oil cost \$3.20, what will a barrel cost?

9. If $\frac{7}{8}$ of a ton of coal cost \$7.70, what will a ton cost?

10. If $\frac{4}{5}$ of the cost of a horse is \$120, what will 12 horses cost at the same rate?

11. What will 21 acres of arable land cost, if $4\frac{1}{2}$ acres cost \$360?

12. M bought 32 cords of wood at the rate of $6\frac{2}{3}$ cords for \$44; what did it cost?

13. What will 80 bushels of potatoes cost at the rate of $12\frac{1}{2}$ bushels for \$10?

14. What will a farm of 120 acres cost at the rate of 8 acres for \$$515\frac{1}{5}$?

15. If 10 yards of muslin cost \$1.20, what will $\frac{2}{3}$ of a yard cost?

16. If 8 barrels of apples cost \$48, what will $\frac{3}{4}$ of a barrel cost?

17. If 12 pounds of beef cost $62\frac{1}{2}$ cents, what will 60 pounds cost?

18. What must I pay for $42\frac{2}{5}$ tons of coal, if $12\frac{1}{3}$ tons cost \$37?

19. What will $100\frac{2}{3}$ bushels of wheat cost at the rate of 5 bushels for \$$6\frac{2}{3}$?

20. If $\frac{4}{5}$ of an acre of land cost \$80, what will $\frac{7}{15}$ of an acre cost?

21. If $6\frac{2}{3}$ pounds of sugar cost $40\frac{5}{6}$ cents, what will $8\frac{3}{4}$ pounds cost?

22. A sold $\frac{2}{7}$ of his land, and had 415 acres remaining; how many acres had he at first?

23. B withdrew $\frac{5}{7}$ of his money from bank, and still had \$4160 on deposit; what was his original deposit?

24. C sold $\frac{3}{10}$ of his cattle, and had 161 remaining; how many had he at first?

MISCELLANEOUS EXAMPLES.

1. Reduce $\frac{24374}{315}$ to a mixed number.
2. Reduce $421\frac{11}{91}$ to an improper fraction.
3. Reduce $\frac{1888}{1952}$ to its lowest terms.
4. Arrange in order of magnitude $\frac{7}{9}$, $\frac{5}{6}$, $\frac{11}{14}$.
5. Multiply $\frac{12}{35}$ by 17.
6. Divide $4\frac{2}{3}$ by 22.
7. Add $\frac{1}{2}$, $\frac{8}{11}$, $\frac{16}{33}$, and $\frac{13}{42}$.
8. Subtract $182\frac{7}{8}$ from $347\frac{13}{14}$.
9. Find the value of $3\frac{2}{3} \times 4\frac{1}{6} \times 4\frac{2}{11}$.
10. Reduce $\frac{568}{639}$ to its lowest terms.

11. Simplify $\dfrac{24 \times 33 \times 65}{363 \times 8 \times 169}$.

12. Find the value of $(8\frac{1}{2} - 2\frac{1}{3}) \times 8\frac{2}{7}$.
13. Add $8\frac{1}{3}$, $9\frac{1}{4}$, $10\frac{1}{6}$, and $11\frac{5}{8}$.
14. Divide $14\frac{5}{6}$ by $7\frac{5}{8}$.
15. Subtract $1009\frac{11}{12}$ from $2001\frac{13}{14}$.

16. Reduce $\dfrac{\frac{2}{3} + \frac{5}{6}}{\frac{3}{4} \text{ of } \frac{8}{9}}$ to a simple fraction.

17. Reduce 35 to a fraction having 101 for a denominator.
18. Reduce $\frac{12827}{12928}$ to its lowest terms.
19. Reduce $625\frac{4}{7}$ to an improper fraction.
20. Reduce $\frac{17}{18}$ to 1098ths.
21. What is the value of $4\frac{7}{9} \div (16\frac{5}{8} - 14\frac{8}{9})$?
22. Multiply $\frac{80}{343}$ of $4\frac{6}{23}$ of $6\frac{2}{15}$ by $\frac{17}{32}$.
23. $\frac{2}{3}$ of $\frac{8}{9}$ is $\frac{16}{17}$ of what number?
24. Divide $14\frac{4}{7}$ by 17.
25. Divide 126 by $13\frac{5}{7}$.
26. Reduce $402\frac{9}{22}$ to an improper fraction.
27. $\frac{15}{16}$ is $\frac{2}{5}$ of $\frac{25}{42}$ of what number?
28. Reduce $\frac{2}{3}$, $\frac{8}{9}$, $\frac{11}{12}$, $\frac{12}{13}$, and $\frac{5}{26}$ to a common denominator.

29. Simplify $\dfrac{4\frac{1}{2} + 3\frac{5}{8} - 2\frac{1}{3}}{7\frac{1}{3} - 3\frac{3}{4} + 2}$.

30. Add $\dfrac{7\frac{5}{8}}{4\frac{3}{8}}$, $5\frac{6}{7}$, $\frac{4}{5}$, and 4.

31. Multiply $14\frac{3}{7}$ by $\dfrac{6\frac{2}{5}}{8\frac{4}{5}}$.

32. Divide 72 by $\frac{2}{3}$ of $\frac{9}{22}$ of $\frac{11}{16}$.
33. Reduce $\frac{5720}{47}$ to a mixed number.
34. Add $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{5}{6}$.
35. Subtract $\frac{4}{5}$ of $18\frac{5}{6}$ from 100.
36. Multiply $19\frac{3}{5}$ by $67\frac{9}{12}$.
37. Divide $12\frac{1}{2}$ by $31\frac{1}{4}$.
38. Add $37\frac{5}{6}$, $43\frac{4}{9}$, $57\frac{3}{4}$, and $\frac{35}{36}$.
39. Reduce $816\frac{5}{11}$ to an improper fraction.
40. Find the value of $(16\frac{2}{5} + 14\frac{1}{4}) \times \frac{7}{9}$.
41. Add $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{9}{10}$, and $\frac{11}{12}$.
42. Subtract $861\frac{5}{12}$ from $1006\frac{2}{41}$.
43. Multiply $1020\frac{5}{6}$ by $127\frac{8}{9}$.
44. Divide $402\frac{11}{12}$ by $27\frac{4}{9}$.
45. Reduce $\frac{15}{6}$ to 786ths.
46. $\frac{5}{7}$ of $4\frac{2}{3}$ is $\frac{5}{8}$ of what number?
47. What is $\frac{5}{8}$ of $\frac{5}{7}$ of $4\frac{2}{3}$?

48. Simplify $\dfrac{18\frac{3}{5}}{4\frac{7}{10}}$.

49. Simplify $\dfrac{42 \times 18 \times 75 \times 6}{25 \times 17 \times 14 \times 9}$.

50. Simplify $\dfrac{3\frac{1}{5} + 6\frac{2}{3} - 7\frac{5}{6} + 5\frac{1}{2}}{4\frac{1}{15} - 2\frac{5}{8} + 6\frac{3}{4} - 1\frac{5}{6}}$.

51. Simplify $\dfrac{(3\frac{1}{2} \times 2\frac{5}{14}) + (6\frac{2}{3} \times \frac{33}{40})}{(16\frac{2}{3} \div 1\frac{1}{3}) - (8\frac{2}{5} \div \frac{21}{25})}$.

MISCELLANEOUS PROBLEMS.

1. What cost $16\frac{1}{2}$ yards of cloth at \$$5\frac{1}{2}$ a yard?

2. At $22\frac{1}{2}$ cents a pound, how many pounds of coffee can be bought for \$47.85?

3. What cost $45\frac{4}{5}$ acres of land at \$$62\frac{1}{2}$ an acre?

4. At $3\frac{1}{4}$ cents a foot, how many feet of boards can be bought for \$36.53?

5. What will a steer which weighs $1112\frac{1}{4}$ pounds cost at $4\frac{2}{3}$ cents a pound?

6. If 40 barrels of apples cost \$$273\frac{1}{3}$, what will 1 barrel cost?

7. At $28\frac{1}{2}$ cents a dozen, what will 650 eggs cost?

8. At \$$13\frac{3}{10}$ a ton, how many tons of hay can be bought for \$275?

9. If $\frac{6}{7}$ of a cord of wood cost \$$6\frac{4}{5}$, what will 10 cords cost?

10. How many pounds of butter, at $32\frac{1}{2}$ cents a pound, must be given for $37\frac{1}{2}$ pounds of sugar, at 6 cents a pound?

11. A sold his house for \$3500, and thus lost $\frac{1}{5}$ of the cost; what was the cost?

12. A farm was sold for \$8400, which was at a gain of $\frac{1}{6}$ of the cost; what was the cost?

13. A horse which cost \$160 was sold for \$120; what part of the cost was lost?

14. A lot which cost \$400 was sold for \$480; what part of the cost was gained?

15. The circumference of the hind wheel of a carriage is $10\frac{1}{2}$ feet, and of the fore wheel $7\frac{5}{7}$ feet; how many revolutions will each wheel make in going a mile (5280 feet)?

16. A and B can do a piece of work in 12 days, and B alone can do it in 20 days; how long will it take A to do it?

17. A cistern can be filled by one pipe in 15 hours, and by another in 20 hours; in what time can the two pipes fill it, flowing together?

18. B owned $\frac{2}{3}$ of a farm, and sold $\frac{4}{5}$ of his share for $6240; what was the farm worth?

19. C's house and barn cost $8425; what was the cost of each if the barn cost $\frac{3}{5}$ as much as the house?

20. A miller sold $\frac{4}{5}$ of all his wheat, and the next day bought 200 bushels; he then found that he had $\frac{2}{5}$ as much as at first; how much had he at first?

21. If a pole $12\frac{1}{2}$ feet long cast a shadow $9\frac{3}{4}$ feet long, how long must a pole be to cast a shadow 21 feet long at the same hour of the day?

22. By what number must $25\frac{4}{5}$ be multiplied so that $\frac{3}{4}$ of the product may be $52\frac{1}{2}$?

23. A fortune was left under the following conditions: A was to receive $\frac{1}{3}$ of it, B $\frac{1}{3}$ of the remainder, and C $\frac{1}{3}$ of what then remained. It was found that A received $10,000 more than C; what was the fortune?

24. A miller bought wheat at $65\frac{3}{4}$ cents a bushel, and sold it at $75\frac{1}{2}$ cents a bushel, gaining $117 by the transaction; how many bushels did he buy and sell?

25. My salary is $\$133\frac{1}{3}$ per month, and my monthly expenses amount to $150; I have previously saved $300; if I take each month from my savings enough to make up the deficiency, how long will they last?

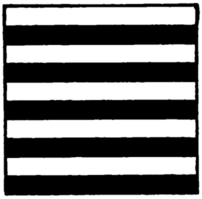
26. A merchant invested $\frac{1}{2}$ of his capital in groceries, $\frac{1}{2}$ of the remainder in dry goods, $\frac{1}{2}$ of what then remained in hardware, $\frac{1}{2}$ of what then remained in notions, and the balance in glassware; what was his capital, if he invested $700 more in groceries than in glassware?

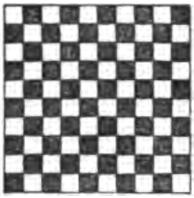
27. A Western farmer, being asked how many sheep he had, said his sheep were in four fields: in the first field there were 108, which was $\frac{3}{5}$ of the number in the second; the number in the second was $\frac{3}{5}$ of the number in the third; and the number in the third was $\frac{3}{5}$ of the number in the fourth; how many sheep had he?

SECTION V.

DECIMAL FRACTIONS.

DEFINITIONS.





1. If a square inch is divided into ten equal parts, what is one of these parts called? What are 2 of these parts called? 3 parts? 7 parts? 9 parts?

2. If one-*tenth* of an inch is divided into ten equal parts, what is one of these parts called? What are 2 of these parts called? 12 parts? 65 parts?

3. If one-*hundredth* of an inch is divided into ten equal parts, what is one of these parts called? What are 8 of these parts called? 15 parts? 125 parts?

4. What is $\frac{1}{10}$ of $\frac{1}{10}$? $\frac{1}{10}$ of $\frac{1}{100}$?

87. These expressions, *one-tenth* ($\frac{1}{10}$), *one-hundredth* ($\frac{1}{100}$), *eight-thousandths* ($\frac{8}{1000}$), *one ten-thousandth* ($\frac{1}{10000}$), etc., are called **Decimal Fractions.**

Decimal fractions are thus seen to be only a special kind of common fractions.

88. A **Decimal Fraction** is usually expressed by placing a point (.), called the *decimal point*, before the numerator, and omitting the denominator. Thus,

$\frac{3}{10} = .3$, $\frac{5}{1000} = .005$,
$\frac{4}{100} = .04$, $\frac{6}{10000} = .0006$,
$\frac{127}{100000} = .00127$, $\frac{348}{1000000} = .000348$, etc.

89. In the common system of notation the value of figures decreases from left to right in a tenfold ratio. If this law be continued below units, the *first* place at the right of units will express *tenths*, the *second* place *hundredths*, the *third* place *thousandths*, etc.

From this it appears that *decimals* are not only closely connected with common fractions, but are also a continuation of the common system of notation. This is more clearly shown in the following

NUMERATION TABLE.

Millions.	Hundred thousands,	Ten thousands,	Thousands,	Hundreds,	Tens,	Units,		Tenths,	Hundredths,	Thousandths,	Ten-thousandths,	Hundred-thousandths,	Millionths.	Is read,
4	0	0	0	0	0	0	.							
	4	0	0	0	0	0	.							
		4	0	0	0	0	.							
			4	0	0	0	.							
				4	0	0	.							
					4	0	.							
						4	.							
							.	4						4 Tenths,
							.	0	4					4 Hundredths,
							.	0	0	4				4 Thousandths,
							.	0	0	0	4			4 Ten-thousandths,
							.	0	0	0	0	4		4 Hundred-thousandths,
							.	0	0	0	0	0	4	4 Millionths.
4	4	4	4	4	4	4	.	4	4	4	4	4	4,	etc.

Integers. Decimals.

90. **A Pure Decimal** is one which consists of decimal figures only; as, .4, .25, etc.

A Mixed Decimal is one which consists of an integer and a decimal; as, 4.25.

A Complex Decimal is one which has a common fraction at the right of the decimal; as, .3$\frac{1}{3}$.

NUMERATION OF DECIMALS.

91. In reading a decimal, first read the *integral* part, if any; then the *decimal* part regarded as an integer; and, third, the name of the right-hand decimal place. To prevent ambiguity, pause after reading each part.

Thus, 5.408 is read five——and four hundred and eight——thousandths.

400.004 is read four hundred——and four——thousandths; and .404 is read four hundred and four——thousandths.

Read the following decimals:

1. .5.
2. .8.
3. .12.
4. .64.
5. .96.
6. .437.
7. .525.
8. .06.
9. .008.
10. .4267.
11. .0261.
12. .0028.
13. .0009.
14. 1.0001.
15. 300.003.
16. .303.
17. 600.005.
18. .605.
19. 42.06.
20. 125.125.
21. 1.205.
22. .000325.
23. 100.001.
24. .000035.
25. 1.2345.
26. .46$\frac{2}{3}$.
27. .085$\frac{1}{3}$.
28. .0$\frac{3}{5}$.

29. Read the following, and notice the relative position of *tens* and *tenths*, *hundreds* and *hundredths*, etc.:

10.1; 100.01; 100.001; 10000.0001; 1000000.000001.

30. Read the following, and notice the effect of moving the decimal point one, two, three, or more places to the left:

516; 51.6; 5.16; .516; .0516; .00516.

31. Read the following, and notice the effect of moving the decimal point one, two, three, or more places to the right:

.4862; 4.862; 48.62; 486.2; 4862.

32. Read the following, and notice the effect of placing one, two, three, or more zeros between the decimal point and the first decimal figure:

26.4; 26.04; 26.004; 26.0004; 26.00004.

33. Read the following, and notice the effect of placing zeros on both sides of the decimal point:

3.3; 30.03; 300.003; 3000.0003.

34. Read the following, and notice the similarity of sound and difference of value:

404000; 400.004; .404; 303000; 300.003; .303.

NOTATION OF DECIMALS.

92. In writing a decimal, *write the numerator as a whole number, and then place the decimal point so that the right-hand figure expresses the denomination required.*

Write in figures—

1. Four tenths. Six tenths. Nine tenths.
2. Five hundredths. Forty-five hundredths.
3. Seven thousandths. Seventy-five thousandths.
4. Eighty-five hundredths. Eight, and five tenths.
5. Sixty, and six hundredths. 125 thousandths.
6. Three hundred and four ten-thousandths.
7. Five, and fifteen thousandths. One millionth.
8. Ninety, and nine hundredths. Four millionths.
9. One hundred and twenty, and four millionths.
10. Sixty-five hundred, and sixty-five hundredths.
11. Seventy-five thousand, and seventy-five thousandths.

12. Three hundred, and three thousandths.
13. Three hundred and three thousandths.
14. Eight tens. Eight units. Eight tenths.
15. Four hundred. Four tens. Four tenths.
16. Six thousand. Six thousandths.
17. Twenty-five tens. Twenty-five tenths.
18. Twenty-five hundred. Twenty-five hundredths.
19. 325 hundredths. 325 tenths.
20. 125 ten-thousandths. 125 millionths.
21. 4211 hundredths. 4211 thousandths.
22. 16, and 25 thousandths.
23. 9, and 9 ten-millionths.
24. 1007 millionths. 14 ten-millionths.
25. Four and one-half tenths.
26. Twenty-four and two-thirds hundredths.
27. One-half tenth. One-third hundredth.

Write the following as decimals:

28. $\frac{4}{10}$.
29. $\frac{25}{100}$.
30. $\frac{625}{1000}$.
31. $86\frac{125}{1000}$.
32. $\frac{625}{10}$.
33. $\frac{4216}{100}$.
34. $\frac{16\frac{2}{3}}{100}$.
35. $\frac{33\frac{1}{3}}{100}$.
36. $216\frac{6}{1000}$.
37. $125\frac{125}{1000}$.
38. $8\frac{16}{1000000}$.
39. $\frac{162}{10000000}$.

Write the following as fractions:

40. .5.
41. .05.
42. .005.
43. .0005.
44. .42835.
45. 4.2835.
46. 42.835.
47. 428.35.
48. 2000.6.
49. 200.06.
50. 20.006.
51. 2.0006.
52. $.33\frac{1}{3}$.
53. $.66\frac{2}{3}$.
54. $.16\frac{2}{3}$.
55. $.37\frac{1}{2}$.

93. Principles.—1. *Annexing ciphers to a decimal does not alter its value.*

Take as an example $.3 = \frac{3}{10}$.

.3 becomes .30 $= \frac{30}{100}$ by annexing one cipher.
.3 becomes .300 $= \frac{300}{1000}$ by annexing two ciphers, etc.

Hence the value is not changed.

2. *Prefixing ciphers to a decimal diminishes its value ten times for every cipher prefixed.*

Take as an example .3 = $\frac{3}{10}$.

.3 becomes .03 = $\frac{3}{100}$ by prefixing one cipher.
.3 becomes .003 = $\frac{3}{1000}$ by prefixing two ciphers, etc.

Hence the value is diminished ten times for each cipher prefixed.

3. *Moving the decimal point to the right increases the value of the decimal ten times for every place passed over.*

Take as an example .333 = $\frac{333}{1000}$.

.333 becomes 3.33 = $\frac{333}{100}$ by moving the decimal point one place.
.333 becomes 33.3 = $\frac{333}{10}$ by moving it two places, etc.

Hence the value is increased ten times for each place passed over.

4. *Moving the decimal point to the left diminishes the value of the decimal ten times for every place passed over.*

Take as an example 33.3 = $\frac{333}{10}$.

33.3 becomes 3.33 = $\frac{333}{100}$ by moving the decimal point one place.
33.3 becomes .333 = $\frac{333}{1000}$ by moving it two places, etc.

Hence the value is diminished ten times for each place passed over.

REDUCTION OF DECIMALS.

CASE I.

94. To reduce a decimal to a common fraction.

WRITTEN EXERCISES.

1. Reduce .75 to a common fraction.

PROCESS.—.75 = $\frac{75}{100}$ = $\frac{3}{4}$, *Ans.*

RULE.

Express the decimal as a common fraction, and reduce it to its lowest terms.

Reduce the following decimals to common fractions:

2. .25.
3. .85.
4. .64.
5. .375.
6. .625.
7. .875.
8. 10.10.
9. 4.3125.
10. 12.9375.
11. 40.3750.
12. 2.00125.
13. 4.0125.

14. Reduce $.16\frac{2}{3}$ to a common fraction.

PROCESS.—$.16\frac{2}{3} = \frac{16\frac{2}{3}}{100} = \frac{1}{6}$, *Ans.*

15. $.3\frac{1}{3}$.
16. $.66\frac{2}{3}$.
17. $.18\frac{3}{4}$.
18. $4.35\frac{1}{3}$.
19. $5.05\frac{5}{6}$.
20. $6.1\frac{1}{3}$.
21. $22.0\frac{1}{3}$.
22. $41.4\frac{1}{6}$.
23. $48.7\frac{7}{9}$.
24. $25.83\frac{1}{3}$.
25. $81.46\frac{2}{3}$.
26. $42.09\frac{1}{11}$.
27. 14.65.
28. .0005.
29. 1.245.
30. .0725.
31. .0225.
32. $.625\frac{1}{3}$.
33. $4.37\frac{1}{2}$.
34. $4.8\frac{1}{3}$.
35. $.03\frac{4}{5}$.
36. $4.00\frac{2}{3}$.
37. $17.00\frac{4}{5}$.
38. $8.66\frac{2}{3}$.
39. $12.04\frac{1}{6}$.
40. $26.03\frac{1}{3}$.
41. 75.75.
42. 400.875.
43. 375.375.
44. 112.0625.
45. 2.0025.
46. $45.45\frac{5}{6}$.

CASE II.

95. To reduce a common fraction to a decimal.

WRITTEN EXERCISES.

1. Reduce $\frac{3}{8}$ to a decimal.

PROCESS.
8)3.000
.375, *Ans.*

$\frac{3}{8}$ equals $\frac{1}{8}$ of 3.

3 equals 30 tenths; $\frac{1}{8}$ of 30 tenths is 3 tenths, and 6 tenths remaining; 6 tenths equals 60 hundredths; $\frac{1}{8}$ of 60 hundredths is 7 hundredths, and 4 hundredths remaining; 4 hundredths equals 40 thousandths; $\frac{1}{8}$ of 40 thousandths is 5 thousandths. Hence $\frac{3}{8}$ equals .375.

RULE.

Annex ciphers to the numerator and divide by the denominator; point off as many decimal places as there are ciphers annexed.

Reduce the following fractions to decimals:

2. $\frac{1}{2}$.
3. $\frac{1}{4}$.
4. $\frac{3}{4}$.
5. $\frac{1}{8}$.
6. $\frac{3}{8}$.
7. $\frac{5}{8}$.
8. $\frac{7}{8}$.
9. $\frac{5}{16}$.
10. $\frac{12}{25}$.
11. $\frac{15}{16}$.
12. $\frac{19}{250}$.
13. $\frac{3}{16}$.
14. $\frac{16}{25}$.
15. $\frac{1}{3}$.
16. $\frac{2}{5}$.
17. $\frac{6}{25}$.
18. $\frac{7}{16}$.
19. $\frac{1}{6}$.
20. $\frac{17}{32}$.
21. $5\frac{1}{4}$.
22. $9\frac{3}{4}$.
23. $7\frac{7}{8}$.
24. $18\frac{3}{5}$.
25. $9\frac{9}{25}$.
26. $4\frac{3}{7}$.
27. $5\frac{2}{3}$.
28. $7\frac{7}{10}$.
29. $8\frac{1}{8}$.
30. $9\frac{1}{9}$.
31. $75\frac{3}{16}$.
32. $4\frac{27}{64}$.
33. $17\frac{17}{125}$.
34. $42\frac{21}{32}$.
35. $105\frac{7}{625}$.
36. $\frac{111}{256}$.
37. $16\frac{1}{64}$.
38. $4.00\frac{8}{5}$.
39. $1.32\frac{5}{7}$.
40. $6.00\frac{31}{8}$.
41. $2.00\frac{12}{5}$.
42. $4.062\frac{1}{2}$.
43. $8.70\frac{3}{4}$.
44. $1.00\frac{1}{2}$.
45. $40.00\frac{17}{5}$.

ADDITION OF DECIMALS.

WRITTEN EXERCISES.

96.—*1.* Add 4.26, 12.005, and 126.4.

PROCESS.

4.26
12.005
126.4
142.665

We write the numbers so that their decimal points are in the same vertical column, and add as in whole numbers, placing the decimal point between units and tenths.

Add—

2. 28.32, 47.16, 35.08, 9.9, 12.001.
3. 126.2, 26.37, 7.425, .6243, 250.1256.
4. .7802, 6.321, 89.42, 172.8, 4216.
5. 40.002, 7.08, 123.4567, 400.005.
6. .9374, 13.21, 45.135, 1.0006, 82.16.
7. 4.2, 16.73, 141.154, 7832.1654.

	12.		*13.*		*14.*		*15.*		*16.*
8.	4.81	+	123.478	+	120.042	+	75.4	=	
9.	63.45	+	2.3	+	74.6	+	62.04	=	
10.	100.027	+	52.67	+	634.842	+	4.7	=	
11.	53.2	+	40.732	+	3.5	+	432.683	=	

17. Find the sum of 9 tenths, 18 hundredths, 125 thousandths, 62 ten-thousandths, 165 tenths, and 125 thousand.

18. Find the sum of five, and twenty-four hundredths, eight, and sixty-two ten-thousandths, ninety, and forty-one millionths, two hundred and five, and one thousand three hundred and seven ten-millionths.

19. A wagon cost $25.37, a horse $180.90, a set of harness $10.05, a mowing-machine $85.37, a cow $28.08, and a pair of mules $225.75; what was the entire cost?

20. What is the sum of $25 and 18 cents, $45 and 27 cents, $610 and 39 cents, $250 and 81 cents, and $1001 and 57 cents?

SUBTRACTION OF DECIMALS.

WRITTEN EXERCISES.

97.—*1.* Subtract 129.486 from 726.278.

PROCESS.	
726.278 129.486 596.792, *Ans.*	We write the numbers so that their decimal points are in the same vertical column, and subtract as in whole numbers, placing the decimal point between units and tenths.

2.	*3.*	*4.*	*5.*
86.71	82.19	126.87	248.0072
5.03	14.21	12.687	39.8979

6.	*7.*	*8.*	*9.*
27.004	12.004	18.	421.
8.2106	1.76234	2.0067	26.070809

10. From 876.472 take 162.587.
11. From 800.719 take 49.62.
12. From 125.6 take 42.3478.
13. From 3.07 take 2.187654.
14. From 10 take 9.768745.
15. From 10.001 take 9.000001.

16. 42.18 − 31.27683 =

17. 40.04 − 31.00202 =

18. .0017 − .000218 =

19. 6.39 − 6.3899 =

20. 4.28 − 3.002786 =

21. 132.06 − 100.0027 =

22. 400. − 362.0041 =

23. 216. − .321862 =

24. From 16 hundredths take 16 millionths.

25. From 10.32 take 17 ten-millionths.

26. From four hundred, and four thousandths take four hundred and four thousandths.

27. A farmer bought 102.26 acres of land from A, 140.216 acres from B, 47.025 acres from C, and enough from D to make in all 400 acres; how much did he buy from D?

28. A coal-dealer bought 1020 tons of coal: he sold A 40.025 tons, B 105.008 tons, C 208.125 tons, D 416.375 tons, and E 50.015 tons; how much was left?

MULTIPLICATION OF DECIMALS.

ORAL EXERCISES.

98.—*1.* How much is $3 \times \frac{1}{10}$? $4 \times .2$? $3 \times .3$?

2. How much is $\frac{1}{10} \times \frac{1}{10}$? $.2 \times .2$? $.3 \times .3$?

3. How much is $\frac{1}{10} \times \frac{1}{100}$? $.2 \times .03$? $.4 \times .06$?

4. How much is $\frac{1}{10} \times \frac{1}{1000}$? $.2 \times .002$? $.6 \times .003$?

5. How much is $\frac{1}{100} \times \frac{1}{100}$? $.02 \times .03$? $.06 \times 04$?

6. When tenths are multiplied by tenths, how many decimal places are there in the product? Tenths by hundredths? Tenths by thousandths? Hundredths by hundredths?

7. $3 \times .2$?

8. $6 \times .4$?

9. $7 \times .3$?

10. $8 \times .6$?

11. $.2 \times .3$?

12. $.5 \times .7$?

13. $.9 \times .8$?

14. $.7 \times .6$?

15. $.7 \times .001$?

16. $.6 \times .002$?

17. $.8 \times .006$?

18. $.9 \times .004$?

19. $.02 \times .03$?

20. $.06 \times .09$?

21. $.08 \times .07$?

22. $.04 \times .09$?

99. ***The product of two decimals contains as many decimal places as there are decimal places in both factors.***

WRITTEN EXERCISES.

1. Multiply 24.6 by .24.

PROCESS.

$$\begin{array}{r} 24.6 \\ \underline{.24} \\ 984 \\ \underline{492} \\ 5.904, \end{array}$$ *Ans.*

1ST. 246 multiplied by 24 is 5904, but tenths multiplied by hundredths give thousandths; hence the result is 5904 thousandths, or 5.904.

2D. $24.6 \times .24 = \frac{246}{10} \times \frac{24}{100} = \frac{5904}{1000} = 5.904$.

RULE.

Multiply as in whole numbers, and point off as many decimal places in the product as there are in both factors.

Prefix ciphers to the product if it does not contain the required number of figures.

2. Multiply 42.8 by .31.
3. Multiply 4.26 by 2.8.
4. Multiply 62.85 by 3.15.
5. Multiply 7.214 by .316.
6. Multiply 40.206 by .027.
7. Multiply .0216 by .0042.

Multiply—

8. .4 by .625.
9. .8 by .068.
10. .9 by .899.
11. .7 by .039.
12. .6 by .079.
13. .26 by .4216.
14. .09 by .0036.
15. .35 by .1024.
16. .73 by .0094.
17. .08 by .6213.
18. .321 by .8432.
19. .027 by .6215.
20. .061 by .0283.
21. .092 by .0063.
22. .007 by .0038.

23. Multiply 1.080502 by 3.216.
24. Multiply .00432 by .00037.
25. Multiply $8\frac{1}{2}$ by $.07\frac{1}{4}$ by 10.
26. Multiply .05 by 14.162 by .02.
27. Multiply $6\frac{2}{5}$ by $7\frac{3}{8}$ by $.8\frac{1}{2}$.
28. Multiply 0.305 by .00046.
29. Multiply .42, 3.04, .125, and .826 by .0031.
30. Multiply 12.63, .081, .073, and .0081 by .00093.
31. Multiply 10, 100, 1000, and 10000 by 8.6213.

DIVISION OF DECIMALS.

ORAL EXERCISES.

100.—*1.* What is $\frac{3}{10} \div 3$? $.6 \div 3$? $.8 \div 4$?

2. What is $\frac{1}{100} \div \frac{1}{10}$? $.48 \div .6$? $.56 \div .7$?

3. What is $\frac{1}{1000} \div \frac{1}{10}$? $.024 \div .8$? $.072 \div .9$?

4. What is $\frac{1}{1000} \div \frac{1}{100}$? $.081 \div .09$? $.063 \div .07$?

5. When hundredths are divided by tenths, how many decimal places are there in the quotient? Thousandths by tenths? Thousandths by hundredths?

6. $.36 \div 9.$
7. $.42 \div 6.$
8. $.64 \div 8.$
9. $.72 \div 9.$
10. $.36 \div .9.$
11. $.42 \div .6.$
12. $.56 \div .7.$
13. $.81 \div .9.$
14. $.035 \div .7.$
15. $.028 \div .4.$
16. $.063 \div .9.$
17. $.049 \div .7.$
18. $.012 \div .06.$
19. $.027 \div .09.$
20. $.032 \div .08.$
21. $.054 \div .06.$

101. *The quotient of two decimals contains as many decimal places as those in the dividend exceed those in the divisor.*

WRITTEN EXERCISES.

1. Divide 2.176 by .34.

PROCESS.
6.4, *Ans.*
.34)2.176
204
136
136

1ST. 2176 divided by 34 is 64; but thousandths divided by hundredths give tenths: hence the quotient is 64 tenths, or 6.4.

2D. $$2.176 \div .34 = \frac{2176}{1000} \div \frac{34}{100} = \frac{2176}{1000} \times \frac{100}{34}$$
$$= \frac{2176}{10 \times 34} = \frac{64}{10} = 6.4.$$

RULE.

Divide as in whole numbers, and point off as many decimal places in the quotient as those in the dividend exceed those in the divisor.

1. If the number of decimal places in the dividend is less than the number of decimal places in the divisor, make them equal before beginning the division.

2. If the quotient does not contain the required number of decimal places, supply the deficiency by prefixing ciphers.

2. Divide 16.5 by .25; and .29808 by 124.2.

PROCESS.

$$\begin{array}{r} \underline{|66}, \textit{Ans.} \\ .25)16.50 \\ \underline{150} \\ 150 \\ \underline{150} \end{array} \qquad \begin{array}{r} \underline{|.0024}, \textit{Ans.} \\ 124.2).29808 \\ \underline{2484} \\ 4968 \\ \underline{4968} \end{array}$$

3. Divide 4.128 by .8.
4. Divide 3.024 by .07.
5. Divide 739.44 by .009.
6. Divide 185.175 by .015.
7. Divide 16.02 by .045.
8. Divide 34.4088 by 1.62.
9. Divide .291624 by 2.32.
10. Divide .0014375 by 6.25.
11. Divide .3234 by 1470.
12. Divide 102.102 by 102.

Divide—

13. .00892 ÷ .04.
14. .01312 ÷ .16.
15. .02268 ÷ .42.
16. .07503 ÷ .61.
17. .02706 ÷ .33.
18. 2.3088 ÷ 6.24.
19. 24.211 ÷ 3.41.
20. 4.92246 ÷ .123.
21. 1.3668 ÷ 2.04.
22. 5.8432 ÷ 47.2.
23. .016 ÷ 400.
24. 160 ÷ .004.
25. .0018 ÷ .06.
26. 800 ÷ .04.
27. 2 ÷ .125.

28. Divide 1249383 by .023.
29. Divide 1 by .0125. By .375.
30. Divide .023 by 10. By 100. By 1000.
31. Divide 16000 by .0004.
32. Divide .00016 by 4000.
33. Divide .2952, 99.63, and 1.3653 by .123.
34. Divide 7.1262, .96942, and 54.891 by 3.21.
35. Divide one, and one hundred-millionth by one ten-thousandth.

APPLICATION OF DECIMALS.

102. The application of the principles of decimals and common fractions gives rise to several abbreviations in multiplication, division, and business transactions, of which the following are the most important:

CASE I.

103. To multiply by an aliquot part of 100.

An **Aliquot Part** of a number is the whole or mixed number which will exactly divide that number. Thus, 50, $33\frac{1}{3}$, 25, 20, $16\frac{2}{3}$, $12\frac{1}{2}$, etc. are aliquot parts of 100.

1. *To multiply by* 25.

This is equivalent to multiplying by $\frac{100}{4}$; hence the

RULE.

Annex two ciphers, or move the decimal point two places to the right, and divide by 4.

25 times—

1. $512 = \frac{51200}{4} = 12800$.
2. 10.24.
3. 3456.
4. 1.23456.
5. 968.424.
6. 12.3284.

2. *To multiply by* $12\frac{1}{2}$.

This is equivalent to multiplying by $\frac{100}{8}$; hence the

RULE.

Annex two ciphers, or move the decimal point two places to the right, and divide by 8.

$12\frac{1}{2}$ times—

7. $512 = \frac{51200}{8} = 6400$.
8. 6.32.
9. 7864.
10. 82.128.
11. 98.7656.
12. 4.248.

3. *To multiply by* $33\frac{1}{3}$.

This is equivalent to multiplying by $\frac{100}{3}$; hence the

RULE.

Annex two ciphers, or move the decimal point two places to the right, and divide by 3.

$33\frac{1}{3}$ times—

13. 612 $= \frac{61200}{3} =$ 20400.

14. 9636.

15. 60.3045.

16. 1357.11.

17. 55.5555.

18. 123.456.

Similar rules may be made for other aliquot parts of 100.

CASE II.

104. To divide by an aliquot part of 100.

1. *To divide by* 25.

This is equivalent to multiplying by $\frac{4}{100}$; hence the

RULE.

Multiply by 4 *and divide by* 100.

Divide 25 into—

1. 98750. $= \frac{98750 \times 4}{100} =$ 3950.

2. 86752.

3. 2161.

4. 762.34.

5. 213.75.

6. 32125.

7. 99975.

2. *To divide by* $12\frac{1}{2}$.

This is equivalent to multiplying by $\frac{8}{100}$; hence the

RULE.

Multiply by 8 *and divide by* 100.

Divide 12½ into—

8. $1012\frac{1}{2} = \frac{1012\frac{1}{2} \times 8}{100} =$ 81.

9. 4625.

10. 345.216.

11. 1784.

12. 1256.

13. 475.

3. *To divide by* 33⅓.

This is equivalent to multiplying by $\frac{3}{100}$; hence the

RULE.

Multiply by 3 *and divide by* 100.

Divide 33⅓ into—

14. $12500 = \frac{12500 \times 3}{100} =$ 375.

15. 4700.

16. 12000.

17. 21.62.

18. 128.5.

19. 895400.

Similar rules may be made for other aliquot parts of 100.

PROBLEMS.

1. What will 72 yards of muslin cost at 12½ cents a yard?

2. What will 32 yards of calico cost at 6¼ cents a yard?

3. What will 150 pounds of coffee cost at 33⅓ cents a pound?

4. What will 48 pounds of butter cost at 33⅓ cents a pound, and 64 dozen of eggs at 25 cents a dozen?

5. What is the cost of 24 yards of cloth at $1.16⅔, and 75 yards of muslin at 12½ cents a yard?

6. What will 216 tons of ore cost at $16⅔ a ton?

7. What will 429 tons of rails cost at $33⅓ a ton?

8. What will 840 bushels of oats cost at 25 cents a bushel?

9. What will 215 pounds of beef cost at 20 cents a pound?

10. What will 126 pounds of sugar cost at 6¼ cents a pound?

11. What will 23 dozen of eggs cost at 25 cents a dozen?

12. What will 90 quarts of berries cost at 12½ cents a quart?

CASE III.

105. To find the cost when the quantity and the price of 100 or 1000 are given.

1. What will 1225 bricks cost at $9 per 1000?

Process.

$$\begin{array}{r} \$9 \\ \underline{1.225} \\ \$11.025, \textit{Ans.} \end{array}$$

If 1000 bricks cost $9, 1225 bricks, which is 1.225 times 1000, will cost 1.225 times $9, or $11.025.

RULE.

Multiply the price by the quantity, and point off two places for the price per hundred or three places for the price per thousand.

What is the cost—

2. Of 5200 shingles at $4.75 per thousand?

3. Of 6875 cigars at $6.50 per thousand?

4. Of 3480 feet of boards at $34 per thousand?

5. Of 5760 bricks at $9.25 per thousand?

6. Of 10824 railroad ties at $15 per hundred?

7. Of 5820 chestnut rails at $4 per hundred?

8. Of 625 pounds of beef at $11.25 per hundred?

9. Of 1425 pounds of freight at $.45 per hundred?

10. Of 276 pounds of pork at $9.50 per hundred?

11. Of 8560 pounds of coal at $6.50 per ton?

Suggestion.—Take half the price per ton, and proceed as before.

12. Of 6780 pounds of hay at $15.50 per ton?

13. Of 16825 pounds of bone-dust at $20.50 per ton?

14. Of 6220 pounds of wool at $540 per ton?

15. Of 4280 pounds of oats at $22.50 per ton?

16. Of 8040 pounds of cast iron at $120 per ton?

MISCELLANEOUS EXAMPLES.

106.—*1.* Reduce $.064\frac{2}{3}$ to a common fraction.
2. Reduce $\frac{125}{512}$ to a decimal.
3. Add .0162 and $\frac{209}{2500}$.
4. Subtract .00865 from 1.02.
5. Multiply 54368 by .0025.
6. Multiply 621.78 by $33\frac{1}{2}$.
7. Divide 18.1771 by 6.7.
8. Divide 86.215 by $12\frac{1}{2}$.
9. Reduce $1.0016\frac{2}{3}$ to a mixed number.
10. Add $18.94 + 2.04\frac{1}{4} + 4.36\frac{2}{5}$.
11. Subtract $.06795\frac{1}{4}$ from 2.00701.
12. Multiply 1.675432 by 10000.
13. Divide 817.1456 by .000001.
14. Multiply 54.063 by $1.11\frac{1}{9}$.
15. Divide 102.056 by $33\frac{1}{3}$.
16. Reduce $12\frac{455}{1280}$ to a decimal.
17. Multiply 682.154 by .0001.
18. Divide 250000 by .00005.
19. Divide .000025 by 50000.
20. Multiply 81.4563 by $333\frac{1}{3}$.
21. Subtract .0406008 from .07.
22. Reduce $.14\frac{2}{7}$ to a common fraction.
23. Reduce $.0011\frac{1}{9}$ to a common fraction.
24. Add $2.84 + \frac{11}{64} + 14.0\frac{12}{25} + 100.001$.
25. Multiply 42.37, .0002, 400, and $\frac{3}{25}$.
26. Reduce $\frac{53}{256}$ to a decimal.
27. Add $1.1\frac{1}{32} + 2.22\frac{1}{16} + 3.333\frac{1}{8} + 4.4444\frac{1}{4}$.
28. Subtract $11.11\frac{1}{5}$ from $22.2\frac{2}{3}$.
29. Subtract 33.3 from 33.333 + 3.3.
30. Multiply the sum of 1000 + .001 by .001.
31. Divide the difference between .1 and .001 by .1.
32. Divide the sum of .5 and .5 by their product; also the product of .5 and .5 by their sum.

MISCELLANEOUS PROBLEMS.

33. A owned 1024 acres of land, and sold .65 of it; how much had he remaining?

34. A farmer raised 872 bushels of grain, and sold B .25 of it and C. $.33\frac{1}{3}$ of the remainder; how much had he left?

35. How much iron in 78165 lb. of ore, if .72 of it is pure iron?

36. How many rods of fence will enclose a field whose sides are 42.56, 75.08, 110.27, and 74.26 rods?

37. If 67 hogsheads of water, each containing 63.5 gallons, are drawn from a reservoir containing 10000 gallons, how much will remain?

38. If 12.56 acres of land cost $1070.74, what will 1 acre cost?

39. If 13.5 yards of cloth are bought for $\$84\frac{3}{8}$, what will 23.75 yards cost?

40. B had $508.75: he spent $120.25 for rent, $216.82 for repairs, and earned $126.71; how much did he then have?

41. A labored $126\frac{3}{4}$ days for $2.35 a day: he received from his employer 15 bushels of wheat at $62\frac{1}{2}$ cents a bushel, and $75.50 in cash; how much was yet coming to him?

42. A bought 5862 bricks at $10.15 per thousand, and 8675 feet of yellow-pine flooring at $42 a thousand; what was his entire bill?

43. A merchant sold some damaged goods for $312.15, on which he lost .25 of the cost; what was the cost?

44. What will 26 loads of lime cost, each containing $15\frac{1}{2}$ bushels, at $22\frac{1}{2}$ cents a bushel?

45. How many pounds of coffee can be bought for $16.25, if $5\frac{1}{2}$ pounds cost $\$1.78\frac{3}{4}$?

46. A bought 10 car-loads of coal, each containing 50.25 tons, at $\$4\frac{2}{5}$ a ton: $\frac{1}{50}$ of it he retained for his own use, and sold the remainder at a gain of $\frac{1}{11}$ of the cost; how much more than the cost of his own coal did he gain?

BILLS AND ACCOUNTS.

107. A **Bill** is a written statement of goods sold, giving the date, quantity, price, and the amount of the whole.

The **Footing of a Bill** is the amount of its items.

A bill is **Receipted** when the person to whom it is due, or his authorized agent, writes the words "Received Payment" and signs his name.

108. An **Account Current** is a full statement of the business transactions between two parties for a certain time.

The amount owed is the **Debt.**

The party owing the debt is the **Debtor.**

The amount due a person is a **Credit.**

The person to whom the credit is due is the **Creditor.**

The **Balance of an Account** is the difference between the amounts of the debit and credit sides.

109. In bills and accounts the following abbreviations are in common use:

Acc't, or $^a/_c$, account.
Am't, amount.
@, at.
Bal., balance.
Bo't, bought.
Co., company.
C. O. D., collect on delivery.
Cr., credit or creditor.
Do., the same.
Dr., debit or debtor.
Fr't, freight.
Inst., this month.
Int., interest.
Mdse., merchandise.
No., number.
Pay't, payment.
Per, by.
Rec'd, received.

In making out a **Bill,** *extend the items and take their sum.*

In making out an **Account,** *find the difference between the debit and credit amounts.*

110. The following bills may be copied, extended, and footed:

1.

New York, Apr. 2, 1894.

Mr. J. W. Lansinger,

Bought of Walter A. Stone.

12 Cane-seated Chairs,	@ $2.25		
8 Looking-glasses,	" 3.50		
25 Picture-frames,	" 1.20		
5 Patent Rockers,	" 4.50		
28 Perforated Seats,	" .32		
15 Boxes of Brass Nails,	" .75		
4 Extension Tables,	" 8.25		
		$160	71

Received Payment,

Walter A. Stone.

2.

Philadelphia, May 1, 1894.

Mr. H. F. Bitner,

Bought of Bullock & Crenshaw.

8 lb. Ammonia,	@ $ .11		
2 oz. Antimony Chloride,	" .20		
2 oz. Cadmium Chloride,	" .20		
2 lb. Mercury,	" .85		
2 ft. Plat. Wire,	" .45		
1 doz. 8-oz. Bottles,	" 3.15		
10 doz. Test-tube Brushes,	" .70		
2 doz. Evap. Dishes,	" 1.44		
		$17	31

Received Payment,

Bullock & Crenshaw.

3.

Baltimore, Aug. 1, 1894.

Mr. W. Noble,

To W. G. Gardner & Co., Dr.

To 35 tons Bro. Coal,	@	$4.29		
“ 8¼ “ Pea “	“	2.72		
“ 4½ “ Nut “	“	5.40		
“ 17½ “ Egg “	“	5.40		
“ 10 “ Stove “	“	4.29		
“ 12¼ “ Mixed “	“	2.72		
			$367	61

Received Payment,

W. G. Gardner & Co.

4.

Chicago, June 1, 1894.

Mr. J. F. Lindsay,

In Acc't with Geo. M. Stinson.

1894.		—*Dr.*—				
Jan.	5	To 2 Stoves,	@	$18.50		
“	“	“ 5 Hoes,	“	.75		
“	“	“ 8 Forks,	“	.80		
“	12	“ 3 Plows,	“	8.25		
“	“	“ 2 Sleds,	“	1.25		
“	“	“ 3 Boxes of Glass,	“	2.50		
					$	
		—*Cr.*—				
Jan.	6	By 20 bu. Potatoes,	@ .80	$16.00		
“	“	“ 40 lb. Butter,	“ .30	12.00		
				Balance,	$53	90

Received Payment,

Geo. M. Stinson.

Make out and receipt the following bills and accounts:

5. Mr. S. M. Boyd, bought of Henry Maynard, 14 yd. of Carpet, @ $.95; 40 yd. of Muslin, @ 12½ cents; 16 yd. of Calico, @ 8¼ cents; 4 pairs of Hose, @ 62½ cents; 12 yd. of Gingham, @ 10½ cents; and 22 yd. of Canton Flannel, @ 10 cents.

6. Mr. A. H. Howard, bought of Geo. P. Long, 15 lb. of Sugar, @ 7 cents; 18 lb. of Coffee, @ 30 cents; 2 lb. of Tea, @ 85 cents; 5 bu. of Potatoes, @ 75 cents; 12 cans of Corn, @ 20 cents; 12 lb. of Butter, @ 30 cents; 14 doz. of Eggs, @ 22 cents; and 18 qt. of Syrup, @ 15 cents.

7. Mr. J. H. Landis, bought of A. W. Groff, 36 bu. of Wheat, @ 77 cents; 50 bu. of Corn, @ 52 cents; 40 bu. of Oats, @ 42 cents; 25 cwt. of Flour, @ $3; 150 bu. of Bran, @ 20 cents; 40 bu. of Cracked Corn, @ 48 cents; 10 bu. of Rye, @ 62 cents; 72 qt. of Corn Meal, @ 4½ cents.

8. H. S. Frew, purchased of Frank Lindsay, Cincinnati, Ohio, Sept. 3, 1894, 55 bu. of White Wheat, @ 54 cents; 175 bu. of Red Wheat, @ 58 cents; 205 bu. of Rye, @ 49 cents; 180 bu. of Barley, @ 42 cents; 318 bu. of Oats, @ 35 cents; and 512 bu. of Corn, @ 50 cents. On the same date Lindsay bought of Frew, 60 yd. of Calico, @ 12½ cents; 500 yd. of Linen Crash, @ 6 cents; 40 pairs of Kid Gloves, @ $1.40; and 50 doz. spools of Cotton, @ 60 cents. Frew paid the balance with his note; receipt accordingly.

9. Jan. 1, 1895, Messrs. Lansinger & Co., bought of Elmer H. Frantz, Philadelphia, 42,000 ft. of White Oak, @ $60 per M.; Jan. 16, 5620 ft. of Walnut, @ $75 per M.; Jan. 17, they paid cash on account, $550; Jan. 29, they bought 8650 ft. of White Pine, @ $40 per M.; Jan. 31, they paid cash on account, $250; Feb. 4, they bought 8050 ft. of White Oak, @ $80 per M.; Feb. 12, 68,260 Shingles, @ $12 per M.; Feb. 15, they sold Frantz, 650 bu. of Wheat, @ 55 cents; Feb. 20, they bought 10800 ft. of Maple, @ $95 per M.: account rendered March 1, 1895.

SECTION VI.

DENOMINATE NUMBERS.

111. A Denominate Number is one in which the unit is a measure; as, 2 feet, 3 bushels, 5 pounds, etc.

112. A Compound Denominate Number is a denominate number composed of two or more units of the same kind of measure; as, 5 ft. 6 in.; 3 bu. 2 pk. 1 qt., etc.

MEASURES.

113. Measures of Value.

Money is the measure of the value of things. It is of two kinds, *Coin* and *Paper Money*.

I. United States Money.

TABLE.

10 mills (m.)	=	1 cent,	ct.
10 cents	=	1 dime,	d.
10 dimes	=	1 dollar,	$
10 dollars	=	1 eagle,	E.

The *coins* of the United States are—

Gold, the *quarter-eagle, half-eagle, eagle*, and *double-eagle.*
Silver, the *dime, quarter-dollar, half-dollar*, and *dollar.*
Nickel, the *five-cent piece.*
Bronze, the *cent.*

NOTE.—The coinage of gold dollars and three-dollar pieces and nickel three-cent pieces was suspended by an act of Congress Sept. 26, 1890.

II. English Money.

English Money is the currency of Great Britain. The unit is the *pound,* whose value is $4.8665.

TABLE.

4 farthings (far.) = 1 penny, d.
12 pence = 1 shilling, s.
20 shillings = 1 pound, £

The following denominations are also used:

5 shillings = 1 crown.
21 shillings = 1 guinea.

The pound when coined is called a *sovereign.*

III. French Money.

The unit is the *franc,* whose value is 19.3 cents. The franc is divided into tenths and hundredths, called, respectively, *decimes* and *centimes.*

IV. German Money.

The unit is the *mark* (*Reichsmark*), whose value is 23.85 cents. The mark is divided into hundredths, called *pfennige.*

ORAL EXERCISES ON VALUE.

1. How many farthings in 6 pence? In 7 d.? In 10 d.?

2. How many pence in 12 far.? In 16 far.? In 20 far.?

3. How many pence in 1 shilling? In 4 s.? In 7 s.? In 10 s.? In 30 s.? In 40 s.?

4. How many shillings in 24 d.? In 36 d.? In 48 d.? In 56 d.? In 65 d.? In 84 d.?

5. How many shillings in £5? In £6? In £8? In £10?

6. How many pounds in 40 s.? In 60 s.? In 100 s.?

7. How many pence in 5 s. 6 d.? In 8 s. 4 d.? In 10 s. 10 d.? In 7 s. 6 d.? In 5 s. 7 d.?

114. Measures of Weight.

I. **Avoirdupois Weight** is used in weighing everything except precious metals and jewels.

TABLE.

16 ounces (oz.) = 1 pound, lb.
100 pounds = 1 hundred-weight, cwt.
20 hundred-weight = 1 ton, T.

At the United States Custom House, and in weighing coal and iron at mines, the following weights are used:

112 pounds = 1 long hundred-weight.
2240 pounds = 1 long ton.

II. **Troy Weight** is used in weighing precious metals.

TABLE.

24 grains (gr.) = 1 pennyweight, pwt.
20 pennyweights = 1 ounce, oz.
12 ounces = 1 pound, lb.

III. **Apothecaries' Weight** is used in mixing medicines.

TABLE.

20 grains (gr.) = 1 scruple, ℈
3 scruples = 1 dram, ʒ
8 drams = 1 ounce, ℥
12 ounces = 1 pound, ℔

Medicines are bought by avoirdupois weight, and are mixed by apothecaries' weight.

A Troy pound = 5760 grains.
An apothecaries' pound = 5760 grains.
An avoirdupois pound = 7000 grains.

ORAL EXERCISES ON WEIGHT.

How many—

1. Ounces in 5 lb. Troy? In 5 lb. avoirdupois?
2. Pounds Troy in 96 oz.? Pounds avoirdupois in 80 oz.?
3. Grains in 5 pwt.? In 8 ℈? In 4 pwt.?
4. Ounces in ½ pound of sugar? In ½ pound of gold?
5. What part of a pound avoirdupois are 4 oz.?
6. What part of a pound Troy are 8 oz.?
7. How many drams in 36 ℈? In 60 ℈?
8. How many grains in 2 lb. Troy? In 3 lb.?
9. How many grains in 2 lb. avoirdupois? In 4 lb.?
10. How many pounds in ½ ton? In ¼ ton?
11. How many scruples in 8 ʒ and 2 ℈?

115. Measures of Length.

I. **Long Measure** is used for measuring lengths or distances.

TABLE.

12 inches (in.) = 1 foot, ft.
3 feet = 1 yard, yd.
5½ yards = 1 rod, rd.
320 rods = 1 mile, mi.

1 mi. = 320 rd. = 1760 yd. = 5280 ft.

II. **Surveyors' Linear Measure** is used by engineers and surveyors in measuring distances.

TABLE.

7.92 inches (in.) = 1 link, li.
100 links = 1 chain, ch.
80 chains = 1 mile, mi.

The unit is a chain called *Gunter's Chain*, which is 4 rods, 66 feet, or 792 inches, long.

ORAL EXERCISES ON LENGTH.

1. How many inches in a yard? In $\frac{1}{2}$ yd.? In $\frac{1}{4}$ yd.? In $\frac{1}{8}$ yd.? In $\frac{3}{4}$ yd.?

2. How many feet in 10 yd.? In 72 in.?

3. How many chains in 3 mi.? In 5 mi.?

4. How many yards in 36 ft.? In 90 ft.?

5. How many rods in a mile? In 3 mi.?

6. How many links in a chain? In 5 ch.?

7. How many yards in a mile? In 3 mi.?

8. How many feet in a mile? In 10 mi.?

9. How many rods in a chain? In 5 ch.?

10. How many feet in a chain? In 8 ch.?

116. Measures of Surface.

A **Surface** is that which has length and breadth only.

A **Square** is a plane surface which has four equal sides and four equal angles.

Square Measure is used in measuring surfaces.

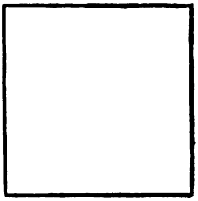


TABLE.

144 square inches (sq. in.)	=	1 square foot,	sq. ft.
9 square feet	=	1 square yard,	sq. yd.
$30\frac{1}{4}$ square yards	=	1 square rod,	sq. rd.
160 square rods	=	1 acre,	A.
640 acres	=	1 square mile,	sq. mi.

A square rod is sometimes called a *perch* (P.). 40 sq. rd. = 1 rood (R.). 4 roods = 1 A. 10 square chains = 1 A. 43560 sq. ft. = 1 A. 6 miles square = 1 township (Tp.).

ORAL EXERCISES ON SQUARE MEASURE.

How many—

1. Square inches in 4 sq. ft.? In 5 sq. ft.?

2. Acres in 20 sq. ch.? In 100 sq. ch.? In 2 sq. mi.?

3. Square feet in 1440 sq. in.? In 720 sq. in.?

4. Square yards in 108 sq. ft.? In 144 sq. ft.?

5. Roods in 200 sq. yd.? In 20 acres? In 20 sq. ch.?

6. Acres in 24 R.? In 2 sq. mi.? In 640 sq. rd.?

7. Square chains in 48 acres? In 120 acres?

8. Square miles in 1920 A.? In 5 townships?

9. Townships in 108 sq. mi.? In 360 sq. mi.?

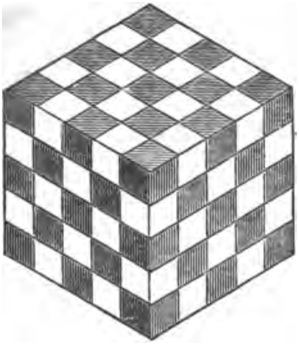
10. What is the difference between 5 square feet and 5 feet square?

117. Measures of Volume.

A **Volume** is that which has length, breadth, and thickness.

A **Cube** is a volume bounded by six equal squares.

Cubic Measure is used for measuring solids.



Cube.

TABLE.

1728 cubic inches (cu. in.) = 1 cubic foot, cu. ft.
27 cubic feet = 1 cubic yard, cu. yd.

A *cord of wood* is a pile 8 ft. long, 4 ft. wide, and 4 ft. high, and contains 128 cu. ft.

A *perch of stone* or of *masonry* is 16½ ft. long, 1½ ft. wide, and 1 ft. high, and contains 24¾ cu. ft.

A cubic yard of earth is called a *load.*

118. Measures of Capacity.

I. **Liquid Measure** is used for measuring liquids.

TABLE.

4 gills (gi.) = 1 pint, pt.
2 pints = 1 quart, qt.
4 quarts = 1 gallon, gal.

In determining the capacity of *reservoirs*, *cisterns*, etc., the following are used:

31½ gallons = 1 barrel, bbl.
63 gallons = 1 hogshead, hhd.

A *liquid* gallon contains 231 cu. in.

NOTE.—The *imperial* gallon of Great Britain contains 277.274 cu. in. Ale, beer, etc. were formerly sold by a gallon of 282 cu. in.

II. **Dry Measure** is used for measuring grain, fruit, vegetables, etc.

TABLE.

2 pints (pt.) = 1 quart, qt.
8 quarts = 1 peck, pk.
4 pecks = 1 bushel, bu.

A bushel contains 2150.42 cu. in.

NOTE 1.—The *imperial* bushel of Great Britain contains 2218.192 cu. in.

NOTE 2.—The gallon, quart, and pint, dry measure, are greater than the corresponding quantities in liquid measure. For example, a quart, dry measure, = $\frac{1}{32}$ of a bushel (2150.42 cu. in.), or 67.2 cu. in.; while a quart, liquid measure, = ¼ of a gallon (231 cu. in.), or 57.75 cu. in.

III. **Apothecaries' Fluid Measure** is used for mixing liquid medicines.

TABLE.

60 minims (♏) = 1 fluidram, fʒ.
8 fluidrams = 1 fluidounce, f℥.
16 fluidounces = 1 pint, O.
8 pints = 1 gallon, Cong.

ORAL EXERCISES ON CAPACITY.

How many—

1. Pints in 10 qt.? In 40 gi.? In 20 qt.?

2. Quarts in 8 gal.? In 32 pt.? In 12 gal.?

3. Quarts in 5 pk.? In 2 bu.? In 10 pk.?

4. Pecks in 6 bu.? In 48 qt.? In 10 bu.?

5. Bushels in 24 pk.? In 60 pk.? In 128 qt.?

6. Minims in 8 fʒ? In 10 fʒ? . In 12 fʒ?

7. Gills in 4 qt.? In 12 pt.? In 10 qt.?

8. Pints in 2 bu.? In 4 pk.? In 72 qt.?

9. When apples are selling at 10 cents a half-peck, at what rate is that per bushel?

119. Measures of Time.

TABLE.

60 seconds (sec.)	=	1 minute,	min.
60 minutes	=	1 hour,	hr.
24 hours	=	1 day,	da.
7 days	=	1 week,	wk.
365 days	=	1 common year,	yr.
366 days	=	1 leap year,	l. yr.
100 years	=	1 century,	C.

The year is divided into 12 *Calendar Months*, as follows:

January,	31 days.	July,	31 days.
February,	28 or 29 days.	August,	31 "
March,	31 days.	September,	30 "
April,	30 "	October,	31 "
May,	31 "	November,	30 "
June,	30 "	December,	31 "

The time required for the earth to revolve around the sun is 365 da. 5 hr. 48 min. 49.7 sec., or nearly 365¼ days. This fractional part of a day is omitted each year until the fourth year, when the time lost amounts to almost a day. This is corrected by adding a day to the month of February. Since the time lost each year is a *little less* than ¼ of a day, by adding a day every fourth year we gain nearly a day every 400 years. To offset this gain every fourth centennial year is made a common year instead of a leap year. Hence

Every year divisible by 4, *except the centennial years, and every centennial year divisible by* 400, *is a leap year; all the others are common years.*

ORAL EXERCISES ON TIME.

How many—

1. Seconds in 10 min.? In 30 min.?

2. Minutes in 2 hr.? In 720 sec.? In 4 hr.?

3. Hours in 4 da.? In 960 min.? In 5 da.?

4. Days in 5 wk.? In 144 hr.? In 240 hr.?

5. Days in 2 common years? In 2 leap years?

6. What months have 30 days each? 31 days each?

7. What part of an hour is 30 min.? 15 min.? 10 min.?

8. Which of the following years are leap years: 1600? 1894? 1800? 1880? 1896? 1900? 2000?

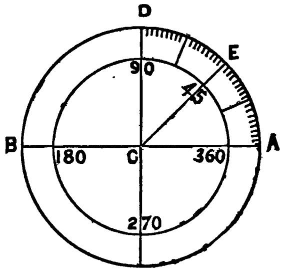
9. When does this century end?

10. How many leap years in a century?

120. Angular Measure.

Angular Measure is used in measuring angles and arcs.

A **Circle** is a plane figure bounded by a curved line every point of which is equally distant from a point within, called the centre.



The **Circumference** of a circle is the bounding line. One-fourth of the circumference is a *quadrant.*

An **Arc** is any part of a circumference.

For convenience of measuring the circumference is divided into 360 equal parts, called *degrees;* each degree into 60 equal parts, called *minutes;* each minute into 60 equal parts, called *seconds.*

TABLE.

60 seconds (″) = 1 minute, ′.
60 minutes = 1 degree, °.
360 degrees = 1 circumference, C.

The length of a degree of longitude at the earth's equator is 69$\frac{1}{6}$ miles.

In astronomy 30° make a **Sign.**

121. Miscellaneous Tables.

NUMBERS.

12 units = 1 dozen.
12 dozen = 1 gross.
12 gross = 1 great-gross.
20 units = 1 score.

PAPER.

24 sheets = 1 quire.
20 quires = 1 ream.
Hence
480 sheets = 1 ream.

122. Miscellaneous Measures.

A bushel of	Barley	weighs	48	pounds.
	Clover-seed	"	60	"
	Corn	"	56	"
	Oats	"	32	"
	Potatoes	"	60	"
	Rye	"	56	"
	Wheat	"	60	"

A barrel of flour weighs 196 pounds.
A barrel of pork weighs 200 pounds.
A firkin of butter weighs 56 pounds.

NOTE.—Some of these weights differ slightly in different States.

ORAL EXERCISES.

1. How many degrees in a quadrant?
2. How many minutes in 5°? In 720″?
3. How many farthings in 2 s.? In 8 d.?
4. How many pence in 5 s.? In 60 far.?
5. How many shillings in £6? In 108 d.?
6. How many eggs in 4½ doz.? In 10½ doz.?
7. How many years in 3 score and 10? In 4 score?
8. How many sheets in 5 quires? In 8 quires?
9. How many quires in 4 reams? In 720 sheets?
10. How many signs in a circumference?
11. How many pounds in 5 bu. of wheat?
12. How many pounds in 6 firkins of butter?

REDUCTION OF DENOMINATE NUMBERS.

123. Reduction Descending.

Reduction Descending is the process of reducing denominate numbers to lower denominations.

CASE I.

124. To reduce a compound denominate number to lower denominations.

WRITTEN EXERCISES.

1. Reduce 10 bu. 3 pk. 7 qt. to quarts.

PROCESS.

$$\begin{array}{lll} \text{bu.} & \text{pk.} & \text{qt.} \\ 10 & 3 & 7 \\ \underline{\ \ 4} & & \\ 43 \text{ pk.} & & \\ \underline{\ \ 8} & & \\ 351 \text{ qt.} & & \end{array}$$

In 1 bu. there are 4 pk., and in 10 bu. there are 10 times 4 pk., or 40 pk. 40 pk. plus 3 pk. are 43 pk. In 1 pk. there are 8 qt., and in 43 pk. there are 43 times 8 qt., or 344 qt. 344 qt. plus 7 qt. are 351 qt. Therefore 10 bu. 3 pk. 7 qt. equal 351 qt.

RULE.

Multiply the number of the highest denomination given by the number of units of the next lower denomination which equals one of the higher, and to the product add the number given of this lower denomination.

Continue in the same manner until the required denomination is reached.

2. Reduce 2 T. 57 lb. 10 oz. to ounces.
3. Reduce 6 lb. 8 oz. 12 pwt. to grains.
4. Reduce 4 ℔ 5 ℥ 3 ʒ 2 ℈ to grains.
5. How many grains in 5 lb. Troy?
6. How many grains in 7 lb. avoirdupois?
7. Reduce 2 mi. 10 rd. 12 yd. to feet.
8. How many inches in 60 ch.?
9. How many feet in 15 miles?
10. Reduce 2 sq. mi. to sq. yd.
11. Reduce 80 sq. rd. 20 sq. yd. to sq. in.
12. Reduce 30 cu. yd. to cu. in.
13. How many cu. ft. in 160 cords?
14. How many cu. ft. in 48 perches?
15. Reduce 110 gal. 3 qt. 2 gi. to gills.
16. How many pints in 5 bu. 3 pk. 4 qt.?
17. How many cu. in. in 12 gal. of water?
18. How many cu. in. in 10 gal. of beans?
19. How many cu. in. in 20 bu.?
20. Reduce Cong. 6 to minims.
21. How many seconds in July?
22. Reduce 15° 40′ 30″ to seconds.
23. How many seconds in 8 signs?
24. Reduce £4 12 s. 8 d. to farthings.
25. How many inches in 40 rd.?
26. How many inches in 2 degrees?
27. Reduce 18 A. 3 R. 14 P. to sq. rd.

CASE II.

125. To reduce a denominate fraction to integers of lower denomination.

WRITTEN EXERCISES.

1. What is the value of $\frac{15}{16}$ of a bushel?

PROCESS.

$\frac{15}{16}$ bu. × 4 = $\frac{15}{4}$ = $3\frac{3}{4}$ pk. $\frac{3}{4}$ pk. × 8 = 6 qt. Hence $\frac{15}{16}$ bu. = 3 pk. 6 qt.

What is the value—

2. Of $\frac{2}{3}$ of a ton?
3. Of $\frac{7}{8}$ of a pound Troy?
4. Of $\frac{11}{15}$ of a rod?
5. Of $\frac{5}{7}$ of a mile?
6. Of $\frac{11}{15}$ of an acre?
7. Of $\frac{2}{3}$ of a gallon?
8. Of $\frac{5}{14}$ of a bushel?
9. Of $\frac{5}{7}$ of a sign?

CASE III.

126. To reduce a denominate decimal to integers of lower denomination.

WRITTEN EXERCISES.

1. Reduce £.645 to integers of lower denomination.

PROCESS.

$$\begin{array}{r} .645\ £ \\ \underline{20} \\ 12.900\ s \\ \underline{12} \\ 10.8\ d. \\ \underline{4} \\ 3.2\ far. \end{array}$$

12 s. 10 d. 3.2 far., *Ans.*

Since there are 20 shillings in a £, we first multiply by 20, and obtain 12.9 s. Omitting the 12 s., we multiply the decimal .9 s. by 12 to reduce it to pence, and obtain 10.8 d. Omitting the 10 d., we multiply the decimal .8 d. by 4 to reduce it to farthings, and obtain 3.2 far. Hence £.645 = 12 s. 10 d. 3.2 far.

What is the value—

2. Of .4025 of a ton?
3. Of .115625 of a lb. Troy?
4. Of .8 of a ℔?
5. Of .6625 of a mile?
6. Of .635 of an acre?
7. Of .745 of a gallon?
8. Of .985 of a bushel?
9. Of .715 of a degree?

127. Reduction Ascending.

Reduction Ascending is the process of reducing denominate numbers to higher denominations.

CASE I.

128. To reduce a denominate number to higher denominations.

WRITTEN EXERCISES.

1. Reduce 413 quarts to bushels.

PROCESS.

$$\begin{array}{r|l} 8 & 413 \\ \hline 4 & 51 + 5 \text{ qt.} \\ \hline & 12 \text{ bu.} + 3 \text{ pk.} \end{array}$$

12 bu. 3 pk. 5 qt., *Ans.*

There are 8 quarts in 1 peck, hence in 413 quarts there are 51 pecks, and 5 quarts remaining; there are 4 pecks in 1 bushel, hence in 51 pecks there are 12 bushels, and 3 pecks remaining. Hence 413 qt. equal 12 bu. 3 pk. 5 qt.

RULE.

Divide the given number by that number which will reduce it to the next higher denomination.

Divide the quotient in the same way, and thus proceed until you reach the required denomination.

The last quotient with the remainders will be the required result.

Reduce—

2. 16209 oz. to cwt.
3. 12845 gr. to lb. Troy.
4. 16284 ft. to miles.
5. 2280 cu. ft. to cords.
6. 528614 sq. ft. to acres.
7. 485 sq. ch. to acres.
8. 497 gills to gallons.
9. 1235 cu. ft. to bushels.
10. 2541 cu. in. to gallons.
11. 1254 cu. in. to gallons.
12. 84621♏ to Cong.
13. 102416 sec. to days.
14. 1020300″ to signs.
15. 2235 far. to £.
16. 27412 gr. to lb. Troy.
17. 302 d. to £.
18. 125682 oz. to tons.
19. 208342 in. to mi.

Reduce—

20. 68142 li. to mi.
21. 824685 cu. in. to cords.
22. 8000 gi. to gallons.
23. 4216 cu. ft. to bushels.
24. 86524″ to degrees.
25. 315478 sq. ft. to acres.
26. 4160 ft. to chains.
27. 22160 ft. to mi.
28. 3275 d. to £.
29. 411405 sec. to da.
30. 360 gross to score.
31. 882046 cu. in. to cu. yd.

CASE II.

129. To reduce a compound denominate number to a fraction of a higher denomination.

WRITTEN EXERCISES.

1. Reduce 4 oz. 16 pwt. 16 gr. to the fraction of a pound Troy.

PROCESS.

4 oz. 16 pwt. 16 gr. = 2320 gr.   1 lb. = 5760 gr. $\frac{2320}{5760}$ lb. = $\frac{29}{72}$ lb., *Ans.*

By reduction we find 4 oz. 16 pwt. 16 gr. equal to 2320 gr., and 1 lb. = 5760 gr.; 1 grain is $\frac{1}{5760}$ of a pound, and 2320 gr. equal 2320 times $\frac{1}{5760}$ = $\frac{2320}{5760}$, which reduced to its lowest terms equals $\frac{29}{72}$.

What part—

2. Of 3 bu. is 1 bu. 3 pk. 4 qt.?
3. Of 4 lb. Troy is 6 oz. 6 pwt. 6 gr.?
4. Of 4 lb. avoirdupois is 3 lb. Troy?
5. Of a 5-inch cube is 5 cubic inches?
6. Of 5 mi. is 2 mi. 240 rd.?
7. Of 60 ch. is 1320 ft.?
8. Of 2 acres is 60½ sq. rd.?
9. Of 45 sq. ch. is 75 P.?
10. Of 2 hhd. is 45 gal. 3 qt.?
11. Of 360 da. is 3 wk. 3 da. 4 hr.?
12. Of a circumference is 30° 30′?
13. Of £2 is 320 far.?

CASE III.

130. To reduce a compound denominate number to a decimal of a higher denomination.

WRITTEN EXERCISES.

1. Reduce 5 oz. 15 pwt. 12 gr. to the decimal of a pound.

PROCESS.

24	12.
20	15.5
12	5.775
	.48125, *Ans.*

There are 24 gr. in 1 pwt., hence $\frac{1}{24}$ of the number of grains equals the number of pwt.; $\frac{1}{24}$ of 12 equals .5, which added to 15 pwt. equals 15.5 pwt.; there are 20 pwt. in 1 oz., hence $\frac{1}{20}$ of the number of pwt. equals the number of oz.; $\frac{1}{20}$ of 15.5 equals .775, which added to 5 oz. equal 5.775 oz.; since there are 12 oz. in 1 lb., $\frac{1}{12}$ of the number of oz. equals the number of lb.; $\frac{1}{12}$ of 5.775 equals .48125. Hence 5 oz. 15 pwt. 12 gr equals .48125 lb.

Reduce—

2. 3 pk. 6 qt. 1 pt. to the decimal of a bu.
3. 4 da. 12 hr. 36 min. 48 sec. to the decimal of a week.
4. 5° 24′ 36″ to the decimal of a sign.
5. 4 rd. 2 ft. 9 in. to the decimal of a mi.
6. 60 sq. rd. 33 sq. yd. 6 sq. ft. to the decimal of an acre.
7. 21 inches to the decimal of a yard.
8. 2 cwt. 37 lb. 8 oz. to the decimal of a ton.
9. £14 17 s. 3 d. to the decimal of a pound.
10. 4 cwt. 65 lb. to the decimal of a ton.
11. 1 R. 37 P. to the decimal of an acre.
12. 10 oz. 5 pwt. 4.8 gr. to the decimal of a pound.

MISCELLANEOUS PROBLEMS.

1. What will 8 barrels of flour cost at 3 cents a pound?
2. What will 8 tons of coal cost at \$.26$\frac{1}{4}$ a cwt.?
3. What will 3 bu. 2 pk. of cherries cost at 6 cents a quart?
4. How many days from Mar. 4 to Dec. 13?

5. What will 20 bu. of seed cost at 8 cents a pint?

6. How many seconds in the year 1896?

7. What will a square chain of land cost at 10 cents a square foot?

8. If a wheel is 10 ft. 5 in. in circumference, how often will it revolve in going 25 miles?

9. A coal-dealer bought 125 tons of coal at $3.50 a long ton, and sold it at $4.25 per short ton; how much did he gain?

10. How many miles in a quadrant of the earth's surface?

11. At 28 cents a pound, how much coffee can be bought for $2560?

12. A farmer had 350 bu. of wheat by measure, which he sold by weight at 62½ cents a bushel; how much did he receive for it if each bushel weighed 57½ pounds?

13. A druggist bought 6 pounds avoir. of medicine at $11.20 a pound, and retailed it so as to realize $15.50 a pound apothecaries' weight; what was his gain?

14. A grocer bought 120 bushels of potatoes at 75 cents a bushel, and retailed them at 15 cents a half-peck; what was his gain, if he lost ⅛ by retail measure?

15. A gardener has 28 bu. 3 pk. 3 qt. of clover-seed and 37 bu. 2 pk. 7 qt. of timothy-seed, with which he wishes exactly to fill equal boxes of the largest possible size without mixing the seeds; what is the capacity of each box, and how many will be required to hold the seed?

16. How far from the earth is a star whose light is 32 years of 365¼ days in reaching the earth, light traveling at the rate of 186,300 miles a second?

17. A daily newspaper, which prints 3 sheets for 1 paper, has 216000 subscribers; how many reams of paper will it require in 313 days, making no allowance for waste?

18. How many kegs, each containing 6 gal. 3 qt. 1 pt., can be filled from 10 hhd. of wine?

19. It requires 2 bu. 1 pk. 1 qt. 1 pt. of seed to sow an acre; how many acres can be sowed with 108 bushels?

20. How much time does a man lose in 40 years, of 365 days each, if he wastes ten minutes a day?

21. A man smokes 5 cigars a day, which he buys at the rate of 5 cents for 2; how much will his cigars cost him in 50 years, 12 of them being leap years?

22. A had a farm of 15 A. 80 P. near the city of Lancaster, which he divided into building lots containing 27 P. 16 sq. yd. 7¼ sq. ft.; how many lots were there?

23. What is the gain or loss on 6 lb. 8 oz. avoir. of drugs bought at $4 an ounce avoir. and sold at $4.40 an ounce apothecaries' weight?

24. How many seconds from 6 A. M. July 4th to 6 P. M. Dec. 25th?

25. If 8 tons of phosphorus are required for tipping 80,000,000 matches, how many grains on each match?

26. If one eagle contains 10 pwt. 18 gr., how many eagles can be coined from 26 oz. 17 pwt. 12 gr. of gold of standard fineness?

27. How many reams of paper will be required to print 10,000 copies of a book of 320 pages, if there are 32 pages printed on each sheet, and no allowance for waste?

28. If a horse eats 1 pk. 2 qt. of oats each day, how many weeks will 6 bu. 2 pk. 2 qt. last him?

29. How many ounces of quinine will be required in the manufacture of 10 gross of 2-grain pills?

30. If the average step of a pedestrian is 2 ft. 3 in., how many such steps will he take in walking around a field the sum of whose sides is .7125 mi.?

31. If the distance to the moon is 238,000 miles, and a railroad connected the earth and the moon, in what year, month, and day would a train starting from the earth at noon April 1, 1894, and traveling 30 miles an hour, reach the moon?

ADDITION OF DENOMINATE NUMBERS.

131. Denominate numbers are added, subtracted, multiplied, and divided on the same general principles as simple numbers.

The only modification that need be made in the operations and rules is that which is rendered necessary by the use of a scale that *varies* instead of one that is *uniform.*

WRITTEN EXERCISES.

1. Find the sum of 18 bu. 3 pk. 5 qt., 15 bu. 2 pk. 7 qt., 20 bu. 1 pk. 4 qt., 11 bu. 2 pk. 6 qt.

PROCESS.

bu.	pk.	qt.
18	3	5
15	2	7
20	1	4
11	2	6
66	2	6

We write the numbers so that units of the same denomination are in the same column.

The sum of the quarts is 22 qt., which by reduction equals 2 pk. and 6 qt.; we write the 6 qt. in the column of quarts, and reserve the 2 pk. to add to the pecks. The sum of the pecks is 10 pk., which equals 2 bu. and 2 pk.; we write the 2 pk. in the column of pecks, and add the 2 bu. to the bushels. The sum of the bushels is 66 bu., which we write in the column of bushels.

2. Add 4 mi. 250 rd. 3 yd. 1 ft. 8 in., 6 mi. 185 rd. 5 yd. 1 ft. 7 in., 7 mi. 255 rd. 4 yd. 6 in., 8 mi. 160 rd. 5 yd. 1 ft. 9 in.

PROCESS.

mi.	rd.	yd.	ft.	in.
4	250	3	1	8
6	185	5	1	7
7	255	4	0	6
8	160	5	1	9
27	213	1½	2	6
			1	6
27	213	2	1	0

When a fraction occurs in any term except the lowest denomination, it should be reduced. Thus,

½ yd. = 1 ft. 6 in. Adding 1 ft. 6 in. to 27 mi. 213 rd. 1 yd. 2 ft. 6 in. gives 27 mi. 213 rd. 2 yd. 1 ft.

3. Add 10 mi. 112 rd. 4 yd. 2 ft. 9 in., 106 mi. 124 rd. 5 yd. 1 ft. 6 in., 54 mi. 205 rd. 3 yd. 2 ft. 8 in., 76 mi. 121 rd. 4 yd. 1 ft. 7 in.

4. Add 112 A. 80 P. 21 sq. yd., 108 A. 75 P. 16 sq. yd., 93 A. 57 P. 12 sq. yd., 5 sq. ft., 115 A. 18 P. 28 sq. yd. 4 sq. ft.

5. Add 22 mi. 92 rd. 5 yd. 2 ft. 7 in., 48 mi. 300 rd. 4 yd. 2 ft. 5 in., 67 mi. 216 rd. 2 yd. 2 ft. 3 in., 54 mi. 214 rd. 4 yd. 2 ft. 8 in., 32 mi. 90 rd. 3 yd. 2 ft. 9 in.

6. Add 15 lb. 9 oz. 17 pwt. 11 gr., 14 lb. 8 oz. 16 pwt. 23 gr., 15 lb. 6 oz. 3 pwt. 18 gr., 12 lb. 10 oz. 19 gr., 24 lb. 3 oz. 16 pwt. 5 gr., 32 lb. 14 pwt.

SUBTRACTION OF DENOMINATE NUMBERS.

WRITTEN EXERCISES.

132.—*1.* Subtract 9 cwt. 85 lb. 6 oz. from 15 cwt. 65 lb. 8 oz.

PROCESS.

cwt.	lb.	oz.
15	65	8
9	85	6
5	80	2

6 oz. from 8 oz. leaves 2 oz. Since 85 lb. is greater than 65 lb., we add 1 cwt., or 100 lb., to 65 lb., making 165 lb.; 85 lb. from 165 lb. leaves 80 lb. Having added 1 cwt. to the minuend, we must add 1 cwt. to the subtrahend, making it 10 cwt. 10 cwt. from 15 cwt. leaves 5 cwt.

2. Subtract 42° 32′ 18″ from 75° 39′ 12″.

3. Subtract 12 mi. 200 rd. 5 yd. 1 ft. 9 in. from 30 mi. 160 rd. 4 yd. 2 ft. 8 in.

4. Subtract 92 A. 150 P. 25 sq. yd. 7 sq. ft. 126 sq. in. from 110 A. 120 P. 20 sq. yd. 6 sq. ft. 118 sq. in.

5. From 132 gal. 1 qt. 1 pt. 1 gi. subtract 118 gal. 3 qt. 1 pt. 3 gi.

133. To find the difference in time between two dates.

There are two methods of finding the difference in time between two dates. The first method is by *compound subtraction,* in which we reckon 30 days to a month and 12 months to a year. The result is expressed in years, months, and days.

When greater accuracy is required we reckon the actual number of days in each month. The result is expressed in days.

1. Find the time from May 1, 1892, to July 4, 1895.

PROCESS.

yr.	mo.	da.
1895	7	4
1892	5	1
3	2	3

Expressing the dates in the *number* of the year, the month, and the day, and subtracting, we find the difference to be 3 yr. 2 mo. 3 da.

2. Find the exact number of days from Jan. 20, 1892, to Apr. 8, 1897.

Since 1892 and 1896 are both leap years, the number of days from Jan. 20, 1892, to Jan. 20, 1897, is 5 × 365 plus 2, or 1827.

Jan.,	11
Feb.,	28
Mar.,	31
Apr.,	8
	78

The number of days left in Jan. is 11. The number of days in Feb. is 28, and in Mar. is 31. From Apr. 1 to Apr. 8 is 8 days. Adding, the number of days from Jan. 20 to Apr. 8 is 78 days.

Then 1827 + 78 = 1905 days, *Ans.*

3. Franklin was born Jan. 6, 1706, and died Apr. 17, 1790; required his age.

4. The Civil War began Apr. 11, 1861, and closed Apr. 9, 1865; how long did it last?

5. General Grant was born Apr. 27, 1822, and died July 23, 1885; required his age.

6. Abraham Lincoln was born Feb. 12, 1809, and died Apr. 15, 1865; required his age.

7. President Garfield was born Nov. 19, 1831, and died Sept. 19, 1881; required his age.

Find the exact time in days from—

8. May 4th to Dec. 18th.
9. Jan. 8, 1888, to Aug. 30, 1888.
10. Dec. 12, 1891, to May 6, 1892.
11. Apr. 1st to Oct. 1st.
12. Feb. 12, 1888, to July 14, 1890.
13. Jan. 6, 1890, to Dec. 25, 1896.
14. Apr. 30, 1890, to Oct. 7, 1900.

MULTIPLICATION OF DENOMINATE NUMBERS.

WRITTEN EXERCISES.

134.—*1.* Multiply £5 6 s. 4 d. by 7.

PROCESS.

£.	s.	d.
5	6	4
		7
£37	4	4, *Ans.*

7 times 4 d. are 28 d., which equals 2 s. and 4 d.; we write the 4 d. under the pence, and add the 2 s. to the next product. 7 times 6 s. are 42 s., which added to the 2 s. equals 44 s., or £2 and 4 s.; we write the 4 s. under the shillings, and add the £2 to the next product. 7 times £5 are £35, which added to the £2 equals £37, which we write with the £.

2. Multiply 10 da. 16 hr. 20 sec. by 12.

3. Multiply 15° 15′ 15″ by 20.

4. Multiply 12 cwt. 25 lb. 12 oz. by 15.

5. Multiply 8 gal. 3 qt. 1 pt. 3 gi. by 24.

6. If 12 men were each paid £25 2 s. 6 d., what was the total amount received by them?

DIVISION OF DENOMINATE NUMBERS.

WRITTEN EXERCISES.

135.—*1.* Divide 28 bu. 3 pk. 4 qt. by 6.

PROCESS.

	bu.	pk.	qt.
6)	28	3	4
	4	3	2, *Ans.*

$\frac{1}{6}$ of 28 bu. is 4 bu. and 4 bu. remaining; 4 bu. equal 16 pk., which added to 3 pk. equal 19 pk.; $\frac{1}{6}$ of 19 pk. is 3 pk. and 1 pk. remaining; 1 pk. equals 8 qt., which added to 4 qt. equal 12 qt.; $\frac{1}{6}$ of 12 qt. is 2 qt.

2. Divide 78 lb. 8 oz. 15 pwt. 6 gr. by 6.

3. Divide 125° 37′ 48″ by 8.

4. 15 farms contain 1262 A. 3 R. 100 P.; what is the average size of a farm?

5. If £301 12 s. were equally divided among 12 men, what would each man receive?

6. Divide 20 gal. 2 qt. 1 pt. by 6 gal. 3 qt. 1 pt.

PROCESS.—$\frac{20 \text{ gal. } 2 \text{ qt. } 1 \text{ pt.}}{6 \text{ gal. } 3 \text{ qt. } 1 \text{ pt.}} = \frac{165 \text{ pt.}}{55 \text{ pt.}} = 3$, *Ans.*

We reduce the denominate numbers to the same denomination and divide as in integers.

7. Divide 50 bu. 3 pk. 1 qt. by 12 bu. 6 qt.

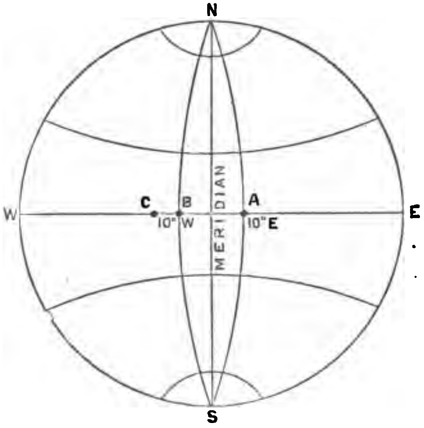
8. Divide 125° 40′ 25″ by 15° 4′ 51″.

9. How many kegs, each containing 6 gal. 3 qt. 1 pt., can be filled from a cask containing 34 gal. 1 qt. 1 pt.?

LONGITUDE.

136. Longitude is the distance east or west from a given meridian.

A *Meridian* is a line drawn from pole to pole through any point of the earth's surface; as, N A S, or N B S.



1. Suppose the line N S to represent any given meridian, and the city A to be 10° east and the city B to be 10° west from the given meridian; how far are the two cities apart?

2. Suppose C is 15° west and B is 10° west from the given meridian; how far are C and B apart?

WRITTEN EXERCISES.

Find the difference—

3. Between 40° E. and 30° W.
4. Between 75° W. and 50° W.
5. Between 80° E. and 110° E.
6. Between 60° 10′ E. and 40° 30′ W.
7. Between 75° 20′ 40″ W. and 25° 40′ 30″ E.
8. Between 85° 10′ 20″ E. and 60° 20′ 30″ E.

9. The longitude of New York is 74° 3′ W., and of Paris 2° 20′ 22″ E.; what is the difference of longitude?

10. The longitude of Boston is 71° 4′ 9″ W., and of San Francisco 122° 23′ W.; what is the difference of longitude?

11. The longitude of St. Louis is 90° 15′ 16″ W., and of St. Petersburg 30° 19′ E.; what is the difference of longitude?

12. The longitude of Berlin is 13° 23′ 53″ E., and of Shanghai 121° 28′ E.; what is the difference in longitude?

LONGITUDE AND TIME.

137. The earth makes a complete revolution on its axis from *west* to east once in twenty-four hours; this causes the sun to *appear* to revolve around the earth from *east* to west in the same time. Therefore, places east of a given meridian, being brought within sight of the sun earlier, have later time, and places west of a given meridian have earlier time.

138. The sun thus appears to travel through 360° in 24 hours, and in 1 hour it travels $\frac{1}{24}$ of 360°, or 15°; in 1 minute it travels $\frac{1}{60}$ of 15°, or 15′; and in 1 second it travels $\frac{1}{60}$ of 15′, or 15″. From this we derive the following

TABLE OF LONGITUDE AND TIME.

15° of longitude = 1 hour of time.
15′ " = 1 minute "
15″ " = 1 second "

ORAL EXERCISES.

1. Over how many degrees of the earth's surface does the sun appear to pass in 1 hr.? 2 hr.? 3 hr.? 4 hr.? 5 hr.?

2. How many degrees of longitude make a difference of 3 hr. of time? 5 hr.? 6 hr.? 7 hr.?

3. In what direction does the sun appear to move?

4. When it is noon at Philadelphia, is it earlier or later east of Philadelphia? West of Philadelphia?

5. When it is noon at Chicago, what is the time 15° east of Chicago? 15° west? 30° west? 30° east? 45° east? 45° west? 15′ east? 15′ west? 30″ east? 30″ west? 60″ east? 90″ west? 10″ east?

6. When it is noon at Chicago, in what direction and how far is the place whose time is 1 P. M.? 11 A. M.? 2 P. M.? 3 P. M.? 10 A. M.? 9 A. M.? 4 P. M.?

7. If I start at Chicago and travel 15° east, what change must I make in my watch? If 15° west? If 30° west? If 45° east? If 120° west? If 90° east?

8. If I start at Chicago and travel until my watch is 1 hour slow, how far and in what direction do I travel? 2 hr. fast? 5 hr. slow? 4 hr. fast?

9. If I start at Chicago and travel 15°, and find that my watch has the correct time of the place, in what direction do I travel?

NOTE.—The time referred to in the above problems is the local or true time of the place, and must not be confused with the *Standard Time* of the United States, which divides the country for the convenience of railroads into four sections. These take their time from the 75th, 90th, 105th, and 120th meridians west from Greenwich, and the time of each section is known as Eastern, Central, Pacific, and Mountain Time, respectively.

WRITTEN EXERCISES.

1. The longitude of Baltimore is 76° 37′ west, and of San Francisco 122° 23′ west; required the difference of time.

PROCESS.

122°	23′	
76	37	
15)45	46	
3	3	4

The difference of longitude is 45° 46′. Since 15 times the number of hr., min., and sec. of time equal the number of °, ′, ″, of longitude, then dividing by 15 we have 3 hr. 3 min. 4 sec.

RULE.

Divide the difference of longitude expressed in °, ′, ″ *by* 15; *the result will be the difference of time in* hr., min., sec.

2. When it is noon at San Francisco it is $13\frac{1}{3}$ min. past 3 P. M. at New York; what is the longitude of San Francisco, that of New York being 74° 3′ west?

PROCESS.

3 hr.	$13\frac{1}{3}$ min.
	15
48°	20′
74	3
122°	23′

The difference of time is 3 hr. $13\frac{1}{3}$ min. Since 15 times the number of hr., min., and sec. equal the number of °, ′, ″, the difference of longitude will be found by multiplying by 15, or 48° 20′. Since New York has later time, San Francisco is west of New York, and the difference of longitude, 48° 20′, must be added to the longitude of New York, making 122° 23′.

RULE.

Multiply the difference of time expressed in hr., min., sec. *by* 15; *the result will be the difference of longitude in* °, ′, ″.

Find the difference in time between—

3. New York, 74° 3′ W., and Paris, 2° 20′ 22″ E.

4. Washington, 76° 56′ W., and St. Louis, 90° 15′ 16″ W.

5. Boston, 71° 4′ 9″ W., and Berlin, 13° 23′ 53″ E.

6. Paris, 2° 20′ 22″ E., and Shanghai, 121° 28′ E.

7. When it is 4 A. M. at Washington, 77° 3′ W., what time is it at Paris, 2° 20′ 22″ E.?

8. When it is 6 P. M. at New York, 74° 3′ W., what time is it at San Francisco, 122° 23′ W.?

Find the difference in longitude between two places whose difference of time is—

9. 5 hr. 10 min. 20 sec.
10. 7 hr. 15 min. 32 sec.
11. 4 hr. 22 min. 8$\frac{2}{3}$ sec.
12. 8 hr. 40 min. 5$\frac{4}{5}$ sec.

13. When it is noon at Vienna it is 20 sec. past 7 P. M. at Shanghai; what is the longitude of Shanghai, that of Vienna being 16° 23′ E.?

14. When it is 10 A. M. in Philadelphia it is about 3 P. M. in London; what is the longitude of London, that of Philadelphia being 75° 9′ 5″ W.?

15. Paris is 2° 20′ 22″ E., and Washington is 76° 56′ W.; what change must I make in my watch in going from Paris to Washington?

16. The longitude of San Francisco is 122° 23′ W., and that of Jerusalem is 35° 32′ E.; when it is 9$\frac{1}{2}$ A. M. at San Francisco, what time is it at Jerusalem?

17. Rome is 12° 27′ E., and St. Louis is 90° 15′ 16″ W.; what change must I make in my watch in going from St. Louis to Rome?

18. The longitude of San Francisco is 122° 23′ W., and that of Shanghai is 121° 28′ E.; what time is it in San Francisco when it is 6$\frac{1}{2}$ A. M., May 5th, at Shanghai?

MISCELLANEOUS PROBLEMS.

1. If 8 boxes of goods weigh 25 cwt. 38 lb. 8 oz., what will 13 boxes weigh?

2. If 12 barrels of flour cost £15 8 s. 4 d., what will 23 barrels cost?

3. What is the exact number of days from May 4th to Dec. 25th?

4. What will $8\frac{2}{3}$ tons of coal cost, if $5\frac{2}{7}$ tons cost $37.74?

5. Add $4.00\frac{5}{8}$ mi. $3.0\frac{3}{11}$ rd. $5.14\frac{2}{3}$ yd. $8.00\frac{2}{3}$ ft.

6. Subtract 8 s. 5 d. from £6 4 s. 2 d., and multiply the result by $5\frac{1}{2}$.

7. How many cubic inches in 18.005 cords of wood?

8. If a mirror costs 4 s. 5 d., how many mirrors can be bought for £3 6 s. 3 d.?

9. If 5 men can do a piece of work in 4 hr. 30 min. 20 sec., how long will it take 8 men to do the work?

10. Find the sum of $\frac{2}{3}$ of 35° 20′ 48″, $\frac{2}{5}$ of 48° 37′ 40″, and $\frac{3}{4}$ of 87° 23′ 40″.

11. A note dated June 12, 1894, was paid Jan. 5, 1895; how long did it run?

12. What will 10 A. 35 P. of land cost at $240 an acre?

13. What will 30 bu. 3 pk. of wheat cost at 55 cents a bushel?

14. What is the distance in feet around a field whose sides are 48 rd. 4 yd. 3 ft., 15 ch. 42 li., and 400 ft. 6 in.?

15. What will 8 T. 12 cwt. of coal cost at $5.25 a ton?

16. How long will it take to empty a stand-pipe which holds 1400 barrels of water with a pump that discharges 50 gal. 2 qt. 1 pt. per second?

17. A quantity of metal weighs 2 T. 68 lb. 12 oz., and an equal quantity of water weighs 2 cwt. 54 lb. $4\frac{3}{4}$ oz.; the metal is how many times the weight of the water?

18. Two men start at different places and travel toward each other until they meet: one finds his watch 1 hr. 15 min. fast, and the other, 3 hr. 32 min. slow; how far and in what direction did each travel?

19. If a man can do a piece of work in 23 hr. 40 min. 25 sec., what part of it can he do in 14 hr. 12 min. 15 sec.?

20. A gentleman who visited the World's Fair at Chicago found that his watch, which had correct local time, was 1 hr. 45 min. fast; did he live east or west from Chicago, and how many degrees?

21. If a bushel of corn weigh 56 lb., what is the value of 6 T. 12 cwt. of corn at 63 cents a bushel?

22. The difference in time between two places on the earth's equator is 4 hr. 35 min. 30 sec.; what is the distance in miles between the places if a degree is equal to 69$\frac{1}{6}$ miles?

23. If the moon revolves around the earth in 27 da. 7 hr. 42 min., over how many degrees of its orbit does it pass in 22 da. 18 hr. 25 min.?

24. A and B bought 400 bu. 3 pk. of wheat for $240.45; if A pays $103.05, and B $137.40, how many bushels should each receive?

25. If the sides of a field are, respectively, 22 ch. 6$\frac{2}{33}$ li., 24 ch. 96$\frac{32}{33}$ li., and 27 ch. 39$\frac{13}{33}$ li., what is the length of the longest rail possible that can be used in making a fence around it without cutting or overlapping the rails, and how many rails will be required if the fence is 5 rails high?

26. If a rail weighs 21 pounds to the foot, how many tons will be required to lay 3 miles of railway?

27. An opera-glass cost $18.50; find its cost in English money if a pound is worth $4.8665.

28. A French-plate mirror cost $40.75; find its cost in German money if a mark is worth 23.85 cents.

29. If a cubic foot of water weighs 1000 oz., and mercury is 13.596 times as heavy as water, what is the weight in pounds of 216 cubic inches of mercury?

30. What decimal of a lb. avoirdupois is a ℔ Troy?

31. A vessel filled with water weighs 60 pounds: a mass of gold is thrown into it which displaces 5 lb. of water; how much do the vessel and its contents now weigh if gold is 19.4 times as heavy as water?

32. Express 6 ℥ 2 ʒ 2 ℈ 4 gr. in avoirdupois weight.

33. If a stick 5 ft. 4 in. high cast a shadow 3 ft. 6 in. long, how high is a building whose shadow is 120 ft. 9 in. at the same time of the day?

34. How many days from the 4th of March, 1889, to the 4th of July, 1890?

35. A locomotive runs 81 miles in 2 hr. 14 min.; in what time will it run from Lancaster to Philadelphia, a distance of 69 miles, at the same rate?

36. If a bushel of wheat weighs 60 pounds, what is the value of A's crop of wheat, which weighs 8 T. 14 cwt., at 56 cents a bushel?

37. If a franc is worth 19.3 cents, what is the value in francs of £3 8 s. 4 d. when a pound is worth $4.8665?

38. A mass of copper weighs 88 cwt. 49 lb. 8 oz., and an equal mass of water weighs 10 cwt. 5 lb. 10 oz.; the copper is how many times as heavy as the water?

39. If A starts at the equator and travels due north 3000 miles, what latitude does he reach, allowing 69⅙ miles to a degree?

40. Reduce 145 rd. 3 yd. 1 ft. 8 in. to the decimal of a mile.

41. What cost 3 bbl. of sugar, each weighing 2 cwt. 12 lb. 10 oz., if 1.26 cwt. cost $7.40?

42. What cost 12 lb. 11 oz. of drugs if 5 lb. 9 oz. cost $22.25?

43. Since noon the sun has seemed to pass over 20° 20′ 20″; what time is it?

44. If B travels 40 mi. 150 rd. 3 yd. in 8 hr. 20 min., how far will he travel in 12 hr. 50 min.?

45. A 5-gallon cask containing vinegar lacks 1½ pints of being full; what is the quantity of vinegar in the cask worth at 8½ cents a quart?

46. A grain-dealer's scales are 25 pounds in a ton below the legal standard; how much does he gain fraudulently from the sale of 5 car-loads of wheat, each containing 240 bags of 174 pounds each, true weight, at 56 cents a bushel, reckoning 60 pounds to the bushel?

SECTION VII.

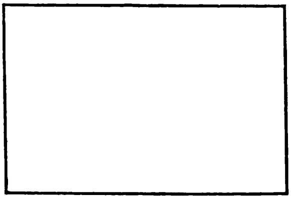
PRACTICAL MEASUREMENTS.

MEASURES OF SURFACES.

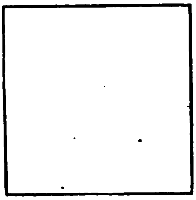
139. A **Surface** is that which has length and breadth without thickness.

THE RECTANGLE.

140. A **Rectangle** is a plane figure having four sides and four right angles; as, a slate, a door, etc. The **Dimensions** of a rectangle are its length and breadth.



A Rectangle.



A Square.

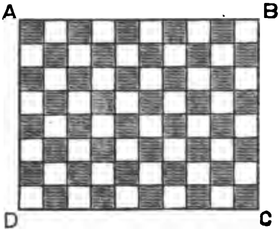
A rectangle has two bases; the side upon which it seems to stand is the *lower base*, and the side opposite the *upper base.*

The *Altitude* of a rectangle is the perpendicular distance between its bases.

A *Square* is a rectangle whose sides are all equal. The *Perimeter* of a rectangle is the sum of its sides.

The *Area* of a rectangle is the number of square units it contains.

Thus, in the rectangle A B C D the *area* is the number of small squares it contains, which is equal to the number in each row multiplied by the number of rows, and this is equal to the number of linear units in the length multiplied by the number in the breadth. Hence the



RULE.

I. *To find the area of a rectangle, multiply the length by the breadth.*

II. *To find either side of a rectangle, divide the area by the other side.*

WRITTEN EXERCISES.

1. How many square feet in a yard 30 feet long and 20 feet wide?

2. How many square yards in a floor 33 feet long and 24 feet wide?

3. How many square rods in a field 120 rods long and 80 rods wide?

4. How many acres in a field 240 rods long and 120 rods wide?

5. How many acres in a field 15 chains long and 8 chains wide?

6. What is the width of a field 80 rods long which contains 12 acres?

7. How many square yards in a floor 16 ft. 6 in. long and 12 ft. 8 in. wide?

8. How many square yards in the sides of a room 24 ft. long, 20 ft. wide, and 12 ft. high?

9. A field is 10 chains long and 60 rods wide; how many acres does it contain?

10. How many chains long must a field 10 ft. wide be to contain an acre?

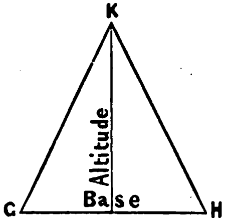
11. What will it cost to pave a sidewalk 75 ft. long and 6 ft. wide at 21 cts. a square foot?

12. What will be the cost of a concrete walk 120 ft. long and 4 ft. wide at 44 cts. a square yard?

13. What will it cost to cement the cellar of a house 40 ft. long and 24 ft. wide at 30 cts. a square yard?

THE TRIANGLE.

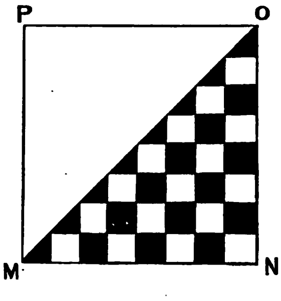
141. A **Triangle** is a plane figure having three sides and three angles; as, C H K.



A Triangle.

The *Base* is the side upon which it seems to stand. The *Altitude* is the perpendicular distance from the vertex, K, to the base.

The *triangle* M N O is seen to be one-half of the rectangle M N O P. If the rectangle is 8 units long and 8 units wide, it will contain 64 square units, and the triangle will contain 32 square units. This is equal to the number of linear units in the base multiplied by one-half the number in the altitude. Hence the



RULE.

I. *To find the area of a triangle, multiply the base by one-half the altitude.*

II. *To find the base or altitude of a triangle, divide the area by one-half the other dimension.*

WRITTEN EXERCISES.

1. What is the area of a triangle whose base is 35 inches and altitude 20 inches?

2. How many square yards in a triangular garden whose base is 45 ft. and whose altitude is 32 ft.?

3. How many acres in a triangular field whose base is 20 chains and altitude 15 chains?

4. A triangular field contains 16 A.; what is its altitude if it is 160 rods long?

5. What is the base of a triangular field whose altitude is 18 feet and whose area is 60 sq. yd.?

6. Find the area of a triangle whose base is 560 rods and altitude 90 chains.

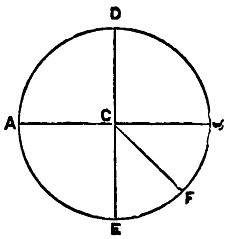
7. Find the area in square feet of a triangle whose base is 16½ feet and altitude 5½ feet.

8. Find the area in square yards of a triangle whose base is 8⅓ yards and altitude 6⅔ feet.

9. What part of an acre is a triangle whose base is 82½ feet and altitude 2 chains

THE CIRCLE.

142. A **Circle** is a plane figure bounded by a curved line every point of which is equally distant from a point within called the *centre*.



The *Circumference* is the bounding line; as, A D B E. Any part of the circumference, as B D, E F, etc., is an *Arc*. The *Diameter* is a straight line passing through the centre and terminating in the circumference; as, A B, D E. The *Radius* is a line drawn from the centre to the circumference; as, C D, C F, etc.

RULE.

I. *To find the circumference of a circle, multiply the diameter by* 3.1416.

II. *To find the diameter of a circle, divide the circumference by* 3.1416 *or multiply the circumference by* .3183.

III. *To find the area of a circle, multiply the circumference by one-fourth of the diameter, or multiply the square of the radius by* 3.1416.

To square a number, multiply it by itself.

WRITTEN EXERCISES.

Find the circumference of a circle whose—

1. Diameter is 10 inches.
2. Radius is 10 inches.
3. Diameter is 31 feet.
4. Radius is 50 rods.

Find the diameter of a circle whose—

5. Circumference is 10 feet.
6. Circumference is 40 inches.
7. Circumference is 80 rods.
8. Circumference is 200 chains.

Find the area of a circle whose—

9. Radius is 10 inches.
10. Diameter is 40 feet.
11. Circumference is 100 rd.
12. Radius is 18 feet.

13. The diameter of a circular fish-pond is 40 feet; what is its area?

14. Find the circumference of a circular window whose diameter is 3 feet.

15. A horse is tied to a stake by a rope 22 feet long; over how much space can he graze?

16. A pond 20 rods in diameter has a walk around it 6 feet wide; what is the area of the walk?

17 What is the area of a circle whose radius is a mile?

18. The area of a circle is 1 A. 154.16 P.; what is its circumference?

19. A wagon-wheel is 3 ft. 6 in. in diameter; how many miles does it travel in revolving 5000 times?

PAINTING, PAPERING, PLASTERING, ETC.

143. *Painting, papering, plastering,* etc. are estimated by the *square foot* or *square yard.*

Roofing is generally estimated by the *square,* which consists of 100 square feet.

Shingles are estimated by the *thousand.* With shingles that average 4 inches in width and are laid—

6	in. to	the weather,	600	shingles cover a	*square.*
7	"	"	515	"	"
8	"	"	450	"	"
9	"	"	400	"	"

WRITTEN EXERCISES.

1. What will it cost to paint a house 40 feet long, 30 feet wide, and 20 feet high, at 15 cents a square yard, no allowance being made for doors or windows?

2. What will it cost to plaster 6 rooms, each 20 ft. long, 15 ft. wide, and 10 ft. high, at 40 cents a square yard?

3. What will it cost to slate a roof 40 ft. 6 in. long, each side being 24 ft. wide, at $16 a square?

4. A roof is covered with shingles put 8 in. to the weather; what is the cost at $12 a thousand if the roof is 80 ft. long and each side is 30 ft. wide?

5. What will it cost to paint a barn 30 ft. long and 20 ft. wide, 16 ft. to the eaves, the gables being 8 ft. high, at 40 cents a square, if it requires 10 gallons of paint at $1.60 a gallon?

6. What will it cost to cover a room 24 feet long and 18 feet wide with carpet 27 in. wide at $1.25 a yard, allowing 5 yards for matching; the strips to be laid lengthwise?

7. What will it cost to cover a hall 33 feet long and 5 feet wide with matting 30 inches wide at 55 cents a yard?

8. What will it cost to cover the roof of a barn 80 feet long and 56 feet wide with shingles which average 4 inches in width and are put 8 inches to the weather, the length of the rafters being $\frac{5}{8}$ of the width of the barn, and the shingles costing $12 per thousand?

9. What will it cost to cover a floor 18 ft. long and 12 ft. 3 in. wide with oil-cloth at 50 cents a square yard?

10. Which will be the cheaper, and how much, to cover a floor 25 feet long and 22 ft. 8 in. wide with matting in strips 2 ft. 10 in. wide, laid lengthwise, at 45 cents a yard, or to cover it with oil-cloth at 50 cents a square yard?

11. What will it cost to plaster a room 18 ft. long, 12 ft. wide, and 10 ft. high, at 41 cents a square yard, allowing 90 sq. ft. for doors and windows?

12. How many rolls of paper, each containing 36 sq. ft., will be required to paper a room 16 ft. long, 15 ft. wide, and 10 ft. high, no allowance being made for doors and windows?

13. What will the paper cost for a room 20 ft. long, 16 ft. wide, and 12 ft. high, with paper 18 in. wide and 8 yd. long, at 20 cents a roll, allowing 100 sq. ft. for doors and windows?

14. What will it cost to paper a room 21 ft. long, 15 ft. wide, and 10 ft. high, with paper 18 in. wide and 8 yd. long, at 24 cents a roll, allowing for 3 windows 6 ft. by 3 ft., and 4 doors 7 ft. by 3½ ft., if the work can be done by 2 men in 1 day at $2.50 each per day?

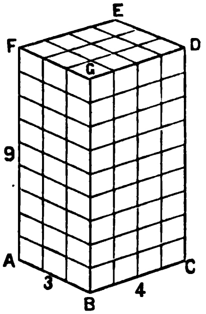
15. A frame barn is covered with shingles put 6 in. to the weather; what is the cost at $11 a thousand if the roof is 60 ft. long, each side being 32 ft., the first course along the eaves being doubled?

16. How many yards of plain matting 30 inches wide will be required to cover a hall 65 feet long and 30 feet wide, there being no allowance for waste and the strips running lengthwise? How many yards if the strips run crosswise?

MEASURES OF VOLUME.

144. A **Volume** is that which has length, breadth, and thickness. A volume is called a *solid.*

THE RECTANGULAR SOLID.



A **Rectangular** solid is a volume bounded by six rectangular faces.

A **Cube** is a volume bounded by six equal squares.

The **Contents** of a volume is the number of cubic units it contains. Thus, in the volume A B C D E F the contents is the number of small cubes it contains, which is equal to the number in the base multiplied by the number of layers; hence the whole number of cubes, or the contents, equals the product of the length, breadth, and thickness. Hence the

RULE.

To find the contents of a rectangular volume, take the product of the length, breadth, and height.

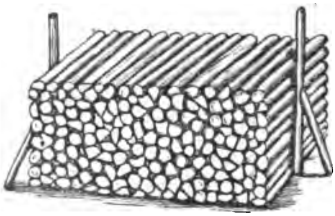
WRITTEN EXERCISES.

1. How many cubic feet in a rectangular block 8 ft. long, 6 ft. wide, and 4 ft. high?

2. How many cubic yards of earth in a cellar 36 ft. long, 24 ft. wide, and 5½ ft. deep?

3. Find the contents of a cube 8 ft. 6 in. on a side.

4. How many cubic feet of water in a rectangular reservoir 80 ft. long, 60 ft. wide, and 10 ft. deep? What is its weight, if a cubic foot of water weighs 62½ lbs.?



5. How many cords in a pile of wood 72 ft. long, 12 ft. wide, and 8 ft. high?

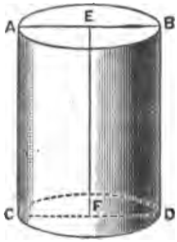
128 cu. ft. = 1 cord.

6. What is the cost of a pile of wood 120 ft. long, 8 ft. wide, and 10 ft. high, at $5.25 per cord?

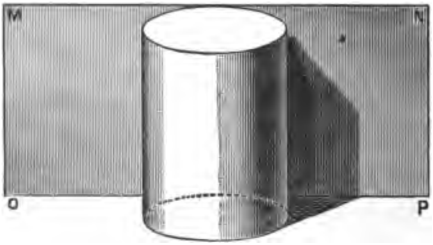
7. What will be the cost of the bark that can be piled in a shed 120 ft. long, 60 ft. wide, and 15 ft. high, at $6½ a cord?

8. What will it cost to dig a cellar for a house 40 ft. long, 32 ft. wide, and 4 ft. 6 in. deep, at $1.50 a cubic yard?

THE CYLINDER.



A Cylinder.



A Cylinder unfolded.

145. **A Cylinder** is a round body with equal parallel circles for its bases, and having a uniform diameter.

The *Altitude* of a cylinder is the perpendicular distance between its bases.

The *Convex Surface* of a cylinder is the curved surface which bounds it.

If the convex surface of a cylinder be unfolded, it will form a rectangle whose length is the circumference of the base, and whose altitude is the height of the cylinder. Hence the

RULE.

To find the convex surface of a cylinder, multiply the circumference of the base by the altitude.

To find the *contents* of a cylinder:

RULE.

To find the contents of a cylinder, multiply the area of the base by the altitude.

WRITTEN EXERCISES.

1. Find the convex surface of a cylinder whose altitude is 12 ft. and the radius of the base 5 ft.

2. How many cubic feet in a cistern 8 ft. deep and 6 ft. in diameter?

3. What is the convex surface of a cylinder whose altitude is 20 ft. and the diameter of the base 20 ft.?

4. Find the convex surface and the contents of a cylinder 5 ft. in diameter and 30 ft. high.

5. How many cubic feet in a log 24 ft. long and 4 ft. in circumference?

6. A bushel measure is a cylinder 18½ in. in diameter and 8 in. deep; find the contents.

7. What will it cost to paint the outside of a stand-pipe 75 ft. high and 10 ft. in diameter at 10 cents a square yard?

8. How many cubic feet of iron in a tube 30 ft. long whose circumference is 37.6992 inches, the thickness of the material being 2 inches?

9. Which is the larger, and how much, a cylinder whose circumference is 12 feet, or a volume whose base is a square 12 ft. in perimeter, both being 10 ft. high?

10. The volume of a cylinder is 2513.28 cu. ft., and the diameter of the base is 20 ft.; what is the altitude, and what is the convex surface?

BINS, TANKS, AND CISTERNS.

146. The *capacity* of bins is estimated by the bushel, and the capacity of tanks and cisterns by the gallon, barrel, or hogshead.

The **Standard Bushel** of the United States is a cylindrical measure 18½ inches in diameter and 8 inches deep.

RULE.

To find the number of bushels in a bin, divide the contents in cubic inches by 2150.42.

Since 1728 is to 2150.42 as 4 is to 5, nearly, *for practical purposes* ⅘ *of the number of cubic feet will equal the number of bushels.*

In the following problems—

1 bushel = 2150.42 cu. in.
1 barrel = 31½ gallons.
1 gallon = 231 cu. in.
1 hogshead = 63 gallons.

WRITTEN EXERCISES.

Estimate, by taking ⅘ of the number of cubic feet as bushels, the capacity of a bin—

1. 8 ft. by 4 ft. by 5 ft.
2. 6 ft. by 3 ft. by 10 ft.
3. 12 ft. by 40 ft. by 8 ft.
4. 20 ft. by 16 ft. by 12 ft.

Calculate exactly the capacity of a bin—

5. 4 ft. by 5 ft. by 6 ft.
6. 8 ft. by 4 ft. by 3 ft.
7. 10 ft. by 12 ft. by 15 ft.
8. 6 ft. by 8 ft. by 10 ft.

9. What is the difference between the exact number of bushels in a bin 3 ft. by 4 ft. by 5 ft. and the number found by estimating ⅘ of the number of cubic feet as bushels?

10. A bin 8 ft. by 10 ft. by 12 ft. is ¾ full of wheat; what is the difference between the exact number of bushels in the bin and the number obtained by estimating as above?

11. A bin 16 ft. long, 10 ft. wide, and 8 ft. high is $\frac{3}{8}$ full of wheat; at 60 cents a bushel how much more is it worth, calculated exactly, than by estimating $\frac{4}{5}$ of the number of cubic feet as bushels?

12. How many barrels of flour can be made from the contents of a bin 16 ft. long, 10 ft. wide, and 8 ft. deep, if it is full of wheat, and if a bushel of wheat makes 50 lb. of flour?

13. How many gallons of water will a cistern 8 ft. deep and 6 ft. in diameter hold?

14. How many barrels of water in a well 4 ft. in diameter, if the water is 15 ft. deep?

15. How many hogsheads of water will a stand-pipe hold whose diameter is 10 ft. and altitude 75 ft.?

16. If a cubic foot of water weighs 1000 oz., how many tons will the water in a stand-pipe whose diameter is 10 ft. and altitude 75 ft. weigh if the stand-pipe be full?

17. If a well holds 20 hhd. of water and is 4 ft. in diameter, how deep is it?

18. What is the depth of a cylindrical measure 4 in. in diameter which will hold a dry gallon of 268.8 cu. in.?

BOARDS AND TIMBER.

147. Boards one inch or less in thickness are estimated by the square foot.

Thus, a board 16 ft. long, 15 in. wide, and 1 in. or less in thickness contains $16 \times \frac{15}{12} = 20$ sq. ft.

When lumber is more than one inch thick the thickness is taken into account, and the lumber is estimated by what is called a **Board-foot,** which is 1 foot square and 1 inch thick.

Thus, a board 16 ft. long, 15 in. wide, and $1\frac{1}{2}$ in. thick contains $16 \times \frac{15}{12} \times 1\frac{1}{2} = 30$ ft.

It is readily seen that 1 **Cubic foot** = 12 **Board-feet.**

RULE.

I. *To find the contents of a board, multiply the length in feet by the width in inches, and divide by* 12.

II. *To find the contents of timber more than an inch thick, multiply the length in feet by the width and thickness in inches, and divide by* 12.

WRITTEN EXERCISES.

How many board-feet in a board—

1. 16 ft. long, 9 in. wide, and 1 in. thick?
2. 18 ft. long, 12 in. wide, and 1½ in. thick?
3. 15 ft. long, 8 in. wide, and 2 in. thick?
4. 20 ft. long, 15 in. wide, and 1½ in. thick?
5. 12 ft. long, 16 in. wide, and 3 in. thick?

How many board-feet in a scantling, etc.—

6. 12 ft. long, 3 in. wide, and 4 in. thick?
7. 24 ft. long, 3 in. wide, and 7 in. thick?
8. 30 ft. long, 5 in. wide, and 6 in. thick?
9. 20 ft. long, 3 in. wide, and 6 in. thick?
10. 15 ft. long and 12 in. square?
11. 18 ft. long and 6 in. square?

12. What will 20 scantling 16 ft. long, 3 in. wide, and 4 in. thick cost, at $25 per thousand?

13. What cost 40 boards 16 ft. long, 15 in. wide, and ½ in. thick, at $30 a thousand?

14. What is the cost of 5 girders, each 50 ft. long, 14 in. wide, and 12 in. thick, at $30 per M.?

15. What will it cost to enclose a rectangular lot 120 ft. long and 90 ft. wide with a tight board fence 6 ft. high, if the boards are worth $24 per M.?

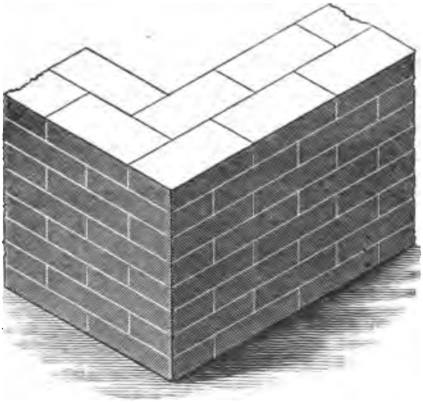
16. What will it cost to lay a floor 80 ft. long and 60 ft. wide at $35 a thousand, adding ⅕ for grooving?

MASONRY AND BRICKWORK.

148. Masonry is commonly estimated by the *perch*, which contains 24¾ cubic feet; in many cases, however, it is estimated by the cubic foot and cubic yard.

149. Brickwork is estimated by the *thousand bricks.*

Bricks vary considerably in their dimensions, an ordinary size being 8 inches long, 4 inches wide, and 2 inches thick. In estimating the number of ordinary bricks required to build a wall, ¼ of an inch for mortar is added to the length and thickness of each brick, making the above dimensions of the face, or surface exposed to view, 8¼ by 2¼ inches, or $\frac{297}{16}$ square inches; hence in a wall one brick in thickness each square foot, or 144 square inches, of wall surface would require as many bricks as $\frac{297}{16}$ is contained times in 144, or a little more than 7 bricks, seven, however, being the number usually taken. If the wall is two bricks in thickness, 14 bricks will be required; and so on. Hence the common



RULE.

To estimate the number of bricks required to build a wall, multiply the number of square feet in the surface by 7 *when the wall is* 1 *brick thick; by* 14 *when the wall is* 2 *bricks thick; and by* 21 *when the wall is* 3 *bricks thick.*

NOTE.—In cases where bricks vary considerably from the ordinary dimensions a more accurate method is to add ¼ of an inch to the length and thickness of a brick, and divide 144 by the product of these increased dimensions for the number of bricks to a square foot of wall surface.

150. In estimating **material** for the walls of a building, whether brick or stone, allowance is made for the doors, windows, and corners. In estimating the cost of **labor** the length of the wall is measured on the outside; the corners are thereby measured twice, but this is considered an offset for the greater labor required in their construction. For the same reason no allowance is made for doors and windows, except in case of special contract, when one-half of the actual space is usually deducted.

NOTE.—In sworn measurements of material in brick walls already built it is customary to count the bricks in a layer and multiply by the number of layers, making proper allowance for doors and windows. Generally, a wall two bricks thick is regarded as a 9-inch wall; one three bricks thick, a 13-inch wall; 4 bricks thick, a 17-inch wall, etc.

RULE.

To find the number of perches of stone in a wall, divide the number of cubic feet in the wall by $24\frac{3}{4}$.

WRITTEN EXERCISES.

1. How many perches of stone in a wall 80 ft. long, 6 ft. high, and 2 ft. thick?

2. How many bricks will be required to construct a wall 40 ft. long, 12 ft. high, and 13 in. thick?

3. How many bricks will it take to build a wall around a square yard 40 ft. on a side, if the wall is 7 ft. high and 3 bricks thick?

4. How many bricks are required to build the walls of a house 36 ft. long, 28 ft. wide, and 22 ft. high, the walls to be 13 in. thick, allowing 500 sq. ft. for doors and windows?

5. What will it cost to lay the bricks in a hall 120 ft. long, 75 ft. wide, 30 ft. high, and 21 in. thick, allowing one-half for doors and windows, there being 12 windows 3 ft. by 12 ft. and 4 doors 5 ft. by 10 ft., at $3 per thousand?

6. How many perches of stone are required to wall a well 50 ft. deep, if the wall is 1 ft. thick and the well is 4 ft. in diameter in the clear?

7. What will be the cost of digging and walling the cellar of a house 24 ft. by 32 ft., the cellar being 5 ft. deep and the wall 8 ft. high and 18 in. thick, if the digging cost 50 cents a cubic yard and the masonry $4 a perch?

MISCELLANEOUS PROBLEMS.

1. A gardener has a flower-bed 8 ft. long and 6 ft. wide, and wishes to make a board-walk around the outside 4 ft. wide; what will it cost at 3 cents a square foot?

2. A farmer has an acre of land 5 ch. long and 2 ch. wide: he wishes to make a tight board fence around it 5 ft. high; what will the boards cost at 2½ cents a square foot?

3. What will it cost to dig a trench 40 rd. long, 18 in. wide, and 2 ft. deep at 10 cents a cubic yard?

4. What will it cost to dig a well 7 ft. in diameter and 42 ft. deep at 50 cents a cubic yard?

5. How many square yards of zinc will be required to line a cylinder 8 ft. in diameter and 40 ft. high, including both ends?

6. How many acres of land in a triangular field whose base is 120 rd. and altitude 80 rd.?

7. Mr. White has a rectangular farm which contains 92 acres; how many rods of fence will be required to enclose it if the width is 20 chains?

8. How much more will it cost to fence a field 20 rd. wide, and containing 40 acres, than to fence a square field of equal area 80 rods on a side, if the fence costs $2 a rod?

9. How many boards 16 ft. long and 15 in. wide will be required to weather-board a barn 80 ft. long, 50 ft. wide, and 20 ft. to the eaves if the two gable ends are 13 ft. high?

10. How many cubic yards of earth must be removed to dig a cellar 40 ft. long, 30 ft. wide, and 6 ft. 5 in. deep?

11. How deep must a cistern 6 ft. in diameter be to contain 40 hhd.?

12. How many cubic feet in a cistern which contains 100 hhd.?

13. A druggist buys 20 lb. of drugs by avoirdupois weight at $10.50 a pound, and sells them at the rate of $1.05 an ounce apothecaries' weight; what is his gain?

14. A man weighs 175 lb. avoirdupois; how much would he weigh by Troy weight?

15. A wagon 8 ft. long and 4 ft. wide is piled with wood to a depth of 6½ ft; what is the cost of the wood at $8.10 a cord?

16. If a vessel holds 600 gallons of water, how many bushels of grain can be put into it?

17. If 5 bushels of seed will sow an acre, how much seed will be required to sow a rectangular piece of land 125 ft. long and 50 ft. wide?

18. How deep must a bin be that is 8 ft. long and 6 ft. 6 in. wide to hold 200 bushels of wheat?

19. Find the cost of 50 white-pine joists, each 16 ft. long, 7 in. wide, and 4 in. thick, at $28 per M.

20. Find the cost of a plank-walk 100 ft. long, 5 ft. 3 in. wide, and 2 in. thick, at $15.50 per M.

21. How many perches of stone will be required to build a monument 120 ft. high and 20 ft. square, with a cylindrical opening through the centre 8 ft. in diameter?

22. If a cubic foot of water weighs 1000 oz., what is the weight of 15 hhd. of water?

23. An irregular stone was thrown into a cylinder 2 ft. 6 in. in diameter, partly filled with water: when the stone was immersed in it, the water in the cylinder rose 10 in.; find the contents of the stone.

24. A circular lake 3 chains in diameter has a walk 8 ft. wide around it; what is the area of the walk?

25. The distance around a circular park is 2 miles; how many acres does it contain?

26. How many bricks are required to build the walls of a house 32 ft. long, 28 ft. wide, and 24 ft. high, the gables being 12 ft. high and the walls 17 in. thick, if there are 24 windows 2 ft. 10 in. by 6 ft. 2 in., and 5 doors 3 ft. 4 in. by 8 ft. 6 in.?

27. What will be the cost of laying the bricks of the preceding problem at $3.50 a thousand, if an allowance of $\frac{1}{2}$ is made for doors, windows, and corners?

28. M and N buy a piece of land containing 14 A. 120 P. for $2380; how much land should each receive, if M pays $952 and N the remainder?

29. If gold is worth $21.50 an ounce Troy, what is the value of a piece of gold weighing 20 lb. 10 oz. avoirdupois?

30. The longitude of Constantinople is 28° 49′ east, and of St. Paul 93° 5′ west; what time is it at Constantinople when it is midnight at St. Paul?

31. The forward wheels of a wagon are 13 ft. 4 in. in circumference, and the hind wheels 18 ft. 6 in.; how many more times will the forward wheels revolve than the hind wheels in going from Philadelphia to New York, a distance of 90 miles?

32. If a piece of metal is 10 in. wide and 4 in. thick, what length of it will be required to make 8 cu. ft.?

33. If 8 T. 12 cwt. 45 lb. of iron cost $517.35, how much will 12 T. 11 cwt. 42$\frac{1}{2}$ lb. cost at the same rate?

34. R owns a piece of land which is 80 ch. 42 li. long and 60 ch. 36 li. wide; how many acres does it contain?

35. Mr. Taylor's house is 36 ft. long and 32 ft. wide: the roof projects 6 in. on each side and 14 in. at the ends, and all the water falling on the roof is conducted to a cistern 7 ft. in diameter; to what depth would it be filled by a shower in which 1$\frac{1}{8}$ in. of rain fell?

SECTION VIII.

PERCENTAGE.

ORAL EXERCISES.

151.—*1.* A FARMER planted 200 trees, and 1 out of each hundred died; how many trees died in all? If 2 out of each hundred had died, how many would have died in all? If 3? If 4? If 5? If 6?

2. If 7 trees out of each hundred die, what part of them die? If 9 out of each hundred die? If 11? If 13? If 17? If 25? If 50? If 75? If 100?

These expressions, 7 hundredths, 9 hundredths, 11 hundredths, etc. of a number are called **Per Cents.**, from the Latin *per centum*, which means *by the hundred.*

3. What per cent. of a number is 3 hundredths of it? 5 hundredths? 6 hundredths? 8 hundredths? 9 hundredths? $\frac{12}{100}$? $\frac{15}{100}$? $\frac{16}{100}$? $\frac{20}{100}$? $\frac{25}{100}$? .06? .09? .12? .18? .45? .80? .85? .90? .94?

4. How many hundredths of a number is 6 per cent.? 8 per cent.? 9 per cent.? 10 per cent.? 12 per cent.? 20 per cent.? 90 per cent.?

The symbol % is generally used instead of *per cent.* Thus, 5% means 5 per cent.

5. How many hundredths of a number is 7%? 14%? 15%? 6%? 5%? 50%? 75%? 80%? 100%?

152. To express a per cent. as a decimal and as a common fraction.

Any per cent. of a number may be expressed as a *decimal* or as a *common fraction.* Thus,

$5\% = .05 = \frac{5}{100} = \frac{1}{20}$.

$6\% = .06 = \frac{6}{100} = \frac{3}{50}$.

$8\% = .08 = \frac{8}{100} = \frac{2}{25}$.

$10\% = .10 = \frac{10}{100} = \frac{1}{10}$.

WRITTEN EXERCISES.

Express each of the following as a decimal and as a common fraction in its lowest terms:

1. 15%, 40%.
2. 20%, 50%.
3. 25%, 75%.
4. 30%, 80%.
5. 9%, 16%.
6. 12%, 18%.
7. $12\frac{1}{2}\%$, $16\frac{2}{3}\%$.
8. 24%, $33\frac{1}{3}\%$.
9. $6\frac{1}{4}\%$, $11\frac{1}{9}\%$.
10. $18\frac{3}{4}\%$, 28%.
11. 90%, 125%.
12. $66\frac{2}{3}\%$, 150%.
13. $8\frac{1}{3}\%$, $37\frac{1}{2}\%$.
14. $62\frac{1}{2}\%$, $87\frac{1}{2}\%$.
15. $\frac{1}{2}\%$, $\frac{1}{4}\%$.

153. To express a common fraction as a per cent.

Any common fraction may be expressed as a per cent. by reducing it to hundredths. Thus,

$\frac{1}{2} = \frac{50}{100} = .50 = 50\%$.

$\frac{3}{4} = \frac{75}{100} = .75 = 75\%$.

$\frac{1}{25} = \frac{4}{100} = .04 = 4\%$.

$\frac{13}{25} = \frac{52}{100} = .52 = 52\%$.

WRITTEN EXERCISES.

1. Express $\frac{2}{3}$ as a per cent.

PROCESS.

3)2.
.66⅔, or 66⅔%, *Ans.*

Dividing 2 by 3 to two places of decimals, we have $\frac{2}{3} = .66\frac{2}{3}$, or $66\frac{2}{3}\%$.

Express each of the following fractions as a per cent.:

2. $\frac{1}{2}$, $\frac{1}{6}$, $\frac{3}{4}$.
3. $\frac{1}{3}$, $\frac{1}{8}$, $\frac{4}{7}$.
4. $\frac{1}{4}$, $\frac{1}{10}$, $\frac{7}{12}$.
5. $\frac{1}{5}$, $\frac{1}{12}$, $\frac{1}{9}$.
6. $\frac{1}{16}$, $\frac{3}{5}$, $\frac{2}{5}$.
7. $\frac{3}{10}$, $\frac{5}{8}$, $\frac{3}{8}$.
8. $\frac{2}{3}$, $\frac{6}{9}$, $\frac{3}{12}$.
9. $\frac{1}{125}$, $\frac{1}{200}$.
10. $\frac{1}{400}$, $\frac{5}{2}$.

The following per cents., with their fractional equivalents, are so frequently used that they should be *committed to memory:*

$6\frac{1}{4}\% = \frac{1}{16}$.	$20\% = \frac{1}{5}$.	$40\% = \frac{2}{5}$.	$70\% = \frac{7}{10}$.
$8\frac{1}{3}\% = \frac{1}{12}$.	$25\% = \frac{1}{4}$.	$50\% = \frac{1}{2}$.	$75\% = \frac{3}{4}$.
$10\% = \frac{1}{10}$.	$30\% = \frac{3}{10}$.	$60\% = \frac{3}{5}$.	$80\% = \frac{4}{5}$.
$12\frac{1}{2}\% = \frac{1}{8}$.	$33\frac{1}{3}\% = \frac{1}{3}$.	$62\frac{1}{2}\% = \frac{5}{8}$.	$87\frac{1}{2}\% = \frac{7}{8}$.
$16\frac{2}{3}\% = \frac{1}{6}$.	$37\frac{1}{2}\% = \frac{3}{8}$.	$66\frac{2}{3}\% = \frac{2}{3}$.	$90\% = \frac{9}{10}$.

DEFINITIONS.

154. Percentage is that process of computation in which the basis of operation is a hundred.

The **Base** is the number of which the per cent. is taken.

The **Rate** is the number of hundredths of the base taken.

The **Percentage** is the result obtained by taking as many hundredths of the base as are indicated by the rate.

Thus, 10% of $40 = ten hundredths of $40 = $4.

In this problem $40 is the *base,* 10% is the *rate,* and $4 is the *percentage.*

The base plus the percentage is the **Amount.**

The base minus the percentage is the **Difference.**

CASE I.

155. Given the base and rate, to find the percentage.

ORAL EXERCISES.

1. What is 10% of 60?

10% of 60 equals $\frac{10}{100}$, or $\frac{1}{10}$, of 60, which is 6, *Ans.*

2. What is $16\frac{2}{3}\%$ of 60?

$16\frac{2}{3}\% = \frac{1}{6}$ (see table). $\frac{1}{6}$ of 60 is 10, *Ans.*

Find—

3. 20% of 50 men.
4. 25% of 120 bu.
5. 50% of 600 yd.
6. 40% of 80 tons.
7. 60% of 300 pupils.
8. $12\frac{1}{2}\%$ of 48 acres.
9. $16\frac{2}{3}\%$ of $600.
10. $33\frac{1}{3}\%$ of 900 feet.
11. $37\frac{1}{2}\%$ of 24 cows.
12. 70% of 120 years.
13. $62\frac{1}{2}\%$ of 64 days.
14. $8\frac{1}{3}\%$ of 72 bu.
15. 5% of $40.
16. 30% of $80.
17. 4% of 200 yd.
18. $6\frac{1}{4}\%$ of 96 lb.
19. 90% of $120.
20. 80% of 50 horses.

WRITTEN EXERCISES.

21. What is 24% of $23?

PROCESS.

$$\begin{array}{r} \$23 \\ \underline{.24} \\ 92 \\ \underline{46} \\ \$5.52, \text{ Ans.} \end{array}$$

24% of $23 equals .24 times $23, which by multiplication is $5.52.

22. What is $55\frac{5}{9}\%$ of 63 bu.?

PROCESS.—$55\frac{5}{9}\% = \frac{55\frac{5}{9}}{100} = \frac{500}{900} = \frac{5}{9}$, and $\frac{5}{9}$ of 63 bu. = 35 bu., *Ans.*

NOTE.—The method of Ex. 21 should be used in all written work when the per cent. can be readily expressed as a pure decimal. The method of Ex. 1 and 22 should be used in all other cases.

RULE.

Multiply the base by the rate.

FORMULA.—$B \times R = P$.

23. A owned 750 acres of land, and sold 15% of it; how much did he sell?

24. A merchant bought goods for $520, and sold them at a gain of 22%; how much did he gain?

25. A farmer raised 4280 bushels of grain, and sold 20% of it; how much did he sell?

26. A house was bought for $5240, and sold at a gain of 25%; what was the selling price?

27. A farm was bought for $10,500, and was sold at a loss of 20%; required the selling price.

28. A merchant failed in business, and was able to pay only 35% of his debts; how much would A lose, to whom he owes $4860?

29. A coal-dealer buys coal at $4.25 a ton, and sells it at a gain of 18%; what is the selling price?

30. A man whose salary is $4500 pays $12\frac{1}{2}$% for board and $17\frac{1}{2}$% for current expenses; how much does he save?

31. A store was sold for $10,800: $41\frac{2}{3}$% of the price was paid in cash; how much remained unpaid?

32. Mr. Bitner had $8600 in bank: he drew out $4\frac{1}{2}$% of it to purchase a library, and 15% of the remainder for living expenses; how much remained in bank?

33. M bought a horse for $200: he sold him to N at an advance of 35%, and N sold him to P at a loss of $22\frac{2}{9}$%; what did the horse cost P?

CASE II.

156. Given the rate and percentage, to find the base.

ORAL EXERCISES.

1. 40 is 25% of what number?

If 40 is 25%, or $\frac{1}{4}$, of some number, $\frac{4}{4}$ of that number is 4 times 40, or 160, *Ans.*

Find the number of which—

2. 60 is 10%.
3. 64 is 20%.
4. 30 is $12\frac{1}{2}$%.
5. 80 is 40%.
6. 125 is 50%.
7. 20 is 5%.
8. 12 is $16\frac{2}{3}$%.
9. 70 is $33\frac{1}{3}$%.
10. 200 is $66\frac{2}{3}$%.

Find the number of which—

11. 50 is $62\frac{1}{2}\%$.

12. 36 is $44\frac{4}{9}\%$.

13. 24 is $37\frac{1}{2}\%$.

14. 42 is $87\frac{1}{2}\%$.

15. 63 is 90%.

16. 56 is 70%.

17. 90 is 30%.

18. 12 is $8\frac{1}{3}\%$.

19. 5 is $6\frac{1}{4}\%$.

WRITTEN EXERCISES.

20. $17.36 is 31% of what sum?

PROCESS.

```
       $56, Ans.
.31)$17.36
     155
     ---
      186
      186
      ---
```

If $17.36 is 31% of some number, then .31 times that number equals $17.36, hence the number is $17.36 ÷ .31, which is $56.

21. 120 is $88\frac{8}{9}\%$ of what number?

PROCESS.—$88\frac{8}{9}\% = \frac{88\frac{8}{9}}{100} = \frac{800}{900} = \frac{8}{9}$.

If $\frac{8}{9}$ of the number is 120, that number is $\frac{9}{8}$ of 120, or 135, *Ans.*

RULE.

Divide the percentage by the rate.

FORMULA.—$P \div R = B$.

22. A farm was sold for $425 less than cost, which was at a loss of 20%; what was the cost of the farm?

23. B owns 420 acres of land, which is 25% of what C owns; how much land has C?

24. A merchant lost $340, which was 17% of all his money; how much money had he?

25. Mr. Noble gained $175 by selling his horse and carriage at a gain of $31\frac{1}{4}\%$; required the selling price.

26. A miller lost 12% by selling a quantity of flour for $616; required the cost of the flour.

SUGGESTION.—The difference ÷ (1 − rate) = base.

27. An army lost 20% of its men in battle, and 25% of the remainder were discharged; how many men were there in the army at first, if there were 33,240 men remaining?

28. A gained 15% of his capital, and then had $4830; what was his capital?

SUGGESTION.—The amount ÷ (1 + rate) = base.

29. By selling my horse for $207 I gained 15%; how much would I have received for him if I had sold him at a loss of 15%?

30. A speculator lost $660 by selling goods at 41¼% below cost; required the selling price.

31. A merchant clears $1240 by selling goods for 51⅔% more than cost; required the selling price.

32. What must I ask for a farm worth $6000, so that I may fall 4% on my asking price and still gain 20%?

33. 40% of the inhabitants of an Asiatic city perished from cholera, leaving 7560 alive; what was the original population?

34. I paid $300, which was 16⅔% of my indebtedness; how much did I then owe?

CASE III.

157. Given the base and percentage, to find the rate.

ORAL EXERCISES.

1. 15 is what per cent. of 60?

15 is ¼ of 60. ¼ equals .25, or 25%.

What per cent.—

2. Of 75 is 15?
3. Of 120 is 30?
4. Of 400 is 160?
5. Of 90 is 30?
6. Of 48 is 6?
7. Of 120 is 12?
8. Of 80 is 40?
9. Of 120 is 90?
10. Of 200 is 120?
11. Of 63 is 27?
12. Of 90 is 15?
13. Of 75 is 50?
14. Of 80 is 32?
15. Of 72 is 27?
16. Of 64 is 40?
17. Of 56 is 49?
18. Of 160 is 10?
19. Of 80 is 180?

WRITTEN EXERCISES.

20. What per cent. of 126 is 90?

PROCESS.

$$\begin{array}{rl} & \underline{.71\frac{3}{7}} = 71\frac{3}{7}\%, \textit{Ans.} \\ 126) & 90.00 \\ & \underline{88.2} \\ & 180 \\ & \underline{126} \\ & \frac{54}{126} = \frac{3}{7} \end{array}$$

If 126 multiplied by some rate is 90, that rate is 90 divided by 126, which is .71$\frac{3}{7}$, or 71$\frac{3}{7}$%.

21. What per cent. of 61¼ is 12¼?

PROCESS.—$\frac{12\frac{1}{4}}{61\frac{1}{4}} = \frac{49}{245} = \frac{1}{5} = 20\%$.

RULE.

Divide the percentage by the base.

FORMULA.—$P \div B = R$.

22. C bought a horse for $210 and sold him at a gain of $42; what was the gain per cent.?

23. A merchant bought goods for $540 and sold them for $500; what was the loss per cent.?

24. A regiment entered battle with 960 men and came out with 912 men; what per cent. were missing?

25. If A's salary is $1500 and his expenses are $1250, what per cent. of his salary does he save?

26. What per cent. do I gain by buying apples at $4.25 a barrel and selling them at $6 a barrel?

27. A merchant buys lead-pencils at ½ a cent apiece and sells them at 3 cents apiece; what per cent. does he gain?

28. B sold his farm for $10,200, which was at a gain of 33⅓ per cent.; what would have been his gain per cent. if he had sold it for $8109?

29. If I gain 15% by selling a carriage for $184, what would I have lost per cent. by selling it for $150?

30. If $\frac{4}{7}$ of the cost of an article is loss, what is the loss per cent.?

31. If a merchant buys goods for $\frac{6}{7}$ of his selling price, what per cent. does he gain?

32. A speculator bought 80 horses at $150 apiece and sold 30 of them at a gain of 16⅔%; at what price per head must he sell the remainder so as to lose but 16⅔% on the transaction?

MISCELLANEOUS PROBLEMS IN PERCENTAGE.

1. What is 40% of £214 5 s.?

2. 10 A. 120 P. is 33⅓% of how much land?

3. 4 lb. 6 oz. is what per cent. of 15 lb.?

4. What is 16⅔% of 116⅔?

5. 82⅓ is 42½% of what number?

6. 212½ is what per cent. of 916⅔?

7. If I buy goods at 20% below cost, and sell them at 20% above cost, what per cent. do I gain?

8. A miller bought 1200 barrels of flour for $4500, and sold 1000 barrels for what they all cost; what was the gain %?

9. A merchant sold goods worth $6544 at a gain of 37½%; required the gain.

10. A firm makes a reduction of 8% for cash; how much is A's bill, who pays $368?

11. How must cloth which cost $2.25 a yard be marked to gain 20%?

12. B invests 45% of his capital in railroad stock, and has $11,000 remaining; what is his capital?

13. B sold cattle at an average price of $35 a head, and thus lost 16⅔%; would he gain or lose, and how much per cent., by selling them at an average price of $44 a head?

14. What must I ask for a house which cost $6400, so that after falling 20% from my asking price I may still gain 12½%?

15. From a tank containing 120 gallons of oil 14 gallons leaked out; what per cent. of the contents remained?

16. Mr. Smith bought a horse for $210, and set such a price on him that after falling $18 he still gained 20%; what per cent. did he abate from the asking price?

17. A merchant who sells goods at a gain of 16% clears $368; what is the cost?

18. The number of boys in a certain school is 75% of the number of girls; what per cent. of the number of boys is the number of girls?

19. A fruit-dealer bought 300 barrels of apples for $660, and lost 20 barrels; at what price per barrel must he sell the remainder to gain 10% on the transaction?

20. What per cent. is gained when chestnuts are bought at $1.60 a bushel and sold at 7 cents a quart?

21. Mr. Sumner sold his horse for $240, and thereby cleared $\frac{1}{6}$ of this money; what would he have gained per cent. if he had sold him for $300?

22. A house was sold for $3160, which was at a loss of 21%; for what should it have been sold to gain 21%?

23. If I buy at 40% below cost and sell at 40% above cost, what per cent. do I gain?

24. Mr. Reed drew 45% of his bank deposit to pay for a house which he bought for $9000; what amount of money remained on deposit?

25. A bought 120 barrels of flour at $4 a barrel, and paid $30 freight for shipment; at what price per barrel must he sell it to gain 20% on the transaction?

26. A book-case was sold for $60, which was at a gain of 12%; what would have been the loss per cent. had it been sold for $40?

27. A farmer sold two cows for $40 apiece: on the one he gained 20%, and on the other he lost 20%; did he gain or lose, and how much?

28. A man is 72 years old, and 12½% of his age is 33⅓% of his son's age; what is the son's age?

29. M drew 33⅓% of his bank deposit, and paid 25% of it for a house worth $5000; what was his original bank deposit?

30. A wagon was sold for $37½, which was at a gain of 33⅓%: another one was bought at $37½, and sold at a loss of 33⅓%; what was the gain or loss?

APPLICATIONS OF PERCENTAGE.

158. Commission.

Commission is the sum paid to an agent for the transaction of business.

The agent is sometimes called a *factor*, a *broker*, a *collector*, or a *commission-merchant.*

The **Base** in commission is the *amount received* when selling and the *amount paid* when buying.

The amount remaining after deducting the commission and other charges is called the **Net Proceeds.**

159. The cases of commission are the same as those of percentage.

WRITTEN EXERCISES.

1. My agent sold goods for $4500: his commission is 3%, and other charges are $15.50; what amount is due me?

PROCESS.

$$\begin{array}{r} \$4500 \\ \underline{.03} \\ \$135.00 \\ \underline{15.50} \\ \$150.50 \end{array}$$

$$\begin{array}{r} \$4500 \\ \underline{150.50} \\ \$4349.50, \textit{ Ans.} \end{array}$$

The commission is .03 times $4500, which is $135. Adding to this the other charges, the entire sum to be deducted is $150.50.

Subtracting this from $4500, the net proceeds are $4349.50.

2. I sent my agent $3075 to invest in flour after deducting his commission at $2\frac{1}{2}\%$; what amount did he invest?

PROCESS.

$$\begin{array}{r|l} & \$3000,\ \textit{Ans.} \\ 1.025)\$3075.000 & \\ 3075 & \end{array}$$

The sum invested, increased by $2\frac{1}{2}\%$ of itself, is $3075.

If $102\frac{1}{2}\%$, or $1.02\frac{1}{2}$ times the *sum invested*, is $3075, that *sum* must be $3075 divided by $1.02\frac{1}{2}$, which is $3000.

3. An agent sold land for $6428, and received as his commission $144.63; what was the rate of commission?

PROCESS.

$$\begin{array}{r|l} & .02\frac{1}{4},\ \text{or } 2\frac{1}{4}\%,\ \textit{Ans.} \\ \$6428)\$144.63 & \\ 12856 & \\ \frac{1607}{6428} = \frac{1}{4} & \end{array}$$

Since $6428 multiplied by some rate is $144.63, that rate is $144.63 divided by $6428, which is $.02\frac{1}{4}$, or $2\frac{1}{4}\%$.

4. An agent sold $4550 worth of goods at 2% commission; what was his commission?

5. An agent's commission at 3% is $216; what amount of goods did he sell?

6. An agent sells $7316 worth of goods, and receives $256.06 as his commission; what is the rate of commission?

7. An agent bought 200 barrels of flour at $4.75 a barrel: his commission was 3%, and other charges were $21.50; what was the entire cost?

8. What is the commission on $9540 at $3\frac{1}{4}\%$?

9. Mr. Brown sent his agent in Chicago $7000 to invest in pork, after deducting his commission at $2\frac{1}{2}\%$; what was his commission?

10. An agent bought sugar, charging $150 commission and $28.45 storage: his entire bill was $5178.45; what was the rate of commission?

11. My agent in St. Paul sells goods to the amount of $4370: his commission is 2%, and freight charges are $65.25; what sum is due me?

12. A factor received $4000 to invest in shoes after deducting his commission at 2% ; what sum was invested?

13. An agent bought 75 horses on a commission of 1½% : the transportation cost $250 ; the entire cost was $20,500 ; what did the agent pay a head for the horses?

14. An agent received a certain sum of money to invest in goods after deducting his commission of 3% : he invested $6250 ; what sum did he receive?

15. I directed my agent in New Orleans to sell my cattle, for which he subsequently received $4800, and to purchase sugar at 5 cents a pound with the proceeds, after deducting his commission of 2% for buying and selling ; how many pounds did he buy?

16. What are the net proceeds from the sale of 425 tons of coal at $4.50 a ton, the commission being 2¼%, and other charges $148.75?

17. A commission merchant received $56.24 for selling flour at 2% commission ; what was the amount of the sale?

18. An agent sold $450 worth of goods on a commission of 1⅕% : he invested the net proceeds in flour, charging the same commission for buying ; what sum was invested in flour?

19. My agent sold 40 horses at $125 a head on 1½% commission, and 60 cows at $25 a head on 1⅘% commission : other charges amounted to $105.16 ; how much was due me?

20. I sold goods on a commission of 3% through an agent who charged me 1¼% : my commission after paying my agent was $140 ; required my entire commission, the agent's commission, and the money paid to my employer.

21. My agent sells 300 barrels of flour at $4.50 a barrel. His charges are as follows : commission 2%, guaranteeing payment 1½%, and freight $125 ; how much is due me?

22. An agent sells goods at 2½% commission : after deducting his commission he remits to his employer $3763.50 ; what is his commission?

23. An agent sold 400 barrels of apples at $4.75 a barrel, charging $2\frac{1}{2}\%$ commission: he invested the proceeds in wheat at 60 cents a bushel, charging 3% commission for buying; how many bushels of wheat did he buy?

24. My agent in Boston sold a shipment of flour, for which he charged me 5% commission: I instructed him to invest the proceeds in shoes, for which I paid him a commission of $2\frac{1}{2}\%$; what did he receive for the flour if he invested $1900 in shoes?

25. I collect a bill of $360, and return my employer $351; what per cent. do I charge for collecting?

160. Taxes.

A **Tax** is a sum of money assessed upon the person, property, or income of an individual for public purposes.

When the tax is assessed upon the person it is called **Poll-tax**, when on property it is called **Property Tax**, and when on the income it is called **Income Tax.**

161. Property is of two kinds—**Real Estate**, or immovable property, as lands, houses, etc.; and **Personal Property**, or movable property, as money, furniture, stocks, etc.

162. The person elected or appointed to estimate the value of the property to be taxed is the **Assessor.**

163. The **Rate of Taxation** is a certain percentage of the taxable property.

WRITTEN EXERCISES.

1. A tax of $7800 is to be raised in a town in which the taxable property is $1,600,000: there are 500 persons who pay a poll-tax of $2 each; what is the rate of taxation? What is A's tax, whose property is valued at $3000, and who pays 1 poll-tax?

PROCESS.

$2
500
$1000, Poll tax.

$7800
1000
1,600,000)$6800.000(.004¼, Rate.
6400000
$\frac{400000}{1600000} = \frac{1}{4}$

$3000
.004¼
$12.750
2.000
$14.750, A's tax.

The poll tax equals 500 times $2, which is $1000.

The amount of tax to be raised on the property is $7800 minus $1000, or $6800.

Dividing $6800 by the value of the property, $1,600,000, we find the rate of taxation, .004¼.

A's tax is $3000 multiplied by .004¼, or $12.75. $12.75 plus his poll tax, $2, makes his entire tax $14.75.

From this example it is evident that the rate of taxation may be found by *deducting the poll tax, if any, from the entire tax to be raised, and dividing the remainder by the value of the taxable property.*

To find the tax of an individual, multiply his taxable property by the rate of taxation and add to the product his poll tax.

2. What is the rate of taxation in a town in which the property is valued at $860,000, and the tax to be raised is $2150?

3. A school district whose taxable property is $540,000 wishes to raise a tax of $810; what is the rate of taxation?

4. In a town whose taxable property is valued at $5,463,000 a tax of $9560.25 is raised; what is the rate of taxation?

5. A tax of $3240 is raised in a town whose taxable property is valued at $440,500; what is the rate of taxation, if there are 398 polls, each paying $1.50?

6. My property is valued at $5600, and I pay a poll tax of $2; what is my entire tax, at the rate of .2¼%?

7. B's real estate is valued at $10,400, and his personal property at $5280; what is his tax, at the rate of $12.50 on $1000?

8. What amount of tax must be assessed in order that $6382.60 may remain after paying 3% commission for collecting?

9. A tax-collector's commission at 2% was $182.50; what amount of tax was assessed, and what were the proceeds?

10. A city whose property is valued at $14,000,000 wishes to assess a tax which will yield $40,950 after paying 2½% commission for collecting; what is the rate of taxation?

11. After deducting his commission of 4% a collector paid to the city authorities $48,206.40; what was the amount of his commission?

12. My property, which cost me $7800, is taxed at ⅔ of its value; what is the rate of taxation, if my tax is $15.60?

13. A city assesses a school tax of $63,000, and pays 2% commission for all taxes actually collected; what amount will be received by the city, if only 90% of the tax can be collected?

14. The taxable property of a city, valued at $4,100,625, is assessed at ⅘ of its value: the amount needed for school purposes is $32,075.30; the State appropriation amounts to $2151.37; the balance in the treasury is $17,064.37; what is the rate of taxation, if 2% commission is paid for collecting?

15. B's tax, at the rate of 3½ mills on a dollar, is $20.30; what is the value of his property, if it is taxed at only ⅘ of its value?

164. Insurance.

Insurance is security against loss by fire, water, storm, accident, death, or other causes.

165. The **Policy** is the written agreement between the insurance company and the person insured.

166. The **Premium** is the sum paid for insurance. It is usually estimated as a certain per cent. of the amount insured.

WRITTEN EXERCISES.

1. My house is insured for $4000 at 1¼%; what is the premium?

PROCESS.

$$\begin{array}{r} \$4000 \\ \underline{.01\frac{1}{4}} \\ \$50.00,\ \textit{Ans.} \end{array}$$

The premium on $4000, at 1¼%, is .01¼ times $4000, which is $50.

2. For what sum must A insure his house, worth $7020, at 2½%, so as to include the premium in case of loss?

PROCESS.

$$\begin{array}{r} \underline{|\$7200},\ \textit{Ans.} \\ .975)\$7020 \\ \underline{6825} \\ 1950 \\ \underline{1950} \end{array}$$

Since the premium is estimated on the amount insured, the cost of the house must be (100% − 2½%) 97½% of the amount insured.

Hence the amount insured is $7020 divided by .975, which is $7200.

3. What is the premium on $2400 at ⅜%? At 1⅜%?

4. What is the premium on $5400 at 1%? At 1½%? At 2%? At 1¾%? At 2¼%?

5. Mr. Long insured his house for $6500 at 1%; what was the premium?

6. B insured his house for $7500, and the contents for $1500, at 2%; what was the premium?

7. Mr. Brown paid $84 for insuring his house for $5600; what was the rate of insurance?

8. C paid $47.75 to insure his house at 1¼%; what sum was named in the policy?

9. The premium for insuring a house at ⅘ of its value at 1¼% is $120; what is the value of the house?

10. B's house, worth $10,000, is insured for ⅘ of its value; what is the rate of premium, if he pays $96 insurance?

11. For what sum must a barn worth $2500 be insured, at 2%, so that the owner in case of loss may receive both the premium and the value of the barn?

12. M's house was insured for ⅚ of its value at 1¼%: the house was burned down, and his loss, including the insurance, was $680; what was the value of the house?

13. A mill is insured for 80% of its value at 1%; what is the value of the mill, if the premium is $72.36?

14. A's store, worth $8500, is insured for ⅘ of its value at 2½%; in case the store is destroyed by fire what is A's actual loss?

15. My house, worth $8000, was insured for ⅞ of its value at 2%: it was destroyed by fire in 1894; what was the actual loss to the company?

16. A house and store worth $9800 is insured at 2%, so that in case of loss both the premium and the value of the property may be covered; what is the premium?

17. If B takes out a life policy for $8000, what is his yearly premium at the rate of $26.50 on $1000?

18. At the age of 28 years I took out an endowment policy for $10,000; what is my yearly premium at the rate of $45.15 on $1000?

INTEREST.

ORAL EXERCISES.

167.—*1.* If you borrow $100, and pay 6% of it every year for the use of the money, what must you pay for 1 year? For 2 yr.? For 3 yr.? For 4 yr.?

2. What must you pay annually for the use of $100 at 5%? At 7%? At 8%? At 9%? At 6½%?

3. What must you pay annually at 5% for $200? For $300? For $400? For $500?

4. What must you pay for the use of $100 at 5% for 1 year? For 2 yr.? For 3 yr.? For 4 yr.?

5. What must you pay for the use of $200 at 6% for 1 year? For 2 yr.? For 3 yr.? For 4 yr.?

DEFINITIONS.

168. **Interest** is money paid for the use of money.

169. The **Principal** is the sum for which interest is paid. The **Amount** is the sum of the principal and interest.

170. The **Rate of Interest** is the interest on one dollar for one year.

NOTE.—In interest a year is 12 mo. of 30 da. each.

171. Simple Interest.

Simple Interest is interest on the principal only.

ORAL EXERCISES.

Common Method.

Find the interest—

1. Of $100 for 2 yr. at 5%.
2. Of $200 for 3 yr. at 6%.
3. Of $400 for 2 yr. at 7%.
4. Of $500 for 3 yr. at 6%.
5. Of $600 for 10 yr. at 6%.
6. Of $400 for 3 yr. at 5%.
7. Of $600 for 5 yr. at 4%.
8. Of $200 for 8 yr. at 7%.
9. Of $1000 for 3 yr. at 9%.
10. Of $900 for 2 yr. at 6%.

WRITTEN EXERCISES.

11. What is the interest of $800 for 3 yr. 10 mo. 15 da. at 7%?

PROCESS.

$800
.07
$56.00, Int. for 1 yr.
$3\frac{7}{8}$
168
49
$217, Int. for $3\frac{7}{8}$ yr.

The interest for 1 yr. at 7% is .07 times $800, which is $56.

3 yr. 10 mo. 15 da. equals $3\frac{7}{8}$ yr.

If the interest for 1 yr. is $56, for $3\frac{7}{8}$ yr. it is $3\frac{7}{8}$ times $56, which is $217.

RULE.

Find the interest for one year, and multiply this by the time expressed in years.

Required the interest—

12. Of $480 for 2 yr. 6 mo. at 7%.
13. Of $500 for 3 yr. 10 mo. at 6%.
14. Of $720 for 5 yr. 9 mo. at 5%.
15. Of $840 for 4 yr. 8 mo. at 8%.
16. Of $960 for 5 yr. 10 mo. at 6%.
17. Of $360 for 2 yr. 7 mo. 15 da. at 6%.
18. Of $1248 for 3 yr. 6 mo. 10 da. at 5%.
19. Of $2496 for 5 yr. 8 mo. 6 da. at 6%.
20. Of $1200 for 4 yr. 5 mo. 20 da. at 8%.

Find the amount—

21. Of $6000 for 1 yr. 6 mo. at 5½%.
22. Of $7200 for 2 yr. 5 mo. at 6%.
23. Of $560 for 4 yr. 8 mo. at 8%.
24. Of $840 for 6 yr. 6 mo. at 6%.

Find the interest at 6% of—

25. $400 from June 20, 1888, to May 1, 1890.
26. $600 from July 5, 1889, to Sept. 12, 1891.
27. $2000 from Mar. 4, 1890, to Aug. 12, 1893.

Find the interest of—

28. $500 at 6% for 8 yr.
29. $200 at 6½% for 8 yr. 6 mo.
30. $425 at 5½% for 4 yr. 8 mo.
31. $660 at 5% for 3 yr. 9 mo.
32. $540 at 7% for 3 yr. 8 mo.
33. $1220 at 8% for 2 yr. 10 mo.
34. $40 at 4% for 5 yr. 6 mo. 18 da.
35. $60 at 6% for 3 yr. 7 mo. 20 da.

36. $120 at 7% for 3 yr. 6 mo. 10 da.
37. $124 at 6½% for 5 yr. 7 mo. 15 da.
38. $240 at 4% for 8 yr. 9 mo. 12 da.
39. $24.50 at 5% for 6 yr. 3 mo. 24 da.

172. Six Per Cent. Method.

At 6% the interest of $1—

For 12 months is .06 (Def. ART. 170).
For 30 days ($\frac{1}{12}$ of 12 mo.) is .005.

Since **6** is ½ of **12**, and **5** is ⅙ of **30**, there are just—

½ as many *cents* as months, and
⅙ as many *mills* as days.

Therefore, to find the interest at 6%,

Regard ½ of the months as cents, ⅙ of the days as mills, and multiply their sum by the principal.

WRITTEN EXERCISES.

1. Find the interest of $500 for 2 yr. 8 mo. 12 da. at 6%.

PROCESS.

2 yr., 2 × .06 = .12
8 mo., ½ of 8 = .04
12 da., ⅙ of 12 = .002
$.162
500
$81.000, *Ans.*

The interest for 2 yr. is $.12.

Regarding ½ of the months as cents, the interest for 8 mo. is $.04.

Regarding ⅙ of the days as mills, the interest for 12 da. is $.002.

The interest on $1 for 2 yr. 6 mo. 12 da. is $.162, and the interest on $500 is 500 times $.162, which is $81.

Find the interest at 6%—

2. Of $450 for 3 yr. 4 mo. 6 da.
3. Of $525 for 2 yr. 6 mo. 12 da.
4. Of $600 for 3 yr. 5 mo. 18 da.

5. Of $125 for 1 yr. 10 mo. 18 da.
6. Of $24.50 for 3 yr. 8 mo. 24 da.
7. Of $1000 for 3 mo. 15 da.
8. Of $280.75 for 5 yr. 21 da.
9. Of $2000 for 4 yr. 10 mo. 24 da.
10. Of $5020 for 5 yr. 5 mo. 6 da.
11. Of $600 for 3 yr. 7 mo. 8 da.
12. Of $50.60 for 48 da.
13. Of $300 for 63 da.

Find the amount at 6%—

14. Of $34.60 for 4 yr. 9 mo.
15. Of $342.65 for 8 mo. 10 da.
16. Of $975.82 for 2 yr. 5 mo. 16 da.
17. Of $516.81 for 4 yr. 7 mo. 18 da.
18. Of $2400 for 10 yr. 10 mo. 10 da.

Find the interest—

19. Of $600 for 2 yr. 10 mo. 12 da. at 5%.

PROCESS.

$$\begin{array}{lr}
\text{2 yr.,} & 2 \times .06 = .12 \\
\text{10 mo.,} & \tfrac{1}{2} \text{ of } 10 = .05 \\
\text{12 da.,} & \tfrac{1}{6} \text{ of } 12 = .002 \\
\hline
 & .172 \\
 & 600 \\
\cline{2-2}
 & 6)\$103.200 \\
\cline{2-2}
 & 17.20 \\
 & 5 \\
\cline{2-2}
 & \$86.00, \textit{Ans.}
\end{array}$$

The interest on $600 for 2 yr. 10 mo. 12 da. at 6% is $103.20, and the interest at 5% is ⅚ of $103.20, which is $86.

20. Of $650 for 2 yr. 10 mo. 12 da. at 5%.
21. Of $3020 for 4 yr. 2 mo. 6 da. at 7%.
22. Of $15.63 for 3 yr. 1 mo. 5 da. at 5½%.
23. Of $2000 for 5 yr. 5 mo. 5 da. at 5%.
24. Of $1625 from June 12, 1890, to Aug. 15, 1892, at 6%.
25. Of $415 from Oct. 10, 1891, to May 15, 1894, at 7%.
26. Of $500 from Apr. 14, 1890, to June 10, 1895, at 5½%.

173. Method by Aliquot Parts.

WRITTEN EXERCISES.

1. What is the interest of $1200 for 4 yr. 5 mo. 15 da. at 5½%?

PROCESS.

	$1200
	.05½
	$66.00 = Int. for 1 yr.
	4
	$264.00 = Int. for 4 yr.
4 mo. = ⅓ yr.	22.00 = Int. for 4 mo.
1 mo. = $\frac{1}{12}$ yr.	5.50 = Int. for 1 mo.
15 da. = ½ mo.	2.75 = Int. for 15 da.
	$294.25, *Ans.*

Find the interest—

2. Of $1080 for 3 yr. 9 mo. at 5%.
3. Of $720 for 2 yr. 8 mo. at 5½%.
4. Of $840 for 4 yr. 7 mo. at 6%.
5. Of $1600 for 2 yr. 6 mo. 15 da. at 7%.
6. Of $3600 for 3 yr. 10 mo. 18 da. at 8%.
7. Of $4200 for 2 yr. 5 mo. 25 da. at 4½%.
8. Of $1800 for 1 yr. 1 mo. 11 da. at 6½%.
9. Of $2100 for 6 yr. 6 mo. 6 da. at 6%.
10. Of $2400 for 2 yr. 2 mo. 12 da. at 5%.

174. Exact Interest.

Exact Interest is interest obtained by reckoning 365 days to the year.

Exact Interest is used by the United States government and by many business men.

Common interest and exact interest for any number of years are the same. They differ only in periods of time less than a year.

Find the interest of $100 for 1 yr. at 5%.

	Common Interest.	Exact Interest.
For 1 yr.	$5	$5
For 1 da.	$\frac{5}{360} = \frac{1}{72}$	$\frac{5}{365} = \frac{1}{73}$
Reducing	$\frac{1}{72} = \frac{73}{5256}$	$\frac{1}{73} = \frac{72}{5256}$

It is thus seen that if $\frac{1}{73}$ of common interest be subtracted from itself, the remainder will be exact interest.

Hence $\frac{1}{73}$ *off from common interest will give exact interest.*

WRITTEN EXERCISES.

1. Find the exact interest of $730 for 2 yr. 8 mo. 18 da. at 6%.

PROCESS.

$730	$730	$31.39
.12	.043	.43
$87.60 = Int. for 2 yr.	2190	30.96
	2920	87.60
	73)$31.390(.43	$118.56, *Ans.*
	292	
	219	
	219	

We find the common interest of $730 for 2 yr. by multiplying by .12, which is $87.60; and for 8 mo. 18 da. by multiplying by .043, which is $31.39.

Since $\frac{1}{73}$ off from common interest gives exact interest, we deduct $\frac{1}{73}$ from $31.39, leaving $30.96, the exact interest for 8 mo. 18 da.

Adding $87.60 and $30.96, the result is $118.56.

Find the exact interest of—

2. $850 for 10 mo. 24 da. at 6%.

3. $1260 for 2 yr. 8 mo. 18 da. at 4%.

4. $2400 for 3 yr. 9 mo. 15 da. at 7%.

5. $1560 from Mar. 1st to Nov. 25th at 8%.

6. $3330 from Jan. 4, 1888, to May 7, 1890, at 5%.

7. $960 from Mar. 12, 1890, to Dec. 20, 1890, at 5½%.

8. $4080 from Dec. 11, 1892, to May 29, 1896, at 7%.

PROBLEMS IN INTEREST.

CASE I.

175. Given the time, rate, and interest, to find the principal.

WRITTEN EXERCISES.

1. What principal will in 2 yr. 6 mo. 12 da. at 6% give $30.40 interest?

PROCESS.

2 yr., $2 \times .06 = .12$
6 mo., ½ of $6 = .03$
12 da., ⅙ of $12 = .002$
Int. of $1 = $.152

$30.40 ÷ .152 = $200, *Ans.*

We find the interest of $1 for 2 yr. 6 mo. 12 da. to be $.152.

If $1 gains $.152, to gain $30.40 it will require as many dollars as .152 is contained times in 30.40, which is $200.

RULE.

Divide the given interest by the interest of $1 *for the given time and rate.*

What principal at interest will gain—

2. $105.60 in 3 yr. 8 mo. at 6%?
3. $11.76 in 4 yr. 8 mo. at 6%?
4. $385 in 5 yr. 10 mo. at 6%?
5. $1496 in 7 yr. 4 mo. at 6%?
6. $45.625 in 2 yr. 6 mo. at 5%?
7. $97.20 in 2 yr. 8 mo. 12 da. at 5%?
8. $113.96 in 3 yr. 4 mo. 21 da. at 7%?

What principal will amount to—

9. $1710.58 in 28 da. at 6%?
10. $249.256 in 4 yr. 6 mo. at 3½%?
11. $1752.84 in 5 yr. 10 mo. 15 da. at 6%?
12. $32.82 in 7 yr. 7 mo. 21 da. at 5%?

CASE II.

176. Given the principal, the rate, and the interest, to find the time.

WRITTEN EXERCISES.

1. In what time will $240 give $50.40 interest at 6%?

PROCESS.

$240
.06
$14.40)$50.40(3½
4320
$\frac{720}{1440} = \frac{1}{2}$
= 3 yr. 6 mo., *Ans.*

We find the interest of $240 for 1 yr. at 6% to be $14.40.

If the interest for 1 yr. is $14.40, to give an interest of $50.40 it will require as many years as $14.40 is contained times in $50.40, which is 3½ years.

RULE.

Divide the given interest by the interest of the principal at the given rate for one year.

In what time will—

2. $940 at 6% give $432.40 interest?
3. $1450 at 6% give $217.50 interest?
4. $60 at 6% give $6 interest?
5. $120 at 7% give $37.10 interest?
6. $480 at 5% give $62 interest?
7. $240 at 6% give $16.92 interest?
8. $720 at 5% give $117.30 interest?
9. $360 at 8% give $312.80 interest?
10. $600 at 7% give $613.30 amount?
11. $960 at 5% give $984.80 amount?
12. $1200 at 4½% give $1203.60 amount?

13. In what time will a principal double itself at 5%? At 8%? At 4%?

14. In what time will a principal treble itself at 6%? At 7%? At 5½%?

CASE III.

177. Given the principal, the time, and the interest, to find the rate.

WRITTEN EXERCISES.

1. At what rate will $640 give $172.80 interest in 4 yr. 6 mo.?

PROCESS.

4 yr., 4 × .06 = .24
6 mo., ½ of 6 = .03
6).27 = Int. at 6%.
$.045 = Int. at 1%.
640
1800
270
$28.800)$172.80(6
172.80

The interest of $640 for 4 yr. 6 mo. at 1% is $28.80.

If the interest at 1% is $28.80, to give $172.80 interest it will require as many per cent. as $28.80 is contained times in $172.80, which is 6%.

At what rate will—

2. $900 gain $198 in 3 yr. 8 mo.?
3. $426 gain $53.25 in 2 yr. 6 mo.?
4. $330 gain $96.25 in 5 yr. 10 mo.?
5. $420 gain $53.90 in 2 yr. 4 mo.?
6. $72 gain $64.26 in 12 yr. 9 mo.?
7. $240 gain $41.18 in 3 yr. 8 mo.?
8. $840 amount to $981.96 in 2 yr. 9 mo. 24 da.?
9. $18.20 amount to $28.43 in 9 yr. 9 mo. 9 da.?
10. $625 amount to $710.62½ in 2 yr. 3 mo. 12 da.?
11. $192 amount to $200.928 in 9 mo. 9 da.?
12. $600 gain $3.525 in 47 da.?
13. $564 gain $64.108 in 11 mo. 11 da.?
14. $744 amount to $764.15 in 10 mo.?

15. At what per cent. will a principal double itself in 4 yr.? In 6 yr.? In 7 yr.?

16. At what per cent. will a principal treble itself in 10 yr.? In 30 yr.? In 5 yr.?

PROMISSORY NOTES.

178. A **Promissory Note** is a written promise to pay a specified person a specified sum at a specified time.

The **Maker** of a note is the person who signs it.

The **Payee** is the person to whom it is payable.

The **Holder** is the person who owns it.

An **Indorser** is a person who writes his name on the back of a note to transfer it or to guarantee its payment in case the maker fails to pay it when due.

The **Face** of a note is the *sum* named in it.

179. A **Negotiable Note** is a note that can be transferred. A note is negotiable when it is made payable to the "bearer" or to the "order" of the payee; when it is made payable to the payee only, it is not negotiable.

NOTE.—In Pennsylvania the law requires the words "without defalcation," in addition to "value received," to make a note negotiable; and in New Jersey, "without defalcation or discount."

180. A note **Matures**, or becomes legally due, at the expiration of the specified time.

NOTE.—In some states *Three Days of Grace* are allowed, and then a note matures three days after the specified time. *In this book days of grace will not be reckoned.* Custom also varies when a note falls due on Sunday or a legal holiday—some states making it payable on the next preceding, and others on the next succeeding, business day.

181. A **Protest** is a written notice sent by a *notary public* to the indorsers, stating that the maker has failed to pay the note. If the notice is not sent on the last day of grace, the indorsers are not held responsible for its payment.

NOTE.—If the words "with interest" are in the note, it draws interest from date; if these words are omitted, the note will not draw interest until after it becomes due. When no rate is mentioned, the legal rate at the place where the note is made is understood.

WRITTEN EXERCISES.

182. The following are the **Principal Forms** of notes:

1.

FORM OF A DEMAND NOTE.

$450. *Philadelphia, Pa., Apr. 2, 1895.*

On demand, I promise to pay Henry King four hundred and fifty dollars, with interest at 6%, *for value received, without defalcation.*

Henry F. Bitner.

This note was paid in 24 days; what was the amount due?

2.

FORM OF A TIME NOTE.

$240. *Pittsburg, Pa., May 4, 1895.*

Sixty days after date, I promise to pay John W. Lansinger, or order, two hundred and forty dollars, with interest at 5%, *for value received, without defalcation.*

H. Justin Roddy.

What is due on this note at maturity?

3.

FORM OF A PRINCIPAL-AND-SURETY NOTE.

$650.25. *Albany, N. Y., June 1, 1895.*

Ninety days after date, I promise to pay W. Hamner Piper, or order, six hundred and fifty $\frac{25}{100}$ *dollars, with interest at* 6%, *for value received, without defalcation.*

J. W. Allen.

Surety, W. F. Beyer.

What is due on this note at maturity?

4.

FORM OF A JOINT NOTE.

Baltimore, Md., July 2, 1895.

$850.

Two months after date, we promise to pay George R. Anderson, or order, eight hundred and fifty dollars, for value received, without defalcation, with interest at 5½%.

John Snyder.
Henry Long.

What is due on this note at maturity?

5. A 90-day note for $1000, with interest, was paid at maturity; what was the amount due?

6. A 60-day note for $960, without interest, was paid in 100 days; what was the amount due?

7. A 3-month note for $2040, with interest, was paid in 120 days; what was the amount due?

8. What is the difference between the interest of two notes for $1200 each, given July 5, 1895, one due 3 months after date and the other 90 days after date?

9. Write a demand note for $520.84, bearing interest at 6%, with yourself as payee and John W. Hayes as maker.

10. Write a thirty-day negotiable note for $400, bearing interest at 6%, with William Y. Jones as maker and Thomas Morton payee. Indorse same to the order of J. C. Printz & Co.

11. Write a principal-and-surety note for $80.50, bearing interest at the legal rate and drawn in favor of Henry A. Jones, with Elwood Baker as maker and Robert C. Lane as surety.

12. Write a joint note, negotiable, bearing interest at 6%; face of the note $1200, time 60 days, payee Howard Smith; makers Horace Fritz and James R. Lind.

13. Face of note $650, maker W. H. Holmes, payee Charles Martin, time 90 days; write a non-negotiable note, without interest.

PARTIAL PAYMENTS.

183. Partial Payments are payments in part on notes.

184. An **Indorsement** is a record of a partial payment, with its date, made on the back of a note.

185. The Merchants' Rule.

When partial payments are made on an interest-bearing note, and settlement is made in a *year or less*, business men usually employ what is known as

THE MERCHANTS' RULE.

Find the amount of the face of the note from its date to the day of settlement; also, the amount of each payment from its date to the day of settlement.

Subtract the sum of the amounts of the payments from the amount of the face of the note.

WRITTEN EXERCISES.

1. What amount is due Nov. 27, 1894, on a note for $800, dated Jan. 15, 1894, with interest at 6%, on which are the following indorsements: May 3, 1894, $300; July 9, 1894, $400?

Process.

Amount of	$800 for 10 mo. 12 da.,		$841.60
"	$300 for 6 mo. 24 da.,	$310.20	
"	$400 for 4 mo. 18 da.,	$409.20	$719.40
		Ans.,	$122.20

The amount of $800 from Jan. 15th to Nov. 27th, or 10 mo. 12 da., is $841.60.

The amount of $300 from May 3d to Nov. 27th, or 6 mo. 24 da., is $310.20.

The amount of $400 from July 9th to Nov. 27th, or 4 mo. 18 da., is $409.20.

The sum of the amounts of the payments is $719.40, which subtracted from the amount of the face of the note leaves $122.20.

2. What is due Dec. 20, 1890, on a note for $1200, dated Feb. 8, 1890, with interest at 5½%, on which is the following indorsement: June 5, 1890, $800?

3. What is due Dec. 24, 1888, on a note for $1600, dated Mar. 1, 1888, with interest at 6%, on which are the following indorsements: May 10th, $500; July 15th, $400?

4. What is due Nov. 27, 1891, on a note for $2000, dated Jan. 1, 1891, with interest at 5%, on which are the following indorsements: Mar. 3, 1891, $600; May 7, 1891, $800?

5. What is due Dec. 12, 1892, on a note for $2500, dated Feb. 6, 1892, with interest at 6%, on which are the following indorsements: Apr. 1, 1892, $800; June 4, 1892, $700; Aug. 10, 1892, $500; Sept. 12, 1892, $300?

6. On a note for $1860, dated Dec. 3, 1894, and bearing interest at 5½%, the following payments were made: Dec. 20, 1894, $500; Jan. 24, 1895, $600; May 10, 1895, $300; Aug. 30, 1895, $250. How much is due Dec. 3, 1895?

7. A note for $3200, dated Jan. 8, 1894, has the following indorsements on it: Feb. 20, 1894, $800; Apr. 20, 1894, $600; June 25, 1894, $500; Aug. 16, 1894, $400; Oct. 25, 1894, $300. What is due Jan. 1, 1895, at 5%?

186. United States Rule.

When partial payments are made on an interest-bearing note, and settlement is not made *within a year*, the amount due is found by the following rule, adopted by the Supreme Court of the United States and nearly all the States:

THE UNITED STATES RULE.

Find the amount of the principal to a time when a payment, or the sum of the payments, equals or exceeds the interest due, and from this amount subtract the payments or the sum of the payments.

Regard the remainder as a new principal, and proceed as before to the time of settlement.

WRITTEN EXERCISES.

1. What is due Aug. 18, 1893, on a note for $400, dated Apr. 1, 1891, with interest at 6%, on which are the following indorsements: Jan. 13, 1892, $50; Sept. 22, 1892, $10; Apr. 25, 1893, $125.

PROCESS.

Principal	$400.00
Int. to first payment, 9 mo. 12 da.	18.80
Amount due Jan. 13, 1892	$418.80
First payment	50.00
New principal	$368.80
Int. to second payment, 8 mo. 9 da.	15.31
Int. to third payment, 7 mo. 3 da.	13.09
	$397.20
Sum of second and third payments	135.00
New principal	$262.20
Int. from Apr. 25th to Aug. 18th, 3 mo. 23 da.	4.94
Amount due Aug. 18, 1893	$267.14

2. A note for $800, dated Apr. 4, 1892, had the following indorsements:

June 4, 1893	$20
Oct. 15, 1893	300
Feb. 20, 1894	350

What was due Aug. 1, 1894, at 6%?

3. What is due May 10, 1891, on a note for $1600, dated Sept. 16, 1888, with interest at 5%, on which are the following indorsements: June 8, 1889, $400; Oct. 15, 1890, $500; Jan. 15, 1891, $300?

4. What is due Oct. 20, 1893, on a note for $2000, dated Mar. 6, 1890, with interest at 5½%, on which are the following indorsements: July 6, 1891, $150; Sept. 25, 1892, $200; May 9, 1893, $250?

5. What is due May 20, 1883, on a note for $2400, dated June 4, 1880, on which are the following indorsements: Aug.

8, 1880, $180; May 2, 1881, $100; July 10, 1882, $50; Jan. 15, 1883, $500?

6. What is due Dec. 31, 1895, on a note for $1800, dated Oct. 10, 1891, with interest at 7%, on which are the following indorsements: Apr. 1, 1892, $55.50; July 21, 1893, $200; June 15, 1895, $200?

7.

$3500. *Chicago, Mar. 6, 1888.*

On demand, I promise to pay George M. Saylor, or order, three thousand five hundred dollars, for value received, with interest at 8%.

H. R. Bozorth.

Indorsed as follows: June 10, 1889, $1250; Sept. 4, 1889, $400; Jan. 15, 1890, $550; May 20, 1890, $600. How much was due Dec. 20, 1890?

THE VERMONT RULE.

Apply the first payment to the liquidation of the interest due on the face of the note or the debt at the time the payment is made. If the payment exceeds the interest, apply the surplus to the reduction of the principal; and so proceed till the time of settlement.

NOTE.—Interest is not allowed on the simple interest remaining unpaid before settlement.

ANNUAL INTEREST.

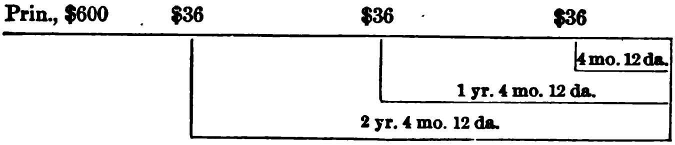
187. Annual Interest is simple interest on the principal and each year's interest, if unpaid, from the time it is due until the day of settlement.

Annual interest is charged when the words "with interest payable annually" are on the note.

WRITTEN EXERCISES.

1. What is due on a note for $600 at 6% for 3 yr. 4 mo. 12 da., interest payable annually?

The interest on $600 for 1 yr. at 6% is $36.



It is seen that the

1st year's interest is unpaid for 2 yr. 4 mo. 12 da.
2d " " " for 1 yr. 4 mo. 12 da.
3d " " " for 4 mo. 12 da.
Hence $36 is on interest for 4 yr. 1 mo. 6 da.

PROCESS.

Principal	$600
Int. of $600 for 3 yr. 4 mo. 12 da. at 6%	121.20
Int. of $36 for 4 yr. 1 mo. 6 da. at 6%	8.86
Amount due . . .	$730.06

2. What is due on a note for $560 for 2 yr. 8 mo., interest payable annually at 6%?

3. What is the annual interest of $1260 for 2 yr. 9 mo. 15 da. at 5%?

4. How much is due Apr. 1, 1894, on a note for $1000, dated July 15, 1891, interest payable annually at 8%, if the yearly interest has been regularly paid?

5. What is the annual interest of $1400 for 5 yr. 6 mo. 18 da. at 5% if the interest for the first two years has been paid?

6. What is due on a note for $840 at 5½% for 4 yr. 8 mo. 18 da., interest payable annually, if the interest for the first year has been paid?

7. What is due on a note for $960 at 4% for 5 yr. 10 mo. 12 da., interest payable annually, if the interest for the first two years has been paid?

COMPOUND INTEREST.

188. Compound Interest is interest on both principal and interest if the interest is unpaid when due.

WRITTEN EXERCISES.

1. What is the compound interest of $300 for 2 yr. at 6%?

PROCESS.

$300 = 1st prin.
18 = Int. for 1st yr.
$318 = 2d prin.
19.08 = Int. for 2d yr.
$337.08 = Comp. amt.
300.00 = 1st prin.
$37.08 = Comp. int., *Ans.*

The interest for 1 yr. is $18, which, added to $300 gives $318, the principal for the second year.

The interest of $318 for 1 yr. is $19.08, which added to $318 gives $337.08, the compound amount for 2 yr.

Subtracting the principal $300 from $337.08 gives the compound interest, $37.08.

RULE.

Find the amount of the principal for the first period of time.

Find the amount of this new principal for the second period, and thus continue until the end of the time.

Subtract the first principal from the last amount, and the result will be the compound interest.

If the interest is compounded semiannually, the rate is one-half the annual rate; if quarterly, one-fourth the annual rate, etc.

Find the compound interest of—

2. $300 for 2 yr. at 6%.
3. $400 for 3 yr. at 5%.
4. $1000 for 4 yr. at 7%.
5. $2000 for 3 yr. at 4%.

6. If the interest on $500 is compounded annually for 3 years, at what rate per cent. would the simple interest on the same principal for the same time equal the compound interest?

The work may be greatly shortened by using the table on page 208.

COMPOUND INTEREST TABLE.

Amount of $1 at Compound Interest from 1 to 20 Years.

Yrs.	1½ per cent.	2 per cent.	2½ per cent.	3 per cent.	3½ per cent.	4 per cent.
1	1.015000	1.020000	1.025000	1.030000	1.035000	1.040000
2	1.030225	1.040400	1.050625	1.060900	1.071225	1.081600
3	1.045678	1.061208	1.076891	1.092727	1.108718	1.124864
4	1.061364	1.082432	1.103813	1.125509	1.147523	1.169859
5	1.077284	1.104081	1.131408	1.159274	1.187686	1.216653
6	1.093443	1.126162	1.159693	1.194052	1.229255	1.265319
7	1.109845	1.148686	1.188686	1.229874	1.272279	1.315932
8	1.126493	1.171660	1.218403	1.266770	1.316809	1.368569
9	1.143390	1.195093	1.248863	1.304773	1.362897	1.423312
10	1.160541	1.218994	1.280085	1.343916	1.410599	1.480244
11	1.177949	1.243374	1.312087	1.384234	1.459970	1.539454
12	1.195618	1.268242	1.344889	1.425761	1.511069	1.601032
13	1.213552	1.293607	1.378511	1.468534	1.563956	1.665074
14	1.231756	1.319479	1.412774	1.512590	1.618695	1.731676
15	1.250232	1.345868	1.448298	1.557967	1.675349	1.800944
16	1.268985	1.372786	1.484506	1.604706	1.733986	1.872981
17	1.288020	1.400241	1.521618	1.652848	1.794676	1.947901
18	1.307341	1.428246	1.559659	1.702433	1.857489	2.025817
19	1.326951	1.456811	1.598650	1.753506	1.922501	2.106849
20	1.346855	1.485947	1.638616	1.806111	1.989789	2.191123

Yrs.	5 per cent.	6 per cent.	7 per cent.	8 per cent.	9 per cent.	10 per cent.
1	1.050000	1.060000	1.070000	1.080000	1.090000	1.100000
2	1.102500	1.123600	1.144900	1.166400	1.188100	1.210000
3	1.157625	1.191016	1.225043	1.259712	1.295029	1.331000
4	1.215506	1.262477	1.310796	1.360489	1.411582	1.464100
5	1.276282	1.338226	1.402552	1.469328	1.538624	1.610510
6	1.340096	1.418519	1.500730	1.586874	1.677100	1.771561
7	1.407100	1.503630	1.605781	1.713824	1.828039	1.948717
8	1.477455	1.593848	1.718186	1.850930	1.992563	2.143589
9	1.551328	1.689479	1.838459	1.999005	2.171893	2.357948
10	1.628895	1.790848	1.967151	2.158925	2.367364	2.593742
11	1.710339	1.898299	2.104852	2.331639	2.580426	2.853117
12	1.795856	2.012197	2.252192	2.518170	2.812665	3.138428
13	1.885649	2.132928	2.409845	2.719624	3.065805	3.452271
14	1.979932	2.260904	2.578534	2.937194	3.341727	3.797498
15	2.078928	2.396558	2.759031	3.172169	3.642482	4.177248
16	2.182875	2.540352	2.952164	3.425943	3.970306	4.594973
17	2.292018	2.692773	3.158815	3.700018	4.327633	5.054470
18	2.406619	2.854339	3.379932	3.996019	4.717120	5.559917
19	2.526950	3.025600	3.616527	4.315701	5.141661	6.115909
20	2.653298	3.207136	3.869684	4.660957	5.604411	6.727500

7. What is the compound interest of $500 for 5 yr. at 5%?

PROCESS.

$$\begin{array}{r} \$.276282 \\ 500 \\ \hline \$138.141000, \textit{ Ans.} \end{array}$$

According to the table, the compound amount of $1 for 5 yr. at 5% is $1.276282, and deducting $1 the compound interest of $1 is $.276282; hence the compound interest of $500 is 500 times $.276282, which is $138.141.

Find the compound interest of—

8. $800 for 10 yr. at 6%.

9. $1200 for 8 yr. at 7%.

10. $2000 for 16 yr. at 10%.

11. $480 for 4 yr. 3 mo. at 6%, payable quarterly.

12. $240 for 8 yr. 6 mo. at 5%, payable semiannually.

13. What is the difference between the simple and the compound interest of $840 for 3 yr. 4 mo. 24 da. at 6%?

14. Find the simple, the exact, the annual, and the compound interest of $365 for 2 yr. 6 mo. at 6%.

DISCOUNT.

189. Trade Discount.

Trade Discount is a reduction made in the price of an article.

Frequently in trade several discounts are made; as, "10 and 5 off," meaning 10% off, and 5% off the remainder.

WRITTEN EXERCISES.

1. What is due on a bill of $400, subject to a discount of 20% and 10%?

PROCESS.

$$\begin{array}{r} 100\% \\ 20 \\ \hline 80\% \\ 8 \\ \hline 72\% \end{array} \qquad \begin{array}{r} \$400 \\ .72 \\ \hline \$288.00, \textit{ Ans.} \end{array}$$

A reduction of 20% will leave 80%; a second reduction of 10% is $\frac{1}{10}$ of 80%, or 8% off, leaving 72%.

72% of $400 is .72 times $400, which is $288.

2. What is due on a bill of $600 at 15% discount and 3% off for cash?

3. What is the cash value of a bill of $84 at a discount of 16⅔%?

4. A bought goods to the amount of $300, and obtained a discount of 20% and 5% off for cash; how much did he pay for the goods?

5. Find the net amount of a bill for $250, subject to a discount of 30% and 10% off.

6. What reduction from the price of an article is 25% and 10% off?

7. What reduction from the price of an article is 5%, 10%, and 20% off?

8. B bought goods to the amount of $240, and obtained a discount of 20% and 12½%; how much did he pay for the goods?

9. What must a merchant ask for an article which cost $40, so that he may fall 20% and still gain 10%?

10. A merchant marks goods 20% above cost, and sells at 12½% below the list price; what per cent. does he gain?

11. At what per cent. above cost must goods be marked so that after falling 20% there may still be a gain of 20%?

12. A merchant marks goods 30% above cost, and then sells them at a discount of 25% from the list price; does he gain or lose, and what per cent.?

13. Find the net amount of a bill for $560, subject to discounts of 20%, 15%, 10%, and 5%.

14. At what per cent. above cost must a merchant mark an article so that he may sell at a discount of 20% from the list price and still have a profit of 40%?

15. At what per cent. above cost must goods be marked so that after falling 10% from the list price there shall still be a gain of 15%?

16. What must I ask for an article which cost $200, so that I may fall 15% from my asking price and still have a profit of 10%?

190. True Discount.

The **Present Worth** of a sum of money due at some future time without interest is such a sum as, being on interest for the given time, will amount to the given sum.

191. The **True Discount** is the difference between the given sum and the *present worth.*

Thus, if $106 is due 1 yr. hence without interest, the *present worth* is $100, since $100 on interest for 1 yr. at 6% will amount to $106. The *true discount* is $106 — $100, or $6. The true discount is, therefore, *the interest of the present worth.*

WRITTEN EXERCISES.

1. What is the true discount of $413 due 3 yr. hence, at 6% ?

PROCESS.

$1.18)$413.00(350
354
590
590

$413
350
63, Discount.

The amount of $1 for 3 yr. at 6% is $1.18.

Hence the present worth of $1.18 is $1, and $413 will give a present worth of as many dollars as $1.18 is contained times in $413, which is $350.

Subtracting the present worth, $350, from the given amount, $413, leaves the *true discount,* $63.

RULE.

To find the present worth, divide the given sum by the amount of $1.

To find the true discount, subtract the present worth from the given sum.

Find the true discount of—

2. $800 due 2 yr. hence at 6%.
3. $400 due 3 yr. 8 mo. hence at 6%.
4. $360 due 2 yr. 10 mo. hence at 5%.
5. $1200 due 4 yr. 6 mo. hence at 8%.
6. $700 due 5 mo. 18 da. hence at 6%.

Find the true discount of—

7. $3000 due 4 mo. 21 da. hence at 5%.

8. $2000 due 1 yr. 6 mo. 12 da. hence at 6%.

9. A bought $300 worth of goods on 6 mo. credit, but was offered 4% discount for cash; what is the gain by paying cash, money worth 6%?

10. B rents his house to A for $300 cash, or $80 a quarter payable in advance; which is the more profitable to A, and how much?

192. Bank Discount.

Bank Discount is a sum of money charged by a bank for the payment of a note before it becomes due. *It is the simple interest on the face of the note for the given time.*

NOTE.—In Pennsylvania, Maryland, and Missouri both the *day of discount* and the *day of payment* are reckoned. A 30-day note in the first two states would be discounted for 31 days, and in Missouri, where three days of grace are allowed, for 34 days.

193. The **Proceeds** of a note is the face less the discount.

In discounting an *interest-bearing* note the discount is estimated on the *amount due at maturity.*

CASE I.

Given the face of the note, the time, and the rate, to find the discount and proceeds.

WRITTEN EXERCISES.

1. What is the bank discount of a three-month note for $720, discounted at 6%?

PROCESS.

3 mo., ½ of 3 = .015
720
300
105
$10.800, *Ans.*

The simple interest of $720 for 3 mo. at 6% is $10.80.

Since bank discount and simple interest are the same, $10.80 is the bank discount required.

RULE.

Find the simple interest on the face of the note for the specified time, for the discount.

Subtract the discount from the face to find the proceeds.

Find the bank discount—

2. Of $400 for 30 da. at 6%.
3. Of $600 for 60 da. at 5%.
4. Of $80.60 for 3 mo. at 7%.
5. Of $780 for 18 da. at 8%.
6. Of $1000 for 2 mo. 12 da. at 6%.
7. Of $950.50 for 90 da. at 7%.

8. Find the proceeds of a 60-day note for $1260, discounted at 5%.

9. Find the bank discount of a note for $1600 due in 3 mo., discounted at 5%.

10. How much should a bank pay for a 4-month note whose face is $2400, if discounted at 6%?

11. Find the proceeds of a note for $5000 payable 3 mo. after date, with interest at 6%, if discounted at 5½%.

12. Find the bank discount of a note for $3600, dated Mar. 6, 1894, and payable 3 mo. after date, with interest at 5%, if discounted May 13, 1894, at 6%.

13. Find the proceeds of a note for $1200, dated July 24, 1894, payable 90 days after date, with interest at 5½%, and discounted Sept. 1, 1894, at 6%.

14. Find the difference between the true discount and the bank discount of $1090, due 1 yr. 6 mo. hence, discounted at 6%.

15. Find the difference between the true discount and the bank discount of a note for $10175, due 90 days hence, discounted at 7%.

16. Find the proceeds of a note for $900, dated July 5, at 90 days, discounted August 14, at 6%.

CASE II.

194. Given the time, the rate, and the proceeds, to find the face.

WRITTEN EXERCISES.

1. What is the face of a note which when discounted for 60 days at 6% yields $800?

PROCESS.

```
                    $1.00
60 da., ⅙ of 60 =    .01        |$808.0808+
Proceeds of $1 =   $.99)$800.0000
                          792
                           800
                           792
                             800
                             792
                               800
                               792
                                 8
```

The interest of $1 for 60 da. at 6% is $.01, which subtracted from $1 gives a proceeds of $.99.

If every dollar of the face gives a proceeds of $.99, to give a proceeds of $800 will require as many dollars as .99 is contained times in $800, which is $808.0808+.

RULE.

Divide the given proceeds by the proceeds of $1 *for the specified time.*

2. I wish to borrow $2000 from a bank for 90 days; for what sum must I give my note, discount at 6%?

3. What must be the face of a note which when discounted at 5% for 3 mo. will yield $240?

4. For what sum must I give my note for 60 days, so that when discounted at 6% it will yield $1200?

5. A broker bought a 90-day note for $40 less than its face; what was the face, discounted at 6%?

6. Mr. Byerly drew his note for 30 days, which when discounted at 1% a month yielded $2800; what was the face?

7. For what sum must a note be drawn for 6 mo. to yield $1800 when discounted at 6%?

8. I wish to borrow $1500 from a bank for 3 mo.; for what sum must I give my note, discounted at 8%?

9. A merchant received a 90-day note in payment for a quantity of flour: he at once had it discounted at 6%, and received $720; for what sum was the flour sold?

10. Mr. Evans presented a 3-mo. note at a bank, which when discounted at 8% yielded $1200; required the face.

11. What is the face of a 90-day note, which when discounted at 1½% a month yields $953.50?

12. A merchant received a 6-mo. note in payment for goods sold: he at once had it discounted at 1% a month, and received $1878; for what sum were the goods sold?

STOCKS AND BONDS.

195. Stock is a term applied to the capital of a company; it is usually divided into equal parts called **Shares**.

196. The **Par Value** of a share is its face value, and the price at which it is selling is its **Market Value.** If it is selling for more than its face value, it is said to be *above par*, or at a *premium;* if it sells for less than its face value, it is said to be *below par*, or at a *discount.*

197. A **Dividend** is a sum paid to the stockholders out of the gains of the company. An **Assessment** is a sum required of the stockholders to meet the losses of the company.

198. Bonds are the interest-bearing notes of a corporation or government. They are of two kinds—*Registered Bonds* and *Coupon Bonds.*

199. Stocks and bonds are usually quoted at their market value, which is a certain per cent. of the par value.

Thus, if stock is quoted at 105, it is selling at 5% above par; if it is quoted at 95, it is selling at 5% below par.

Bonds are usually named from the rate of interest they bear and the year in which they mature.

The following are the quotations of the United States bonds in the market of March 5, 1894:

	Bid.	Asked.
U. S. 4's, 1907, reg.	112¾	113¼
U. S. 4's, 1907, coup.	114	114¾
New 5's, reg., 1904	117¼	117⅝
New 5's, coup., 1904	117⅜	117½
Currency 6's, 1895	102	
" 6's, 1896	104	
" 6's, 1897	107	
" 6's, 1898	110	

200. Brokerage is a commission charged by a broker for buying and selling stocks and bonds.

The brokerage is usually ⅛% of the par value, and will be so regarded in this book.

The brokerage being estimated on the par value, it may be added to or subtracted from the market value of a share.

Thus, if stocks are bought at 108½, and the brokerage is ⅛%, the stocks cost 108⅝; if they are sold at 108½, the proceeds of a share are 108⅜.

WRITTEN EXERCISES.

1. What will 40 shares of New Jersey Central stock cost at 113½, brokerage ⅛%?

Process.

$$\begin{array}{r} \$113\tfrac{1}{2} \\ \underline{\tfrac{1}{8}} \\ 113\tfrac{5}{8} \\ \underline{40} \\ \$4545 \end{array}$$

One share will cost $113½ plus $⅛ brokerage, or $113⅝, and 40 shares will cost 40 times $113⅝, which is $4545.

2. What will 88 shares of New Jersey Central cost at 115½, brokerage ⅛%?

3. What is realized from the sale of 105 shares of Chicago and Northwestern at 104¼, brokerage ⅛%?

4. What must I pay for 15 shares of Del., Lack. and Western at 165¼, brokerage ⅛%?

5. Find the proceeds from the sale of 60 shares of New York Central at 100¼, brokerage ⅛%.

6. Find the cost of 80 shares of Delaware and Hudson stock at 140¼, brokerage ⅛%.

7. Find the cost of $16000 U. S. 4's, 1907, at 112¾, brokerage ⅛%.

8. What is the cost of $10000 U. S. New 5's, coup., 1904, at 117⅜, brokerage ⅛%?

9. How many shares of U. S. 4's, 1907, at 112¾, can be bought for $5643.75, brokerage ⅛%?

10. How many shares of Lake Shore and Michigan at 126⅞ can be bought for $15875, brokerage ⅛%?

11. What is the cost of 125 U. S. 6's at 104, brokerage ⅛%?

12. B sold stock at 97½, brokerage ⅛%, receiving $6232; how many shares did he sell?

13. How many shares of bank stock at 122⅛ can be bought for $4156.50, brokerage ⅛%?

14. How many shares at 110⅜ can be bought for $4088.50, brokerage ⅛%?

15. How many shares must a broker sell to realize $10.50, commission at ⅛%?

16. A broker realizes $37.62½ per week from the sale of stock at ⅛%; how many shares does he sell, and what is their market value at 105 per share?

17. If 90 shares of Penna. R. R. stock are bought for $9405, including a brokerage of ⅛%, what is the market value of the stock?

18. B realized \$11337.50 from the sale of \$10000 worth of stock, after paying $\frac{1}{8}\%$ brokerage; what was the quoted price of the stock?

19. If my quarterly dividend is \$92.50 at $2\frac{1}{2}\%$, how many shares do I own?

20. Mr. Martin will sell 72 shares of bank stock for \$8685, including the brokerage at $\frac{1}{8}\%$; what premium does he ask?

21. If \$10000 worth of stock is bought for \$9737.50, including the brokerage at $\frac{1}{8}\%$, what is the discount?

22. Mr. Lyte bought 240 shares of stock at $2\frac{3}{4}\%$ below par, and sold them at $5\frac{1}{2}\%$ above par; what was his gain if he paid brokerage on both transactions at $\frac{1}{8}\%$?

23. My broker bought 64 shares of Jersey Central at $114\frac{1}{2}$, and sold it at $121\frac{1}{4}$; what was my profit and what was his commission, brokerage $\frac{1}{8}\%$?

24. What annual income would I receive by investing \$2882.75 in 6% stock, bought at $110\frac{3}{4}$, brokerage $\frac{1}{8}\%$?

PROCESS.

```
$110.75
    .125
$110.875)$2882.750(26 shares.
          221750
           665250
           665250
                     $6
                     26
                   $156, Ans.
```

The cost of 1 share, including the brokerage, is \$110.875; then for \$2882.75 I can buy 26 shares.

The income on 1 share is \$6, and on 26 shares it is 26 × \$6, which is \$156.

25. What annual income will be realized from investing \$7200 in 5% stock at $112\frac{3}{8}$, brokerage $\frac{1}{8}\%$?

26. What annual income will be realized from investing \$7155 in 4% bonds at $99\frac{1}{4}$, brokerage $\frac{1}{8}\%$?

27. What annual income will be realized from investing \$141000 in U. S. 5's at $117\frac{3}{8}$, brokerage $\frac{1}{8}\%$?

28. B paid \$10989 for U. S. 6's at $110\frac{7}{8}$, brokerage $\frac{1}{8}\%$; what was his income?

29. What amount must be invested in $6\frac{1}{2}\%$ stock at $121\frac{5}{8}$, brokerage $\frac{1}{8}\%$, to realize an annual income of \$1365?

PROCESS.

\$6.5)\$1365.0(210 shares.	\$$121\frac{5}{8}$	\$121.75
130	$\frac{1}{8}$	210
65	\$$121\frac{3}{4}$	121750
65		24350
00		\$25567.50, *Ans.*

1 share yields \$$6\frac{1}{2}$; to yield \$1365 will require 210 shares.

1 share costs \$$121\frac{3}{4}$; 210 shares will cost 210 times \$$121\frac{3}{4}$, which is \$25567.50.

30. What amount must be invested in $4\frac{1}{2}\%$ stock at $103\frac{3}{8}$, brokerage $\frac{1}{8}\%$, to realize \$477?

31. What amount must be invested in U. S. 5's at $117\frac{3}{8}$, brokerage $\frac{1}{8}\%$, to secure an annual income of \$150?

32. What sum must I invest in U. S. 6's at $111\frac{7}{8}$, brokerage $\frac{1}{8}\%$, to secure an annual income of \$420?

33. What sum must be invested in U. S. 5's at $116\frac{1}{2}$, brokerage $\frac{1}{8}\%$, to secure an annual income of \$160?

34. What sum must be invested in U. S. 4's at 114, brokerage $\frac{1}{8}\%$, to secure an annual income of \$1000?

35. If I buy 5% stock at $83\frac{5}{24}$, brokerage $\frac{1}{8}\%$, what per cent. does the investment pay?

PROCESS.

\$$83\frac{1}{3}$)\$5.00(.06, or 6%
500

1 share costs \$$83\frac{1}{3}$; then \$$83\frac{1}{3}$ multiplied by some rate gives \$5. That rate is \$5 ÷ \$$83\frac{1}{3}$, which is .06, or 6%, *Ans.*

36. If 6% stock is bought at 120, what per cent. does the investment yield?

37. If 7% stock is bought at $139\frac{7}{8}$, brokerage $\frac{1}{8}\%$, what is the rate of income?

38. What must be paid for 5% bonds to yield 6% interest on the investment, brokerage not considered?

39. What rate of interest do I realize on my investment by buying Georgia 6's at $112\frac{3}{4}$; brokerage $\frac{1}{8}\%$?

40. What is the annual income from investing $4446 in 5½% stock, at 92½, brokerage ⅛%?

41. What sum must be invested in 6¼% stock at 106¼ to yield an annual income of $525, brokerage not considered?

42. Which is the better investment, 7% stock at 122½, or 6% stock at 105¾, brokerage ⅛%?

43. What must I pay for stock which pays a 9% dividend to realize 6% on my investment?

44. What rate of interest is made by buying stock which pays a 4% semiannual dividend at 159⅞, brokerage ⅛%?

45. How much must be invested in 4½% stock at 92⅜, brokerage ⅛%, to yield an annual income of $166.50?

MISCELLANEOUS PROBLEMS.

1. A sold his house for $4000, and thereby cleared 20% of this amount; what would he have gained % by selling it for $3600?

2. What must I ask for an article worth $18, so that I may fall 10% on my asking price and still gain 20%?

3. What is the gain per cent. by buying an article at 20% below cost and selling it at 12% above cost?

4. An agent sold goods on 4% commission: after deducting his commission he sent his employer a check, which being discounted at 6% for 30 days yielded $3000; what was the amount of the sales?

5. I gave a note worth 1% premium for 40 shares of stock at 3¼% discount; what was the face of the note?

6. My agent bought goods on 3% commission: I sent him a note for $2400 due 60 days hence, which when discounted at 6% paid both the commission and the cost of the goods; find the commission.

7. I sent my agent a note for 90 days with which to purchase goods on 2½% commission: he had the note discounted at a bank, and after retaining his commission bought $1800 worth of goods; what was the face of the note?

8. My house is insured for ¾ of its real value at 1⅛% : in a recent fire it was burned down, and I lost $2481; what was the value of the house?

9. What is the interest of $325.62 for 4 yr. 6 mo. 17 da. at 6⅔%?

10. Find the exact interest of $780 from June 14, 1891, to Jan. 22, 1892, at 7%.

11. Find the annual interest of $460 for 3 yr. 6 mo. 10 da. at 5%.

12. A paid B a debt which had been standing for 3 yr. 2 mo. 29 da. at 6%: the amount was $750; what was the original debt?

13. How long must $800 be on interest at 5½% to give $210 interest?

14. At what rate will $905.16 for 2 yr. 3 mo. 4 da. give $112.57 interest?

15. A 90-day note for $450, with interest, was paid in 50 days; what amount was due if discounted at 6%?

16. What is the difference between the simple interest and compound interest of $500 for 5 yr. 5 mo. 5 da. at 5%?

17. What is the true discount of $800, due 4 yr. 4 mo. 4 da. hence at 6%?

18. What are the proceeds of $240, due 60 days hence, discounted at a bank at 5%?

19. For what sum must I give my note for 90 days, bearing interest at 6%, which when discounted at 5½% will yield $600?

20. What must I pay for Pennsylvania 6's to realize 7% on my investment, brokerage ⅛%?

21. At what price must 4½% stock be bought to yield the same per cent. on the investment that a 6% stock yields when bought at 110¾?

SECTION IX.

RATIO AND PROPORTION.

201. Ratio.

The **Ratio** of two similar numbers is the quotient obtained by dividing the first by the second.

Thus, the ratio of 8 to 4 is $\frac{8}{4}$, or 2; it may also be expressed 8 : 4.

The **Terms** of a ratio are the numbers compared.

The first term of the ratio is called the **Antecedent**, and the second term the **Consequent**.

Since a ratio can be expressed as a fraction,

Both terms of a ratio may be multiplied or divided by the same number without changing its value.

A **Simple Ratio** is the ratio of two numbers; as, 8 : 4. A **Compound Ratio** is the product of two or more simple ratios; as, (4 : 6) × (8 : 3), or $\frac{4}{6} \times \frac{8}{3}$.

WRITTEN EXERCISES.

What is the ratio of—

1. 12 to 4?
2. 4 to 40?
3. $\frac{2}{3}$ to $\frac{1}{3}$?
4. $\frac{2}{3}$ to $\frac{5}{6}$?
5. 44 to 10?
6. 16 to 72?

7. 8 bu. : 56 bu?
8. $\frac{2}{3}$ yd. : $\frac{5}{6}$ yd.?
9. $\frac{3}{4}$ yd. : $\frac{3}{4}$ ft.?
10. \$2½ : \$15?
11. £8 : 5 s.?
12. $\frac{12}{35} : \frac{18}{49}$?

13. What is the ratio of 4 lb. 8 oz. to 1 lb. 2 oz.?

14. What is the ratio of £5 8 s. to £3 4 s.?

15. Which ratio is the greater, 49 : 56 or 64 : 72?

16. The antecedent is 8 and the consequent is 16; what is the ratio?

17. The consequent is 17 and the ratio is 34; what is the antecedent?

18. The antecedent is 512 and the ratio is 8; what is the consequent?

202. Simple Proportion.

A **Proportion** is an equality of ratios.

Thus, 4 : 8 = 5 : 10 is a proportion, and may be written $\frac{4}{8} = \frac{5}{10}$.

A **Simple Proportion** is a proportion in which both ratios are *simple* (Art. 201); as, 6 : 3 = 8 : 4.

A *Proportion* may be *read* in two ways; thus, 4 : 8 = 5 : 10 is read "the ratio of 4 to 8 equals the ratio of 5 to 10;" or simply, "4 is to 8 as 5 is to 10."

203. The **Terms** of a proportion are the four numbers used in the comparison.

In a proportion—

The **Antecedents** are the first and third terms.
The **Consequents** are the second and fourth terms.
The **Extremes** are the first and fourth terms.
The **Means** are the second and third terms.
The **Couplets** are the ratios compared.

Thus, in the proportion 4 : 8 = 5 : 10,

4 and 5 are the antecedents. 8 and 10 are the consequents.
4 and 10 are the extremes. 8 and 5 are the means.
4 and 8 are the *first* couplet. 5 and 10 are the *second* couplet.

PRINCIPLES.

204.—1. *In every proportion the product of the means is equal to the product of the extremes.*

Thus, in the proportion $4 : 8 = 5 : 10$,

$$8 \times 5 = 4 \times 10.$$

2. *Either mean equals the product of the extremes divided by the other mean.*

Thus, in the proportion $4 : 8 = 5 : 10$,

$$8 = \frac{4 \times 10}{5}, \text{ or, } 5 = \frac{4 \times 10}{8}.$$

3. *Either extreme equals the product of the means divided by the other extreme.*

Thus, in the proportion $4 : 8 = 5 : 10$,

$$4 = \frac{8 \times 5}{10}, \text{ or, } 10 = \frac{8 \times 5}{4}.$$

WRITTEN EXERCISES.

Find the term that is wanting in—

1. $4 : 6 = 8 : (\ )$.
2. $9 : 5 = (\ ) : 15$.
3. $8 : (\ ) = 6 : 12$.
4. $(\ ) : 5 = 8 : 20$.
5. $(\ ) : 8 = 10 : 40$.
6. $9 : (\ ) = 7 : 21$.
7. $6 ; 5 = (\ ) : 11$.
8. $8 : 4 = 12 : (\ )$.
9. 10 lb. : 6 lb. = 6 : ( ).
10. 140 men : 8 men = 70 : ( ).
11. 5 gal. : 120 gal. = 8 : ( ).
12. 420 oz. : 630 oz. = 12 : ( ).
13. 570 yd. : 19 yd. = 116 : ( ).
14. 625 ft. : 25 ft. = 1200 : ( ).
15. $640 : $3200 = 75 : ( ).

PROBLEMS IN PROPORTION.

1. If 12 yards of cloth cost \$54, what will 22 yards cost?

It will be found most convenient to arrange the terms of the proportion so that the required result shall be in the *fourth term*, and the quantity which is of the same kind as the answer in the *third term*. If we let x represent the required result, the proportion will be of the form—

$$(\ ):(\ ) = \$54 : x.$$

The other two quantities, 12 yards and 22 yards, will form the first ratio; and since 22 yards will cost more than 12 yards, we place the greater number in the second term; for, if the fourth term is to exceed the third, the second term must exceed the first.

The proportion will then be

$$12 \text{ yd.} : 22 \text{ yd.} = \$54 : x.$$

To avoid multiplying *yards* by *dollars*, we consider all the terms abstract. Thus,

$$12 : 22 = 54 : x.$$

By Art. 204, Prin. 3, either extreme is equal to the product of the means divided by the other extreme.

Whence $$x = \frac{22 \times 54}{12} = 99.$$

The cost of 22 yards is \$99, *Ans.*

2. If a pound of tea lasts 8 persons 105 days, how long will it last 21 persons?

Since the answer is to be days, we put 105 days as the third term.

It is evident that a pound of tea will last 21 persons *less* time than it will last 8 persons; hence we put 21 for the first term and 8 for the second; for, if the fourth term is to be less than the third, the second term must be less than the first; and we have

$$21 : 8 = 105 : (?).$$

Whence $$(?) = \frac{105 \times 8}{21} = 40.$$

The required time is 40 days.

RULE.

Put the quantity which is of the same kind as the answer for the third term.

Place the greater of the other two numbers in the second term when the result is to be greater, and in the first term when the result is to be less, than the third term, and the remaining number in the unoccupied term.

Divide the product of the means by the given extreme.

3. If 15 cords of wood cost $112.50, what will 34 cords cost?

4. If 12 men can do a piece of work in 53 days, how long will it require 22 men to do it?

5. What will 75 acres of land cost, if 22 acres cost $3520?

6. If 42 bushels of potatoes cost $25, what will 52 bushels cost?

7. If 40 men can do a piece of work in 289 days, in what time can 17 men do it?

8. If 3½ tons of coal cost $18, what will 14 tons cost?

9. If 7½ tons of coal can be bought for $46⅞, how many tons can be bought for $150?

10. If 8 horses consume 14 tons of hay in a certain time, how much will 51 horses consume in the same time?

11. If 16 horses consume a quantity of oats in 142 days, how long will it last 40 horses?

12. If a pole 57 feet high casts a shadow 42 ft. long, how long will be the shadow of a pole 114 ft. high?

13. A failed in business, and can pay only 62½ cents on each dollar of his indebtedness; how much will B receive, whose claim against A is $825.75?

14. If 250 men can do a piece of work in 16⅔ days, how long will it require 42 men to do it?

15. If 12½ tons of hay cost $180.25, what will 81½ tons cost?

16. A regiment of 960 men has provision for 40 days; how long will it last if the regiment is reinforced by 240 men?

17. A walked 156 rods in 13 minutes; how long will it take him to walk a mile?

18. If 96 men can dig a ditch in 41$\frac{2}{3}$ days, how long will it require 57 men to dig it?

19. A garrison of 560 men has provision for 240 days: a reinforcement is received, and the provision lasts but 80 days; how many men in the reinforcement?

20. If 125 acres of land cost $10500, what will 17 acres cost at $4 less an acre?

21. If 42 men can do a piece of work in 64 days, how long will it take to do one-half of it if 12 men refuse to work?

22. If 54 men can do a piece of work in 96 days, how long will it take to complete the work if 8 men leave when it is one-third done?

23. If it requires 145 days to complete a piece of work by working 10 hours a day, how long will it take by working 7$\frac{1}{2}$ hours a day?

24. If a field 60 rods long and 20 rods wide cost $500, what will a field 15 chains long and 8 chains wide cost?

25. A family of 10 persons have provision for 212 days: at the end of 42 days 2 persons leave; how long will the remainder of the provision last?

26. A's rate of working is to B's as 4 : 7; how long will it take A to do what B does in 63 days?

27. If 5 bu. 3 pk. of potatoes cost $4.60, what will 2 bu. 1 pk. cost at the same rate?

28. If a lot of ground 50 feet wide by 126 feet deep contains 700 square yards, how deep must a lot be that is 75 feet wide to contain the same area?

29. My watch gains 3 min. 10 sec. a day; if it is set right on Monday morning at 6 A. M., what is the time by my watch on the following Saturday afternoon at 4 P. M., correct time?

205. Compound Proportion.

A Compound Proportion is a proportion in which one or both ratios are *compound* (ART. 201); as,

$$\left.\begin{array}{r}8:16\\4:\ \ 3\end{array}\right\} = 6:9.$$

PROBLEMS IN COMPOUND PROPORTION.

1. If 8 men build 240 feet of wall in 60 days of 10 hours each, how many feet of wall will 12 men build in 80 days of 8 hours each?

Since the answer is to be *feet*, we make 240 feet the third term.

If 8 men build 240 feet of wall, 12 men will build more wall, other things being equal; hence the answer would be *greater* than the third term, and the first ratio is 8 : 12.

Again, if 240 feet of wall are built in 60 days, in 80 days more wall can be built, other things being equal; hence the answer would be *greater* than the third term, and the second ratio is 60 : 80.

Also, if 240 feet of wall are built by working 10 hours a day, a less number of feet of wall will be built by working 8 hours a day, other things being equal; hence the answer would be *less* than the third term, and the third ratio is 10 : 8.

The work therefore stands as follows:

$$\left.\begin{array}{r}8:12\\60:80\\10:\ \ 8\end{array}\right\} = 240:x.$$

Whence $$x = \frac{\overset{4}{\cancel{12}} \times \overset{4}{\cancel{80}} \times \cancel{8} \times \overset{24}{\cancel{240}}}{\cancel{8} \times \underset{3}{\cancel{60}} \times \cancel{10}} = 384 \text{ ft., } \textit{Ans.}$$

2. If 40 men earn $1280 in 16 days, how much will 36 men earn in 31 days?

3. If 12 men dig a trench 40 rods long in 24 days of 10 hours each, how many rods can 16 men dig in 18 days of 9 hours each?

4. If 42 men can build a wall 56 rods long in 96 days, how many men will be required to build 63 rods in 84 days?

5. If $500 in 4 yr. at 6% gain $120 interest, how much will $560 gain in 10 yr. at 5%?

6. If a man travel 360 miles in 15 days of 7 hours each, how far will he travel in 14 days of 10 hours each?

7. If a man travel 540 miles in 20 days of 10 hours each, how many hours a day must he travel to cover 630 miles in 25 days?

8. If 10 planks 18 ft. long, 16 in. wide, 2 in. thick cost $12, what cost 25 planks 16 ft. long, 15 in. wide, and 3 in. thick?

9. If 30 men, working 10 hours a day, dig in 20 days a trench 400 rods long, 4 ft. wide, and 5 feet deep, in how many days can 36 men, working 8 hours a day, dig a trench 240 rods long, 5 feet wide, and 6 feet deep?

10. If a piece of iron 7 feet long, 4 inches wide, and 6 inches thick weighs 600 pounds, how much will a piece of iron weigh that is 16 feet long, 8 inches wide, and 4 inches thick?

11. If 10 men can build 120 rods of fence in 12 days of $9\frac{1}{3}$ hours each, how many hours a day will 16 men have to work to build 300 rods of fence in 25 days?

12. If 24 men can build a wall 72 rods long, 6 feet wide, and 5 feet high in 60 days of 10 hours each, how many days will it take 32 men to build a wall 96 rods long, 4 feet wide, and 8 feet high, working 8 hours a day?

13. If from 40 cows, each giving 12 quarts of milk daily, 15 cheeses of 40 pounds each are made in 15 weeks, how many cows will produce 25 cheeses of 60 pounds each in 12 weeks, if they give 10 quarts each daily?

14. If 12 compositors in 30 days of 10 hours each set up 25 sheets of 16 pages each, 32 lines on a page, in how many days 8 hours long can 18 compositors set up, in the same type, 64 sheets of 12 pages each, 40 lines on a page?

PARTNERSHIP.

206. A **Partnership** is an association formed by two or more persons for the transaction of business.

The *Partners* are the persons forming the partnership.

The **Capital** is the money invested.

The **Assets** or **Resources** of a partnership are its property of any kind.

The **Liabilities** are its debts.

207. Simple Partnership.

In *Simple Partnership* each partner's capital is invested for the same time, and the profits or losses are divided in proportion to their capital.

WRITTEN EXERCISES.

1. A, B, and C form a partnership: A puts in $5000, B $4000, and C $3000; if they gain $3840, what is each one's share of the gain?

The capital invested is $5000 + $4000 + $3000, or $12000. If $12000 gain $3840, $1 will gain $.32. Then,

A's share is 5000 × $.32 = $1600;
B's share is 4000 × $.32 = $1280;
C's share is 3000 × $.32 = $960, *Ans.*

2. A and B formed a partnership: A put in $4200 and B $3400; they gained $1824; what was each man's share of the gain?

3. A, B, and C engaged in business with a capital of $13000: A put in $5100, B $4800, and C the remainder; they gained $5200; what was each one's share of the gain?

4. Four men engaged in business: L put in $5000, M $6000, N $7000, and O $8000; they lost $3380; what was each one's share of the loss?

5. A and B engaged in business: A put in 5 times as much capital as B; if they gained $1434, what was each one's share of the gain?

6. E failed in business, owing A $214, B $316, and C $470; what was the share of each creditor, if the resources of E amounted to $350.25?

7. M, N, and P engaged in the hardware business with a joint capital of $7280: M put in twice as much as N, and P put in $1040; if they gained $1820, what was each one's share of the gain?

8. A, B, and C engaged in business, A putting in $\frac{2}{3}$ as much capital as B, and B $\frac{2}{3}$ as much as C; if they gained $3610, what was each one's gain?

9. A man dying willed $5000 to each of two daughters, $4000 to each of three sons, and $8000 to his widow: on settling the estate it was found to be $25,000; how should it be divided?

10. A, B, and C engaged in business with a joint capital of $25,000: at the end of the year A's share of the gain was $1620, B's $2214, and C's $2416; required the capital of each.

11. Three farmers raised 500 bushels of wheat on a tract of land cultivated by them conjointly: they agreed that A should take $\frac{1}{4}$, B $\frac{1}{5}$, and C $\frac{1}{6}$ of it; but after this division there is a remainder; how should it be divided?

12. A, B, and C formed a partnership to operate a dairy: A contributed $500, B $700, and C 24 cows; the first year they gained $1728, of which C received $768; what were A's and B's gain and the average value of C's cows?

13. M, N, and O buy a property for $12,000: M contributes $4500, N $3000, and O the remainder; the rents received from this property amount to $100 per month, and the taxes and other expenses to $180 per annum; what is each man's yearly share of the proceeds?

208. Compound Partnership.

In **Compound Partnership** each partner's capital is invested for unequal periods of time, and the profits or losses are divided in proportion to the capital and the length of time it is invested.

WRITTEN EXERCISES.

1. A, B, and C form a partnership: A puts in $300 for 5 months, B $500 for 6 months, and C $700 for 7 months; they gain $1974; what is each partner's share of the gain?

Investing—

$300 for 5 mo. is the same as $1500 for 1 mo.
$500 " 6 " " " $3000 "
$700 " 7 " " " $4900 "

If the gain of $1500 + $3000 + $4900, or $9400, for 1 mo. is $1974, the gain on $1 for 1 mo. is $.21. Then

A's share is 1500 × $.21 = $315,
B's " 3000 × $.21 = $630,
C's " 4900 × $.21 = $1029, *Ans.*

2. M and N formed a partnership: M put in $2300 for 8 months and N $3200 for 7 months; if they gained $1264.80, what was each one's share of the gain?

3. R and S formed a partnership: R put in $1100 for 9 months, S $1300 for 11 months; they lost $726; what was each man's share of the loss?

4. A, B, and C entered into partnership: A invested $325 for 10 months, B $450 for 13 months, and C $300 for 14 months; they gained $798; what was each man's gain?

5. R, S, and T hire a pasture for $115.20: R puts in 10 cows for 3 months, S 12 cows for 3½ months, and T 16 cows for 4½ months; what should each man pay?

6. A commenced business with $800 capital: at the end of 4 months he took in B, with $1000 capital, and at the end of

8 months he took in C, with $1200 capital; at the end of the year they had gained $2912; required the gain of each.

7. M and N formed a partnership for one year: M put in $1600 at the beginning of the year and $900 at the end of 7 months; N put in $1200 at the beginning of the year and $1200 at the end of 8 months; they gained $4719; required the gain of each.

8. A and B engaged in business for one year: A put in $2000 and B $3500; at the end of 8 months A put in $500 and B drew out $900; they gained $8372; what was each man's share of the gain?

9. A, B, and C agreed to do a piece of work for $65.50: A labored 8 days of 10 hours each, B 7 days of 9 hours each, and C 14 days of $8\frac{1}{2}$ hours each; how much should each receive?

10. M, N, and P entered into partnership for one year: M put in $1500, and at the end of 7 months added $900; N put in $2000, and at the end of 9 months took out $500; P put in $1800, at the end of 5 months put in $600, and at the end of 9 months took out $1100; they gained $4650.60; what was each man's share of the gain?

11. A, B, and C entered into partnership for one year: at the beginning of the year A put in $2000, B $2200, and C $2460; at the end of 5 months A added $800, C $600, and B took out $400; at the end of 8 months A took out $500, C $1240, and B added $1200; they gained $8476; what was each man's share of the gain?

EQUATION OF PAYMENTS.

209. Equation of Payments is the process of finding at what time several sums due at different times may all be paid together, with justice to both debtor and creditor.

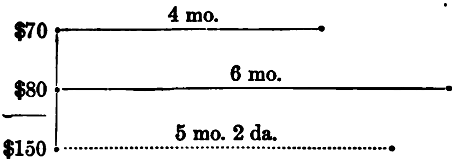
The time thus found is the *Equated Time.*

210. The **Focal Date** is the date from which we begin to reckon in averaging an account.

WRITTEN EXERCISES.

1. A owes B $70 due in 4 months, and $80 due in 6 months; what is the average term of credit?

ILLUSTRATION.



PROCESS.

70 × 4 = 280
80 × 6 = 480
150)760(5 mo.
750
$\frac{10}{150}$ = 2 da.
5 mo. 2 da., *Ans.*

A credit on $70 for 4 mo. is equivalent to a credit on $1 for 280 mo.

A credit on $80 for 6 mo. is equivalent to a credit on $1 for 480 mo.

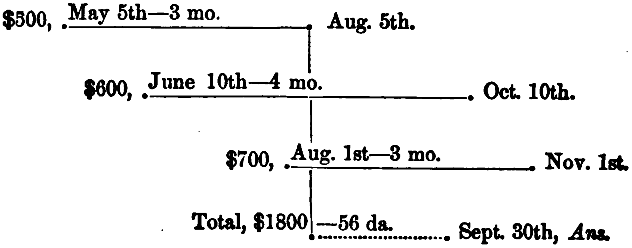
Adding, A is entitled to a credit on $1 for 760 mo.; hence he is entitled to a credit on $150 for as many months as 150 is contained times in 760, which is $5\frac{1}{15}$ months, or 5 mo. 2 da.

2. I purchased the following bill of goods:

May 5, a bill of $500 on 3 mo. credit,
June 10, " $600 on 4 "
Aug. 1, " $700 on 3 "

When can I settle these bills in one payment without loss to either party?

ILLUSTRATION.



PROCESS.

500 × 00 = 00000
600 × 66 = 39600
700 × 88 = 61600
1800)101200(56 +
90
112
108

56 da. after Aug. 5, which is Sept. 30, *Ans.*

These bills begin at different dates and are due at different dates; hence, to average them, we select the earliest date at which any of the bills become due (Aug. 5) from which to reckon times.

The second falls due Oct. 10, and the third Nov. 1.

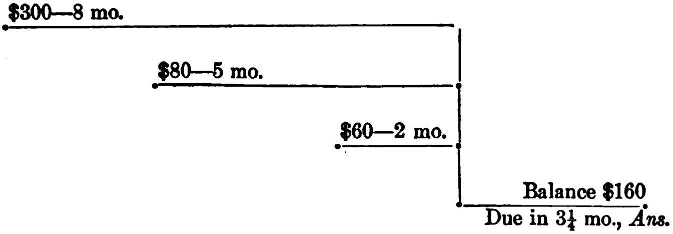
From Aug. 5 to Oct. 10 is 66 da.

From Aug. 5 to Nov. 1 is 88 da.

Proceeding as in Ex. 1, we have a credit of 56 days after Aug. 5, which is Sept. 30.

3. A owes $300 due in 8 months; if he pays $80 in 3 months and $60 in 6 months, when should he pay the balance?

ILLUSTRATION.



PROCESS.

80 × 5 = 400
60 × 2 = 120
$300 $140 160)520(3¼ mo.
140 480
$160 $\frac{40}{160} = \frac{1}{4}$

By paying $80 5 mo. before it is due and $60 2 mo. before it is due, A loses an equivalent of $1 for 520 mo.

Hence he is entitled to the use of the balance, $160, long enough after it becomes due to equal $1 for 520 mo., which is $\frac{1}{160}$ of 520 mo., or 3¼ mo.

4. M owes $400 due in 5 mo., $500 due in 6 mo., and $600 due in 7 mo.; what is the average term of credit?

5. Mr. Bowman bought goods as follows: Mar. 15, $300 on 2 mo. credit; Apr. 10, $400 on 3 mo. credit; May 5, $500 on 4 mo. credit; what is the equated date of payment?

6. B sells goods to C on credit of 60 days as follows: Jan. 14, 1894, a bill of $2000; Feb. 10, 1894, a bill of $1500; Mar. 25, 1894, a bill of $3000; find the equated time and date of payment.

7. B borrowed $1200 for 6 mo.: at the end of 2 mo. he paid $400, and at the end of 4 mo. he paid $600; when should the balance be paid?

8. What is the average time of paying $120 due in 5 mo, $150 due in 6 mo., and $160 due in 4 mo.?

9. What is the date of paying $60 due May 18, $130 due June 24, and $180 due Aug. 1?

10. I gave B $12000 for 10 mo., $\frac{1}{2}$ of which he paid in 4 mo., and $\frac{1}{2}$ of the remainder in 6 mo.; how long may the balance remain unpaid?

11. A merchant owes $2200 due in 9 mo.; if he pays $800 in 3 mo., $600 in 5 mo., and $400 in 7 mo., how long in justice may the balance remain unpaid?

12. I sold goods to Mr. Martin as follows:

Oct. 14, 1893, a bill of $600 on 3 mo. credit;
Nov. 20, 1893, " $500 on 4 "
Dec. 16, 1893, " $400 on 3 "
Jan. 24, 1894, " $350 on 6 "

Find the date of payment.

13. A owes $800 due in 5 mo., $1200 due in 7 mo.; if at the end of 5 mo. he pays $1400, when should the balance be paid?

14. Three 90-day notes bear date as follows: Apr. 10, 1894, $480; May 12, 1894, $520; June 14, 1894, $640. What is the average date of payment, counting three days of grace?

15. A sells B goods as follows: $200 in 90 days, $300 in 60 days, $400 in 30 days, $500 in 10 days, and $600 cash; what is the average term of credit?

16. A merchant owes $3300 due in 8 months; if he pays $1200 in 3 mo., $1000 in 4 mo., and $500 in 6 mo., when should he pay the balance?

AVERAGING OF ACCOUNTS.

211. Averaging of Accounts is the process of finding the time when the balance of an account may be paid with justice to both debtor and creditor.

WRITTEN EXERCISES.

1. When is the balance of the following account due?

Dr. *John H. Landis.* *Cr.*

1893.					1893.				
Mar.	15	To Flour,	$120	00	Apr.	25	By Cash,	$100	00
Apr.	20	" " 2 mo.	200	00	May	24	" Draft, 15 da.	180	00
May	28	" " 1 mo.	300	00	June	16	" Cash,	200	00

PROCESS.

Mar. 15,	120 × 00 =	0000	Apr. 25,	100 × 41 =	4100
June 20,	200 × 97 =	19400	June 11,	180 × 88 =	15840
June 28,	300 × 105 =	31500	June 16,	240 × 93 =	22320
	$620	50900		520	42260
	520	42260			
	$100	8640			

8640 ÷ 100 = $86\frac{2}{5}$. In practice 86 da.
Mar. 15 + 86 da. = June 9th, *Ans.*

The payment for the flour sold Apr. 20th on 2 mo. credit is due June 20th, and that sold May 28th on 1 mo. credit is due June 28th. The 15-day draft of May 24th is due 18 days after May 24th, or June 11th.

It will be seen that Mar. 15th is the earliest date at which any one of the items is due. Using this as the focal date, we find the sum of the products on the debit side to be equivalent to $1 for 50900 days, and the sum of the products on the credit side to be equivalent to $1 for 42260 days. Subtracting 42260 from 50900, there is a balance on the debit side equivalent to the use of $1 for 8640 days.

In order to make the products on the debit side equal to the products on the credit side, it is evident that the balance of $100 must be paid after the focal date, Mar. 15th. Now, for the credit of $100 to be equivalent to $1 for 8640 days, it must have a credit of 86 days.

Hence the account can be settled by paying $100 86 days after Mar. 15th, or on June 9th.

2. When is the balance of the following account due?

Dr. *William C. Ford.* *Cr.*

1892.					1892.				
Aug.	16	To Car'ge, 60 da.	$250	00	Sept.	20	By Cash,	$100	00
Oct.	2	" Mdse.,	200	00	Sept.	25	" Draft, 30 da.	300	00

PROCESS.

Oct. 15, 250 × 25 = 6250	Sept. 20, 100 × 00 = 0000
Oct. 2, 200 × 12 = 2400	Oct. 25, 300 × 35 = 10500
$450 8650	$400 10500
400	8650
$50	1850

1850 ÷ 50 = 37 da. 37 da. back from Sept. 20th is Aug. 14th, *Ans.*

Solving this problem as Ex. 1, we find that the balances are on opposite sides of the account.

Now, in order to make the products of both sides equal the credit of the balance, $50, for 37 days must be subtracted from the sum of the products on the credit side. This indicates that it should have been paid 37 days before the focal date, Sept. 20th, or on Aug. 14th.

It is evident from these examples *that if the two balances are on the same side of the account, the equated time is after the focal date; if they are on opposite sides of the account, the equated time is before the focal date.*

3. Find the equated time for paying the balance of the following account:

Dr. *Samuel P. Hambleton.* *Cr.*

1890.				1890.			
Aug.	18	To Mdse.,	$400	Sept.	1	By Cash,	$350
Sept.	24	" "	800	Oct.	12	" "	650

4. Find the equated time for paying the balance of the following account:

Dr. *John L. Shroy.* *Cr.*

1891.				1891.			
Jan.	16	To Mdse.,	$600	Jan.	18	By Cash,	$400
Feb.	1	“ “ 30 da.	500	Mar.	16	“ Draft, 15 da.	600
Apr.	10	“ “	700	May	1	“ Cash,	500

5. Find the equated time for paying the balance of the following account:

Dr. *Geo. W. Bickel.* *Cr.*

1892.				1892.			
May	4	To Mdse., 20 da.	$800	June	1	By Draft, 2 mo.	$700
June	6	“ “	500	July	2	“ Cash,	500
July	10	“ “ 30 da.	600	Aug.	3	“ Draft, 30 da.	400

6. Find the equated time for paying the balance of the following account:

Dr. *J. L. Lindsay.* *Cr.*

1893.				1893.			
Mar.	4	To Mdse., 30 da.	$600	Apr.	5	By Draft, 30 da.	$1000
Apr.	1	“ “	800	May	14	“ “ 15 da.	300
June	12	“ “ 20 da.	500	July	1	“ Cash,	500
Aug.	18	“ “	400	Aug.	30	“ Draft, 20 da.	300

NOTE 1.—Any other date than the earliest might be selected as the focal date. If we reckon from the last date at which any item becomes due, as a focal date, and the balances are on the same side of the account, the equated time is *before* the focal date; if they are on opposite sides, the equated time is *after* the focal date.

NOTE 2.—An account may be averaged by finding the interest at any per cent. on each item, and dividing the balance of interest by the interest on the balance of the account for one day; the quotient will be the number of days to be added to or subtracted from the focal date.

SECTION X.

INVOLUTION AND EVOLUTION.

INVOLUTION.

212. Involution is the process of raising a number to any required power.

213. The **Power** of a number is the product obtained by using the number several times as a factor.

The *Power* is indicated by a small figure, called an **Exponent,** placed at the right of and a little above the number.

Thus, $4 = 4^1$, the *first* power of 4.

$4 \times 4 = 4^2 =$ 16, the *second* power of 4, or the square of 4.

$4 \times 4 \times 4 = 4^3 =$ 64, the *third* power of 4, or the cube of 4.

$4 \times 4 \times 4 \times 4 = 4^4 =$ 256, the *fourth* power of 4, etc.

ORAL EXERCISES.

1. Square 2, 3, 4, 5, 6, 7, 8, 9, 10.
2. Cube 2, 3, 4, 5, 6, 7, 8, 9, 10.
3. Square 20, 25, 30, 40, 50, 60, 70.
4. Cube 20, 30, 40, 50, 60, 70, 80.
5. Find the value of 4^2, 4^3, 4^4, 5^2, 5^3, 8^3.
6. Square .2, .3, .4, .6, .8, .9, .03, .05.
7. Cube .2, .3, .4, .5, .6, .7, .8, .9.

8. Find the value of $(\frac{2}{3})^3$.

We have $(\frac{2}{3})^3 = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27}$.

Hence *to raise a fraction to any power, raise both numerator and denominator to the required power.*

9. Find the value of $(\frac{3}{4})^2$, $(\frac{5}{8})^3$, $(\frac{1}{4})^2$, $(\frac{2}{5})^3$, $(\frac{7}{8})^2$.
10. Find the value of $(.4)^2$, $(\frac{2}{9})^3$, $(\frac{1}{6})^3$, $(\frac{10}{12})^2$, $(\frac{5}{9})^3$.
11. Find the value of $5^2 \times 5^3$.

We have $5^2 \times 5^3 = 5 \times 5$ times $5 \times 5 \times 5 = 5^5$.

Hence *the product of any two powers of a number equals a power of the number denoted by the sum of the exponents.*

What power is—

12. $8^2 \times 8^3$?
13. $7^3 \times 7^2$?
14. $9^4 \times 9^5$?
15. $3^5 \times 3^6$?
16. $(\frac{2}{3})^4 \times (\frac{2}{3})^5$?
17. $4^2 \times 4^3 \times 4^5$?
18. $2^2 \times 2^3 \times 2^4$?
19. $5^3 \times 5^3 \times 5^3$?
20. $6^{10} \times 6^{12}$?
21. $12^2 \times 12^3$?
22. 18×18^6?
23. $20^3 \times 20^8$?

24. What power is $(4^2)^3$?

We have $(4^2)^3 = 4^2 \times 4^2 \times 4^2 = 4^6$.

Hence *a power of a number raised to any power equals a power of the number denoted by the product of the exponents.*

What power is—

25. $(2^2)^2$?
26. $(3^3)^3$?
27. $(4^2)^3$?
28. $(5^3)^2$?
29. $(5^2)^5$?
30. $(7^3)^4$?
31. $(6^3)^5$?
32. $(8^3)^4$?
33. $(12^3)^4$?
34. $(15^4)^4$?
35. $(20^2)^4$?
36. $(30^6)^3$?

NOTE.—By means of this principle we may abbreviate the process of involution. Thus,

We may raise 5 to the 6th power by squaring its cube, or to the 9th power by cubing its cube, etc.

$$5^6 = (5^3)^2 = (125)^2 = 15625.$$
$$5^9 = (5^3)^3 = (125)^3 = 1953125.$$

WRITTEN EXERCISES.

Find the value of—

1. 15^2.
2. 45^2.
3. 83^2.
4. 99^2.
5. 305^2.
6. 12^3.
7. 25^3.
8. 85^3.
9. 999^2.
10. 99^3.
11. 67^3.
12. 2.5^3.
13. 65^2.
14. 34^3.
15. $\left(\frac{15}{22}\right)^3$.
16. $\left(2\frac{1}{2}\right)^2$.
17. $\left(6\frac{2}{3}\right)^2$.
18. $\left(3\frac{1}{3}\right)^3$.
19. $.99^2$.
20. $.09^3$.
21. $.15^3$.
22. $.001^3$.
23. $.999^3$.
24. $.001^2$.
25. $.999^2$.
26. 75^2.
27. 125^2.
28. $.0001^3$.
29. 218^2.
30. 362^2.
31. 6.05^2.
32. 42.1^2.
33. 2.37^2.
34. 1001^2.
35. 10.01^2.
36. 1.001^2.

SQUARING NUMBERS GEOMETRICALLY.

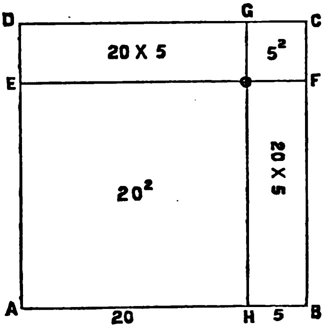
37. Square 25.

25 equals 20 + 5.

Let the line A H be 20 units long, and H B 5 units long.

Construct a square on A H and one on A B, as indicated in the figure. Then A B C D will represent the square of 25, and will consist of

$$20^2 + 2 \times (20 \times 5) + 5^2 = \\ 400 + 200 + 25 = 625.$$



That is, *the square of any number composed of two figures is equal to the square of the tens, plus twice the product of the tens into the units, plus the square of the units.*

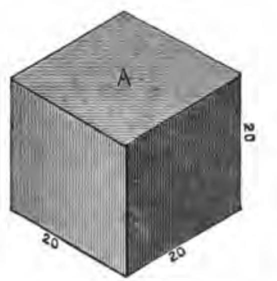
Draw a figure and square the following numbers:

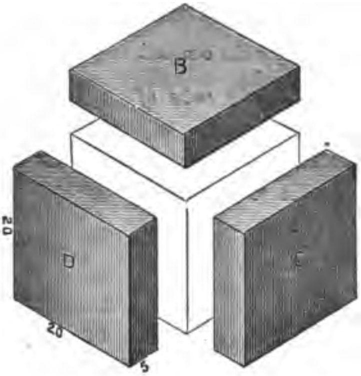
38. 32.
39. 45.
40. 48.
41. 53.
42. 63.
43. 72.
44. 84.
45. 92.
46. 123.
47. 245.
48. 357.
49. 582.

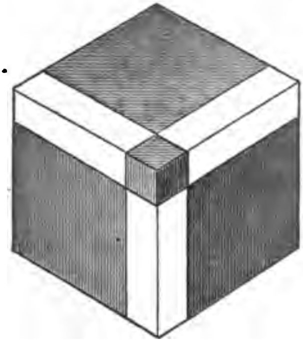
CUBING NUMBERS GEOMETRICALLY.

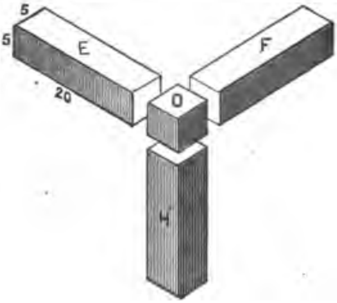
50. Cube 25.

25 equals 20 + 5.









Process.

$$20^3 = 8000$$
$$3 \times 20^2 \times 5 = 6000$$
$$3 \times 20 \times 5^2 = 1500$$
$$5^3 = 125$$
$$25^3 = 15625$$

Let A represent a cube 20 units on a side. Its contents are 20^3.

Let D represent an addition to one side of the cube. Its contents are $20 \times 20 \times 5$, or $20^2 \times 5$. It will be seen that there are three of these blocks, whose contents are $3 \times 20^2 \times 5$.

Let E represent a corner piece which is $20 \times 5 \times 5$, or 20×5^2. It is seen that there are three of these, whose contents are $3 \times 20 \times 5^2$.

Finally, a small cube, O, which is 5 units on a side and whose contents are 5^3, will complete the new cube.

We shall then have

$$20^3 + 3(20^2 \times 5) + 3(20 \times 5^2) + 5^3,$$

or 15625 for the contents of a cube whose side is 25.

Hence, *the cube of any number consisting of two figures is equal to the cube of the tens, plus three times the tens squared into the units, plus three times the tens into the units squared, plus the units cubed.*

Cube the following numbers geometrically:

51. 12.
52. 15.
53. 18.
54. 24.
55. 35.
56. 43.
57. 54.
58. 67.
59. 72.
60. 84.
61. 92.
62. 95.

EVOLUTION.

214. Evolution is the process of finding a root of a number.

215. A **Root** of a number is *one* of the *equal* factors of that number.

A *Root* is indicated by the symbol $\sqrt{}$, called the **Radical Sign.**

Thus,

$25 = 5 \times 5$, hence $\sqrt{25} = 5$, the *square* root of 25.
$64 = 4 \times 4 \times 4$, hence $\sqrt[3]{64} = 4$, the *cube* root of 64.
$81 = 3 \times 3 \times 3 \times 3$, hence $\sqrt[4]{81} = 3$, the *fourth* root of 81.

216. The **Index** of the root is the number written in the angle of the radical sign to indicate what root of the number is required.

217. A Perfect Power is a number of which the required root can be found exactly.

Thus, 25 is a *perfect square,* since $5^2 = 25$.

And 27 is a *perfect cube,* since $3^3 = 27$.

ORAL EXERCISES.

The following table should be *committed to memory.*

Square Root.		Cube Root.	
$\sqrt{4} = 2$	$\sqrt{36} = 6$	$\sqrt[3]{8} = 2$	$\sqrt[3]{216} = 6$
$\sqrt{9} = 3$	$\sqrt{49} = 7$	$\sqrt[3]{27} = 3$	$\sqrt[3]{343} = 7$
$\sqrt{16} = 4$	$\sqrt{64} = 8$	$\sqrt[3]{64} = 4$	$\sqrt[3]{512} = 8$
$\sqrt{25} = 5$	$\sqrt{81} = 9$	$\sqrt[3]{125} = 5$	$\sqrt[3]{729} = 9$

Find the value—

1. Of $\sqrt{100}$.
2. Of $\sqrt{121}$.
3. Of $\sqrt{169}$.
4. Of $\sqrt{144}$.
5. Of $\sqrt[3]{1000}$.
6. Of $\sqrt{.09}$.
7. Of $\sqrt{.16}$.
8. Of $\sqrt{.25}$.
9. Of $\sqrt[3]{8000}$.
10. Of $\sqrt{.36}$.
11. Of $\sqrt[3]{27000}$.
12. Of $\sqrt{.49}$.
13. Of $\sqrt{400}$.
14. Of $\sqrt[3]{64000}$.
15. Of $\sqrt[3]{.008}$.
16. Of $\sqrt{4900}$.
17. Of $\sqrt[3]{.027}$.
18. Of $\sqrt[3]{512000}$.
19. Of $\sqrt{900}$.
20. Of $\sqrt[3]{729000}$.
21. Of $\sqrt[3]{.343}$.
22. Of $\sqrt{.81}$.
23. Of $\sqrt[3]{.064}$.
24. Of $\sqrt{.64}$.
25. Of $\sqrt[3]{.125}$.
26. Of $\sqrt[3]{343000}$.
27. Of $\sqrt{8100}$.
28. Of $\sqrt[3]{.512}$.
29. Of $\sqrt{6400}$.
30. Of $\sqrt[3]{.729}$.
31. Of $\sqrt[3]{.216}$.
32. Of $\sqrt{1600}$.
33. Of $\sqrt[3]{125000}$.
34. Of $\sqrt{2500}$.
35. Of $\sqrt[3]{216000}$.
36. Of $\sqrt{3600}$.

37. Find the value of $\sqrt[3]{\frac{125}{512}}$.

PROCESS.

$$\sqrt[3]{\frac{125}{512}} = \frac{\sqrt[3]{125}}{\sqrt[3]{512}} = \frac{5}{8}, \textit{ Ans.}$$

Find the value—

38. Of $\sqrt{\frac{9}{25}}$.
39. Of $\sqrt{\frac{25}{36}}$.
40. Of $\sqrt{\frac{16}{49}}$.
41. Of $\sqrt{\frac{25}{64}}$.
42. Of $\sqrt[3]{\frac{343}{729}}$.
43. Of $\sqrt{\frac{100}{169}}$.
44. Of $\sqrt[3]{\frac{27}{512}}$.
45. Of $\sqrt{\frac{121}{144}}$.
46. Of $\sqrt[3]{\frac{64}{343}}$.
47. Of $\sqrt{\frac{81}{100}}$.
48. Of $\sqrt[3]{\frac{512}{729}}$.
49. Of $\sqrt{\frac{64}{81}}$.
50. Of $\sqrt{12\frac{1}{4}}$.
51. Of $\sqrt[3]{4\frac{17}{27}}$.
52. Of $\sqrt{6\frac{1}{4}}$.

EVOLUTION BY FACTORING.

218. When the number is a perfect power and the factors are readily found, the root may be obtained as follows:

WRITTEN EXERCISES.

1. Find the square root of 3025, and the cube root of 3375.

PROCESS.

$$\begin{array}{r|l} 5 & 3025 \\ 5 & 605 \\ 11 & 121 \\ & 11 \end{array} \qquad\qquad \begin{array}{r|l} 3 & 3375 \\ 3 & 1125 \\ 3 & 375 \\ 5 & 125 \\ 5 & 25 \\ & 5 \end{array}$$

Hence $\sqrt{3025} = 5 \times 11 = 55$, *Ans.*

Hence $\sqrt[3]{3375} = 3 \times 5 = 15$, *Ans.*

RULE.

Resolve the number into its prime factors, and take the product of one *of every* two *equal factors for the square root; the product of* one *of every* three *equal factors for the cube root, etc.*

Find the—

2. $\sqrt{441}$.
3. $\sqrt[3]{2744}$.
4. $\sqrt{1225}$.
5. $\sqrt[3]{9261}$.
6. $\sqrt{6561}$.
7. $\sqrt[3]{970299}$.
8. $\sqrt[4]{50625}$.
9. $\sqrt[5]{7776}$.
10. $\sqrt[3]{1728}$.
11. $\sqrt[3]{15625}$.
12. $\sqrt[4]{2401}$.
13. $\sqrt[5]{59049}$.
14. $\sqrt[6]{46656}$.
15. $\sqrt[4]{65536}$.
16. $\sqrt[6]{262144}$.

SQUARE ROOT.

219. By involution—

$1^2 = 1.$ $10^2 = 100.$ $100^2 = 10000.$

$9^2 = 81.$ $99^2 = 9801.$ $999^2 = 998001.$

The square of 1, the smallest number of *one* figure, is 1; the square of 9, the largest number of *one* figure, is 81; hence the square of a number consisting of *one* figure must contain *one* figure or *two* figures.

The square of 10, the smallest number of *two* figures, is 100; the square of 99, the largest number of *two* figures, is 9801; hence the square of a number of *two* figures must contain *three* figures or *four* figures—that is, *twice two,* or *twice two less one.*

The same may be shown for the square of a number consisting of any number of figures. Hence the

220. Principle.—*The square of a number contains twice as many figures as the number itself, or twice as many less one.*

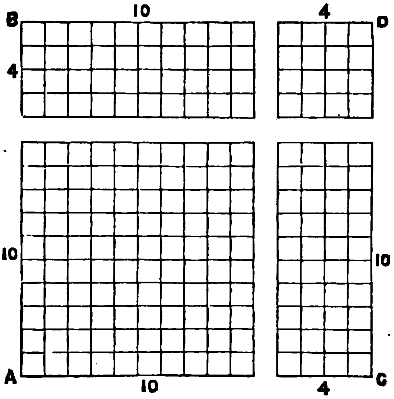
WRITTEN EXERCISES.

1. Find the square root of 196.

Process.

$$
\begin{array}{rr|l}
 & & 10 \\
 & 1'96 & \underline{4} \\
 & 10^2 = 1\ 00 & 14,\ \textit{Ans.} \\
\hline
2 \times 10 = 20 & 96 & \\
\underline{4} & & \\
24 & \underline{96} & \\
\end{array}
$$

Illustration.



Pointing the number off into periods of two figures each, by ART. 219 we find that there are two figures in its square root; hence the root will consist of tens and units.

The greatest number of tens whose square is contained in 196 is 1 ten, or 10 units. This is shown in the illustration by the square A, whose sides are 10 units and whose area is 10^2, or 100 square units. Subtracting 100 from 196, the remainder is 96. This we see consists principally of two rectangles B and C, each of which is 10 units long. Since they nearly complete the square, their area is nearly 96 square units. If we divide 96 by their length, we shall find their width. The length of both rectangles is $2 \times 10 = 20$. Dividing 96 by 20, we find the width to be 4 units. Adding the length of the small square D, whose sides are 4 units, the entire length of the remaining surface is 24 units. Multiplying this by the width, 4, the area is $24 \times 4 = 96$ square units. Subtracting, nothing remains. Hence the square root of 196 is 14.

In practice we omit the ciphers and condense the process, as in the next example.

2. Find the square root of 8464.

FULL FORM.

$$\begin{array}{lr|ll}
 & & 84'64 & \underline{|\,90 + 2 = 92} \\
 & 90^2 = & \underline{81\ 00} & \\
\text{Trial divisor, } 2 \times 90 = & 180 & 3\ 64 & \\
 & \underline{2} & & \\
\text{Complete divisor,} & \underline{182} & \underline{3\ 64} &
\end{array}$$

IN PRACTICE.

$$\begin{array}{r|l}
 & 84'64\ \underline{|\,92}\ \textit{Ans.} \\
 & 81 \\ \hline
182 & 364 \\
 & \underline{364}
\end{array}$$

From this example we derive the following

RULE.

Begin at the unit's place and separate the number into periods of two figures each.

Find the largest number whose square is contained in the left-hand period; write it as the first figure of the root, subtract its square from the left-hand period, and annex the next period to the remainder.

Double the root found, for a trial divisor; divide the remainder, omitting the last figure, by this divisor, and annex the quotient to the root, and also to the trial divisor.

Multiply the complete divisor by the second term of the root, and subtract the product from the remainder.

If other periods remain, double the root already found, for the next trial divisor, and proceed as before.

When a cipher occurs in the root, annex 0 to the trial divisor and bring down the next period (see Ex. 3).

3. Find the square root of 254016.

PROCESS.

$$\begin{array}{r|l} & 25'40'16\ \underline{|\ 504} \\ & 25 \\ \hline 1004 & 4016 \\ & \underline{4016} \end{array}$$

The second trial divisor is 10, and the second remainder is 40. Since 10 is not contained in 4, we place a 0 in the root and annex a 0 to the trial divisor, and bring down the next period.

Find the square root of—

4. 256.
5. 441.
6. 625.
7. 2025.
8. 2704.
9. 4489.
10. 7056.
11. 8836.
12. 15129.
13. 12321.
14. 12544.
15. 41616.
16. 104976.
17. 262144.
18. 368449.
19. 767376.
20. 103041.
21. 180625.
22. 315844.
23. 944784.
24. 277729.
25. 519841.
26. 998001.
27. 1104601.
28. 4016016.
29. 5499025.
30. 6270016.

SQUARE ROOT OF DECIMALS.

221. By involution—

$.1^2 = .01.$ $.01^2 = .0001.$ $.001^2 = .000001.$
$.9^2 = .81.$ $.99^2 = .9801.$ $.999^2 = .998001.$

We thus see *that the square of a decimal contains twice as many decimal places as the decimal itself;* hence *to extract the square root of a decimal, we point off the decimal into periods of two figures each, beginning at the decimal point.*

31. Find the square root of .6889, and of 33.4084.

PROCESS.

$$\begin{array}{r|l} & .68'89\;\underline{|.83},\ \textit{Ans.} \\ & 64 \\ \hline 163 & 489 \\ & 489 \\ \hline \end{array} \qquad \begin{array}{r|l} & 33.40'84\;\underline{|5.78},\ \textit{Ans.} \\ & 25 \\ \hline 107 & 840 \\ & 749 \\ \hline 1148 & 9184 \\ & 9184 \\ \hline \end{array}$$

Find the square root of—

32. .0289.
33. .4225.
34. .6561.
35. .8464.
36. 21.16.
37. 547.56.
38. .003969.
39. 1.679616.
40. 204.7761.
41. .00009801.
42. .000001.
43. 1.0201.

SQUARE ROOT OF IMPERFECT SQUARES.

222. When a number is an imperfect square, periods of ciphers may be annexed and the process continued as far as desired.

44. Find the square root of 14 to three places of decimals.

PROCESS.

$$\begin{array}{r|l} & 14.00'00'00\;\underline{|3.741+},\ \textit{Ans.} \\ & 9 \\ \hline 67 & 500 \\ & 469 \\ \hline 744 & 3100 \\ & 2976 \\ \hline 7481 & 12400 \\ & 7481 \\ \hline \end{array}$$

Find the square root of the following numbers to four places of decimals:

45. 2.
46. 3.
47. 5.
48. 6.
49. 7.
50. 8.

SQUARE ROOT OF FRACTIONS.

223.—*51.* Find the square root of $\frac{3}{4}$.

$$\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{\sqrt{4}} = \frac{\sqrt{3}}{2} = \frac{1.732+}{2} = .866+, \textit{ Ans.}$$

If the denominator is a perfect square and the numerator is not, divide the approximate square root of the numerator by the square root of the denominator.

52. Find the square root of $\frac{2}{3}$.

$$\sqrt{\frac{2}{3}} = \sqrt{\frac{6}{9}} = \frac{\sqrt{6}}{\sqrt{9}} = \frac{2.449+}{3} = .816+, \textit{ Ans.}$$

If the denominator is not a perfect square, reduce the fraction to an equivalent fraction having a perfect square for a denominator.

Find the square root of the following fractions to three places of decimals:

53. $\sqrt{\frac{1225}{4489}}$.

54. $\sqrt{\frac{289}{2704}}$.

55. $\sqrt{\frac{11}{16}}$.

56. $\sqrt{\frac{12}{25}}$.

57. $\sqrt{\frac{3}{8}}$.

58. $\sqrt{\frac{5}{12}}$.

59. $\sqrt{.4}$.

60. $\sqrt{.9}$.

APPLICATIONS OF SQUARE ROOT.

224. The Square.

The side of a square is equal to the square root of its area.

WRITTEN EXERCISES.

1. A square farm contains 40 acres; how long is it?

2. What will it cost to fence a square field containing 10 acres at $1.20 a rod?

3. It requires 1250 bricks, each 8 in. long and 4 in. wide, to cover a square pavement; how long is it?

4. Find the length of a square court which requires 6889 marble blocks, each 9 inches square, to cover the floor.

5. Find the dimensions of a rectangular field which is twice as long as it is wide, and contains 80 acres.

6. If it costs $144 to put a wire fence around a field 96 rods long and 24 rods wide, how much less will it cost to fence a square field of equal area?

7. A general drew up his army of 4325 men in the form of a square, and had 100 men over; required the number of men on one side of the square.

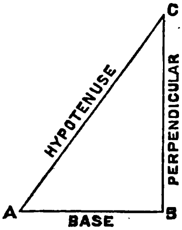
8. A general drew up his army in the form of a square, and found he lacked 209 men to complete the square: he then diminished the side of the square by 3 men, and found he had 1000 men remaining; how many men in the army?

9. A square lake which contains 10 acres has a gravel walk 8 ft. wide around it; how many square yards in the walk?

225. The Right Triangle.

A **Right Triangle** is a triangle which has one right angle.

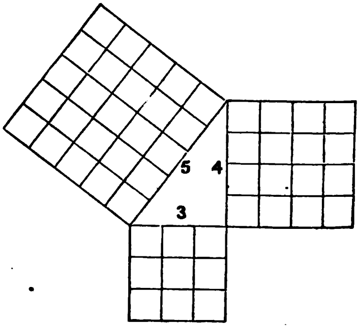
226. The **Hypotenuse** is the side opposite the right angle. Either of the two sides forming the right angle may be taken as the **Base**, and the other side as the **Perpendicular.**



It is proved in Geometry that—

In any right triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Hence *the square of either side equals the square of the hypotenuse diminished by the square of the other side.*



WRITTEN EXERCISES.

1. The base of a right triangle is 24 inches, and the perpendicular is 18 inches; required the hypotenuse.

We have $\sqrt{18^2 + 24^2} = \sqrt{324 + 576} = \sqrt{900} = 30$.

Hence the hypotenuse is 30 in., *Ans.*

2. The hypotenuse of a right triangle is 75 ft., and the base is 60 feet; required the perpendicular.

We have $\sqrt{75^2 - 60^2} = \sqrt{5625 - 3600} = \sqrt{2025} = 45$.

Hence the perpendicular is 45 ft., *Ans.*

3. The hypotenuse of a right triangle is 50 ft., and the perpendicular is 30 ft.; required the base.

4. The base of a right triangle is 120 inches, and the hypotenuse is 40 ft.; required the perpendicular.

5. Find the distance between the opposite corners of a field 45 rods long and 20 rods wide.

6. A square farm contains 90 acres; required the distance between the opposite corners.

7. B, standing 36 ft. from a tree, shoots a bird from its top; how high is the bird from the ground, if the ball travels 60 ft. and starts 6 ft. from the ground?

8. A ladder 52 ft. long stands against a building; how far must it be drawn out at the bottom, that the top may be lowered 8 ft.?

9. A room is 40 ft. long, 25 ft. wide, and 12 ft. high; how far is it from an upper corner to the lower opposite corner?

10. The diagonal of a square is 25 feet; find the approximate value of its side to three places of decimals.

11. A tree was broken off 24 ft. above the ground, and fell so that its top struck 32 feet from the foot of the tree, the end resting on the stump; what was the height of the tree?

12. The area of a square is 6 A. 64 P.; find the length of the diagonal.

227. Similar Figures.

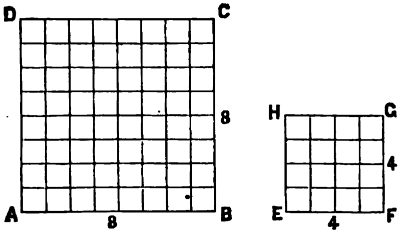
Similar Figures are those which have the same *form*, but differ in *area*.

It is proved in Geometry that—

1. *The areas of similar figures are to each other as the squares of their like dimensions.*

2. *The like dimensions of similar figures are to each other as the square roots of their areas.*

These principles may be illustrated thus:



Area A B C D : area E F G H = length AB^2 : length EF^2.

Or, 64 : 16 = 8^2 : 4^2.

Or, $\sqrt{\text{Area ABCD}} : \sqrt{\text{area EFGH}}$ = length A B : length E F.

$\sqrt{64} : \sqrt{16} = 8 : 4.$

WRITTEN EXERCISES.

1. The area of a triangle whose base is 32 in. is 768 sq. in.; what is the base of a similar triangle whose area is 48 sq. in.?

Process.

$768 : 48 = 32^2 : x^2$

$$x^2 = \frac{48 \times 1024}{768} = 64$$

$x = 8$

Since the triangles are similar, their areas are to each other as the squares of their bases; hence we have the proportion $768 : 48 = 32^2 : x^2$.

Solving the proportion, $x = 8$.

2. The length of a rectangular field containing 12 acres is 48 rods; what is the length of a similar field containing 48 acres?

3. There are two circular lakes: one is 4 miles in diameter and the other 20 miles; the second is how many times as large as the first?

4. A has a circular garden 11 feet in diameter, and wishes to lay out another 16 times as large; what must be the diameter of the latter?

5. B has a field 60 rods wide and 72 rods long, which contains 27 acres; required the dimensions of a similar field which contains $18\frac{3}{4}$ acres.

6. If it costs $60 to paint a house 24 ft. long and 20 ft. wide, what will it cost to paint with the same kind of color a house whose dimensions are three times as large?

7. If a horse tied to a stake by a rope 10.09 rd. long can graze upon 2 acres of land, how long must the rope be that he may graze upon $12\frac{1}{2}$ acres?

CUBE ROOT.

228. By involution—

$$\begin{aligned} 1^3 &= 1, & 10^3 &= 1000, \\ 3^3 &= 27, & 30^3 &= 27000, \\ 9^3 &= 729, & 99^3 &= 970299, \text{ etc.} \end{aligned}$$

The cube of 1, the smallest number of *one* figure, is 1; the cube of 3 is 27; the cube of 9, the largest number of *one* figure, is 729; hence the cube of a number consisting of *one* figure contains *one* figure, *two* figures, or *three* figures.

The cube of 10, the smallest number consisting of *two* figures, is 1000; the cube of 30 is 27000; the cube of 99, the largest number of *two* figures, is 970299; hence the cube of a number of *two* figures contains *four*, *five*, or *six* figures; that is, *three times two*, or *three times two less one* or *less two.*

The same may be shown for the cube of a number consisting of any number of figures. Hence the

Principle.—*The cube of a number contains three times as many figures as the number itself, or three times as many less one, or three times as many less two.*

WRITTEN EXERCISES.

1. Find the cube root of 15625.

FIG. 1.

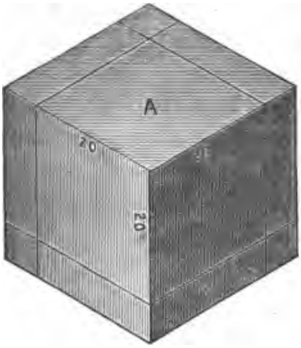
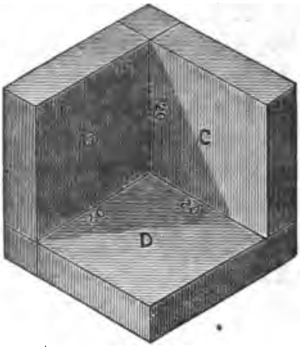


FIG. 2.



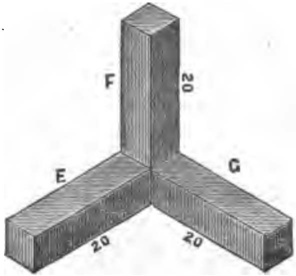


FIG. 3.

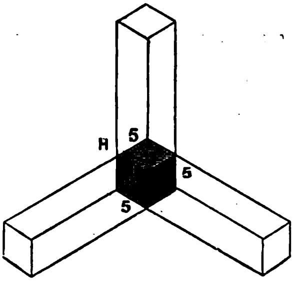


FIG. 4.

Process.

	15′625	20
$20^3 =$	8000	5
$3 \times 20^2 = 1200$	7625	
$3 \times 20 \times 5 = 300$		
$5^2 = 25$		
1525	7625	

Pointing the number off into periods of three figures each, by Art. 220 we find that there are two figures in its cube root; and hence the root will consist of tens and units.

The greatest number of tens whose cube is contained in 15625 is 2 tens, or 20 units. This is shown in Fig. 1 by the cube A, whose sides are 20 units and whose volume is 20^3, or 8000. Subtracting 8000 from 15625, the remainder is 7625. This we see by Fig. 2 to consist principally of three equal square solids, B, C, and D, each of which is 20 units square. Since they nearly complete the cube, their volume is nearly 7625. If we divide 7625 by their surface, we shall find their thickness. The surface of each square solid is 20^2, and that of the three solids is $3 \times 20^2 = 1200$ square units. Dividing 7625 by 1200, we have a quotient of 5 units; hence the thickness of these square solids is 5 units.

Removing these three square solids, there are three rectangular solids remaining, E, F, G (Fig. 3), each being 20 units long and 5 units thick; hence the surface of a face is $20 \times 5 = 100$ square units, and that of the three is 300 square units.

There remains a small cube, H (Fig. 4), whose sides are 5 units, and the surface of one face of which is 5^2, or 25 square units.

Taking the sum of these three surfaces, we have 1525 square units, and multiplying this by the thickness, 5, we have 7625 cubic units; subtracting, there is no remainder. Hence the cube root of 15625 is 20 + 5, or 25.

Omitting the ciphers, the process will stand as follows:

2. Find the cube root of 778688.

Process.

	778′688	92, *Ans.*
	729	
Trial divisor, $3 \times 9^2 = 243$	49688	
$3 \times 9 \times 2 = 54$		
$2^2 = 4$		
Complete divisor, 24844	49688	

From this example we derive the following

RULE.

Begin at the unit's place and separate the number into periods of three figures each.

Find the largest number whose cube is contained in the left-hand period; write it as the first figure of the root; subtract its cube from the left-hand period, and annex the next period to the remainder.

Take three times the square of the root found, for a **Trial Divisor**; *divide the remainder, omitting the last two figures, by this divisor, and annex the quotient to the root.*

Add to the trial divisor three times the product of the first and second terms of the root, **written one figure to the right,** *and also the square of the second term of the root,* **written one figure to the right**; *their sum will be the* **True Divisor.**

Multiply the complete divisor by the second term of the root, and subtract the product from the remainder.

If other periods remain, take three times the square of the root already found for the next trial divisor, and proceed as before.

When a cipher occurs in the root, annex two ciphers to the trial divisor and bring down the next period.

3. Find the cube root of 41063625.

PROCESS.

```
       | 41'063'625 | 345, Ans.
       | 27
-------|------
27     | 14063
 36    |
  16   |
-------|
3076   | 12304
-------|---------
3468   |  1759625
 510   |
   25  |
-------|
351925 |  1759625
```

Find the cube root of—

4. 32768.
5. 79507.
6. 157464.
7. 274625.
8. 438976.
9. 614125.
10. 804357.
11. 1860867.
12. 12326391.
13. 130323843.
14. 34328125.
15. 145531576.
16. 264609288.
17. 354894912.
18. 1879080904.

SHORT METHOD OF CUBE ROOT.

229. The following **Short Method** of cube root is the shortest and most convenient method yet used. The abbreviation consists in obtaining the successive trial divisors by using the previous work. It is readily explained either by the blocks or by the algebraic formula.

19. Find the cube root of 48228544.

PROCESS.

	48′228′544	364, *Ans.*
	27	
27, T. D.	21228	
54, 1st C.		
36, 2d C.		
3276, C. D.	19656	
36		
3888	1572544	
432		
16		
393136	1572544	

We find the number of figures in the root and the first and second figures of the root as before.

In finding the second *trial divisor*, 3888, we add the *first correction* (54), the *second correction* (36), the *complete divisor* (3276), and the *second correction* (36) **repeated**, as indicated by the bracket in the process, and proceed as before.

It is not necessary to give a full rule for this method, as it is merely a modification of the previous method. It will be easily remembered by means of the following statement:

Find the first trial divisor by the usual method.

The second trial divisor is obtained by adding to the first complete divisor the two corrections required to form it, together with the square of the second term.

This method holds good for any trial divisor, and will be found to save a great deal of labor in extracting the cube root.

20. **Find the cube root of 45156047481.**

PROCESS.

	45′156′047′481	3561, *Ans.*
	27	
27	18156	
45		
25		
3175	15875	
25		
3675	2281047	
630		
36		
373836	2243016	
36		
380208	38031481	
1068		
1		
38031481	38031481	

Find the cube root of—

21. 40001688.
22. 176558481.
23. 377933067.
24. 181321496.
25. 277167808.
26. 410172407.
27. 12895213625.
28. 160279981568.
29. 428490531819.

CUBE ROOT OF DECIMALS.

230. By involution—

$$.1^3 = .001. \qquad .01^3 = .000001.$$
$$.9^3 = .729. \qquad .99^3 = .970299.$$

We thus see *that the cube of a decimal contains three times as many decimal places as the decimal itself;* hence to extract the cube root of a decimal *we point off the decimal into periods of three figures each, beginning at the decimal point. The number of periods to the right of the decimal point shows the number of places in the decimal portion of the root.*

The method of ART. 229 may be used to find the cube root of a decimal if the decimal point is inserted in its proper place in the root.

30. Find the cube root of .614125.

Process.

	.614′125	.85, *Ans.*
	512	
192	102125	
120		
25		
20425	102125	

Find the cube root of—

31. .079507.
32. .166375.
33. .300763.
34. .636056.
35. .001953125.
36. .082881856.
37. .000046656.
38. .000970299.
39. 36.926037.

CUBE ROOT OF IMPERFECT CUBES.

231. When a number is an imperfect cube, periods of ciphers may be annexed and the process continued as far as desired.

40. Find the cube root of 5.

Process.

	5.000′000′000	1.709+, *Ans.*
	1	
3	4000	
21		
49		
559	3913	
49		
86700	87000000	
4590		
81		
8715981	78443829	
	8556171	

Find the cube root of—

41. 3.
42. 4.
43. 25.
44. .27.
45. 65.
46. 90.

CUBE ROOT OF FRACTIONS.

232.—*47.* Find the cube root of $\frac{7}{8}$.

$$\sqrt[3]{\frac{7}{8}} = \frac{\sqrt[3]{7}}{\sqrt[3]{8}} = \frac{1.912+}{2} = .956+, \textit{Ans.}$$

If the denominator is a perfect cube and the numerator is not, divide the approximate cube root of the numerator by the cube root of the denominator.

48. Find the cube root of $\frac{2}{9}$.

$$\sqrt[3]{\frac{2}{9}} = \sqrt[3]{\frac{6}{27}} = \frac{\sqrt[3]{6}}{\sqrt[3]{27}} = \frac{1.817+}{3} = .605+, \textit{Ans.}$$

If the denominator is not a perfect cube, reduce the fraction to an equivalent fraction having a perfect cube for a denominator.

Find the cube root of—

49. $\frac{1728}{15625}$.

50. $\frac{19683}{166375}$.

51. $\frac{3}{9}$.

52. $\frac{5}{4}$.

53. $\frac{2}{3}$.

54. $\frac{3}{4}$.

APPLICATIONS OF CUBE ROOT.

233. The Cube.

The **Edge** of a cube is equal to the cube root of its contents.

WRITTEN EXERCISES.

1. Find the edge of a cubical box which contains 216000 cubic inches.

2. Find the entire surface of a cubical chest whose contents are 15625 cubic inches.

3. What is the edge of a cube which contains as much as a solid 7 ft. long, 42 in. wide, and 21 in. high?

4. A miller has a cubical box with which he takes toll; what are its dimensions, if it contains $\frac{1}{10}$ of a bushel?

5. What are the dimensions of a cubical box that will hold 20 bushels?

6. What is the depth of a cubical cistern that will hold 1000 gallons of water?

7. B has a cubical bin whose contents are 175616 cubic inches; what will it cost to line the bottom and sides, at 10 cents a square foot?

234. Similar Volumes.

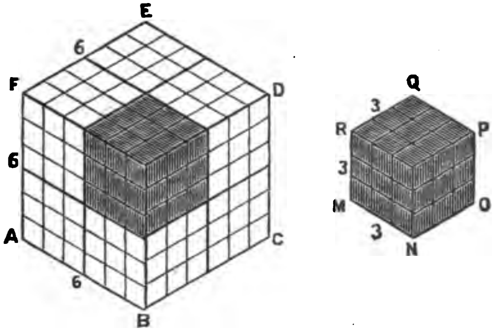
Similar Volumes are those which have the same *form*, but differ in volume.

It is proved in Geometry that—

1. *Similar volumes are to each other as the cubes of their like dimensions.*

2. *The like dimensions of similar volumes are to each other as the cube roots of their volumes.*

These principles may be illustrated thus:



$$\begin{array}{lcccl} \text{Vol. } ABCDEF & : & \text{vol. } MNOPQR & = & AB^3 : MN^3. \\ 216 & : & 27 & = & 6^3 : 3^3. \\ \sqrt[3]{\text{Vol. } ABCDEF} & : & \sqrt[3]{\text{vol. } MNOPQR} & = & AB : MN. \\ \sqrt[3]{216} & : & \sqrt[3]{27} & = & 6 : 3. \end{array}$$

WRITTEN EXERCISES.

1. If a ball 4 in. in diameter weighs 10 lb., what does a ball 12 in. in diameter weigh?

PROCESS.

$4^3 : 12^3 = 10 : (?)$.

$$\frac{1728 \times 10}{64} = 270 \text{ lb.}$$

Since the cubes of the like dimensions of similar solids are to each other as their volumes, then 4^3, the diameter of the first cubed, is to 12^3, the diameter of the second cubed, as 10 lb., the weight of the first, is to the weight of the second. From which we find the weight of the larger ball to be 270 lb.

2. If a ball 5 in. in diameter weighs 24 lb., what is the diameter of a similar ball which weighs 81 lb.?

3. If a stack of hay 8 ft. in diameter weighs 15 tons, what is the diameter of a similar stack which weighs 120 tons?

4. If a globe of gold 2 inches in diameter is worth $500, what is the value of a globe 6 inches in diameter?

5. The diameters of two similar cisterns are to each other as 1 to 8; what is the relation of their contents?

6. If a bin 10 ft. long holds 50 bushels, how long is a similar bin which holds 400 bushels?

7. If a bin 15 ft. long and 10 ft. wide contains 800 cubic feet, what are the dimensions of a similar bin which contains 2700 cubic feet?

8. If a bar of metal 3 ft. long, 2 ft. wide, and 1 ft. thick weighs 2700 lb., what are the dimensions of a similar bar which weighs 6400 lb.?

9. If the surfaces of two spheres are to each other as 4 to 9, what is the relation of their contents?

10. If the volumes of two spheres are to each other as 250 to 128, what is the relation of their surfaces?

11. If a log 3 ft. in diameter contains 70 cubic feet, what is the diameter of a log of the same length that contains 210 cubic feet?

SECTION XI.

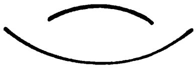
MENSURATION.

235. **Mensuration** is the process of computing the lengths of lines, the areas of surfaces, and the volumes of solids.

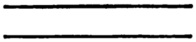
LINES.

236. A **Line** is that which has length only.

Straight Line.



Curved Lines.



Parallel Lines.

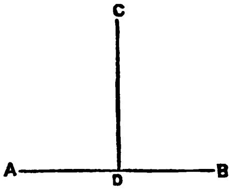
A *Straight Line* is a line that does not change its direction.

A *Curved Line* is a line that changes its direction at every point.

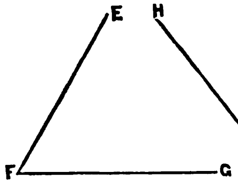
Parallel Lines are lines which have the same direction.

ANGLES.

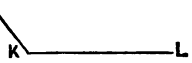
237. An **Angle** is the figure formed by two straight lines which meet at a point; as, CDA, CDB, EFG, HKL.



Right Angles.



Acute Angle.



Obtuse Angle.

When one straight line meets another, making the adjacent angles equal, the lines are *perpendicular* to each other, and the angles are *Right Angles.*

An *Acute Angle* is an angle less than a right angle. An *Obtuse Angle* is an angle greater than a right angle.

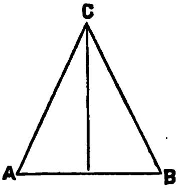
SURFACES.

238. A **Surface** is that which has length and breadth only.

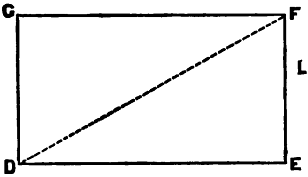
A **Plane Surface,** or a **Plane,** is a surface in which, if any two points are taken, the straight line which joins these points will lie wholly in the surface.

239. A **Plane Figure** is a plane surface bounded by lines either straight or curved.

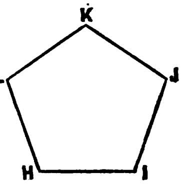
240. A **Polygon** is a plane figure bounded by straight lines.



Triangle.



Quadrilateral.



Pentagon.

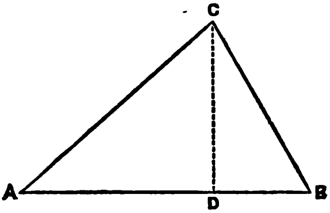
241. A *Polygon* of three sides is called a *Triangle;* one of four sides, a *Quadrilateral;* one of five sides, a *Pentagon;* one of six sides, a *Hexagon,* etc.

The lines AB, BC, CA; DE, EF, FG, GD; and HI, IJ, JK, KL, LH, of the above polygons are called the *sides,* and in any polygon the sum of the sides is called the *perimeter.*

The *diagonal* of a polygon is a line joining any two vertices not consecutive.

The *area* of a plane figure is the number of square units it contains.

242. A **Triangle** is a polygon of three sides; as, A B C.

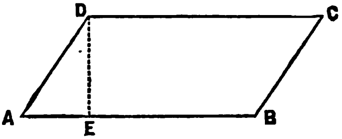


Triangle.

The *base* of a triangle is the side upon which it seems to stand; as, A B. The point opposite the base is called the *vertex;* as, C.

The *altitude* of a triangle is the perpendicular drawn from the vertex to the base; as, C D.

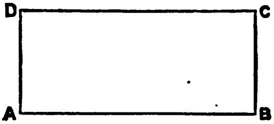
243. A **Parallelogram** is a quadrilateral whose opposite sides are parallel; as, A B C D.



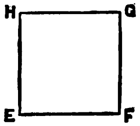
Parallelogram.

The *base* of a parallelogram is the side upon which it seems to stand; as, A B. The *altitude* is the perpendicular distance between the base and the side opposite; as, D E.

A **Rectangle** is a parallelogram whose angles are all right angles; as, A B C D.



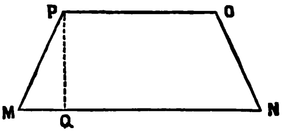
Rectangle.



Square.

A **Square** is a rectangle whose sides are all equal; as, E F G H.

A **Trapezoid** is a quadrilateral two of whose sides are parallel; as, M N O P. The two parallel sides are called the *bases* of the trapezoid; as, M N and P O. The *altitude* is the perpendicular distance between the bases; as, P Q.



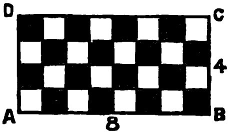
Trapezoid.

AREA OF POLYGONS.

244. The Rectangle.

Let the figure A B C D represent a rectangle 8 inches long and 4 inches wide. What is its area?

If we divide the base and altitude into inches, and draw lines through these points of division parallel to the sides, the rectangle will be divided into equal squares. There will be 8 square inches in one row; and since there are 4 such rows, there will be 4 times 8 square inches, or 32 square inches.



From this we derive the

RULE.

To find the area of a rectangle, multiply the base by the altitude.

WRITTEN EXERCISES.

1. What is the area of a rectangle 20 ft. long and 16 ft. wide?

2. What is the area of a rectangle 12 ch. long and 5 ch. wide?

3. How many square yards in a rectangle 40 ft. long and 18 ft. wide?

4. How many acres in a rectangle 40 rd. long and 12 rd. wide?

5. How many acres in a rectangle 36 chains long and 10 chains wide?

6. Find the area of a square whose sides are each 8 ft. 6 in.

7. Find the length in rods of a rectangular field whose area is 16 acres and width 40 rd.

8. A rectangular yard 40 ft. long and 30 ft. wide is surrounded by a walk 4 ft. wide; how many square feet in the walk?

9. If a rectangular field is 80 rd. long and 60 rd. wide, how much is the field worth, at $140 an acre?

10. A city lot is 5 rd. wide and 18 rd. long; what is it worth, at $1600 an acre?

11. There is a hollow square whose outer side is 120 rd. and the inner side 40 rd.; how many acres does it contain?

12. A rectangular field is 120 rd. long and 40 rd. wide; how many more acres will a square field of equal perimeter contain?

13. How many more feet in two sides of a square acre than in its diagonal?

14. There is a rectangular field whose sides are 125 rd. and 20 rd., respectively; what is the side of a square field of equal area?

245. The Parallelogram.

If the rectangle (Fig. 1) be divided by the line A E, and the triangle A D E be placed at the right of the figure, we shall have a parallelogram of the form A B D E (Fig. 2).

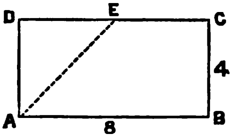


FIG. 1.

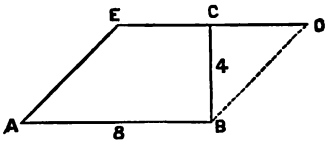


FIG. 2.

It is evident that the base, the altitude, and the area remain the same.

Hence the

RULE.

To find the area of a parallelogram, multiply the base by the altitude.

WRITTEN EXERCISES.

1. What is the area of a parallelogram whose base is 32 in. and altitude 25 in.?

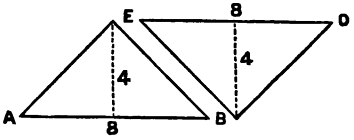
2. How many acres in a parallelogram whose base is 120 rd. and altitude 15 rd.?

3. How many acres in a farm 320 rd. long and 45 rd. wide?

4. How many acres in a parallelogram 60 ch. long and 45 ch. wide?

246. The Triangle.

If the parallelogram (Fig. 2, Art. 245) be divided by a line drawn from B to E, it will form two equal triangles. It is evident that the width and altitude of each triangle are the same as the width and altitude of the parallelogram, but the area of each triangle is only *one-half* of the area of the parallelogram. Hence the



RULE.

To find the area of a triangle, multiply the base by half the altitude.

WRITTEN EXERCISES.

1. Find the area of a triangle whose base is 120 ft. and altitude 60 ft.

2. How many acres in a triangular field whose base is 80 rd. and altitude 60 rd.?

3. How many acres in a triangular field whose base is 40 ch. and altitude 30 ch.?

4. What is the altitude of a triangle whose base is 360 ft. and area 1 acre?

Note.—In Geometry it is proved that if the three sides of a triangle are given and not the altitude, the area can be found as follows:

From half the sum of the three sides subtract each side separately; multiply the half sum and the three remainders together, and extract the square root of the product; the result will be the area required.

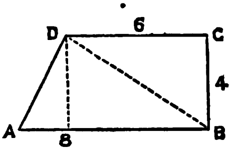
5. What is the area of a triangle whose sides are, respectively, 6, 8, and 10 ft.?

6. What is the area of a triangle whose sides are each 90 ft.?

247. The Trapezoid.

If the triangle A D E be taken from the parallelogram (Fig. 2, Art. 245), the trapezoid A B C E will remain.

The trapezoid A B C D consists of two triangles, A B D and B C D.



The area of A B D $= \frac{1}{2}$ A B $\times$ B C $= \frac{8}{2} \times 4$.

The area of B C D $= \frac{1}{2}$ D C $\times$ B C $= \frac{6}{2} \times 4$.

Adding, the area of the trapezoid $= \frac{1}{2}(8 + 6)\,4 = 28$ sq. in.

From this explanation we derive the

RULE.

To find the area of a trapezoid, multiply the sum of the bases by half the altitude.

WRITTEN EXERCISES.

1. Find the area of a trapezoid whose bases are 44 and 36 in. respectively, and altitude 20 in.

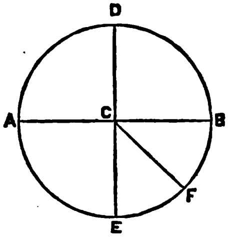
2. Find the area in acres of a trapezoid whose bases are 80 and 50 rd. respectively, and altitude 40 rd.

3. The bases of a trapezoid are 32 and 24 ch. respectively, and the altitude is 40 ch.; how many acres in its area?

4. Find the side of a square equal in area to a trapezoid whose bases are 60 and 30 in. respectively, and altitude 20 in.

248. The Circle.

A Circle is a plane figure bounded by a curved line, every point of which is equally distant from a point within called the *centre.*



The bounding line A D B E is called the *circumference.*

The *radius* is the distance from the centre to the circumference; as, C D, C F, etc.

The *diameter* is a straight line drawn through the centre and terminating in the circumference; as, A B, D E.

If we take a circle 6 inches in diameter and measure accurately the distance around it, we shall find the circumference to be about 18.8496 inches. Dividing the circumference, 18.8496 in., by the diameter, 6 in., we have a quotient of 3.1416.

Hence the

RULE.

I. *To find the circumference of a circle, multiply the diameter by* 3.1416.

II. *To find the diameter of a circle, divide the circumference by* 3.1416.

NOTE.—This quotient, 3.1416, is represented by the Greek letter π, called *pi.*

$$\text{The diameter} = \frac{\text{circumference}}{3.1416} = \text{circumference} \times \frac{1}{3.1416}.$$

$$\text{But } \frac{1}{3.1416} = .3183. \quad \text{Hence}$$

III. *To find the diameter of a circle, multiply the circumference by* .3183.

If the circumference be represented by C and the diameter by D, these rules may be briefly expressed, as follows:

$$C = \pi D; \quad D = \frac{C}{\pi};$$

$$C = 2\pi R; \quad D = C \times .3183.$$

WRITTEN EXERCISES.

Find the circumference when the—

1. Diameter is 10 in.
2. Diameter is 40 rd.
3. Diameter is 305 ch.
4. Diameter is 456 ft.

Find the diameter when the—

5. Circumference is 50 in.
6. Circumference is 100 ft.
7. Circumference is 314.16 rd.
8. Circumference is 6283.2 ch.

9. What is the circumference of the earth, its diameter being 7960 miles?

10. What is the circumference of the planet Jupiter, its diameter being 85000 miles?

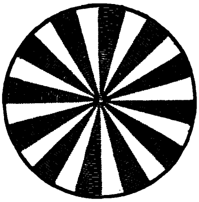
11. How often does a car-wheel revolve in an hour, running at the rate of 30 miles an hour, if the wheel is 3 ft. in diameter?

12. What is the radius of a wheel which makes 17600 revolutions in going 40 miles?

13. How much farther would a man travel in going around a circular field 100 yards in diameter than in going around a square field whose area is 5625 square yards?

14. A cart whose wheels are 5 ft. in diameter and 5 ft. apart is turned around, making a complete circle: it is observed that the inner wheel has made one revolution; what is the circumference made by the outer wheel?

15. If in the last problem the inner wheel makes 4 revolutions, what is the circumference made by the outer wheel?



249. A circle may be regarded as made up of a large number of triangles, the sum of whose bases forms the circumference of the circle, and whose altitude is the radius of the circle. Hence the

RULE.

I. *To find the area of a circle, multiply the circumference by half the radius.*

$\text{Area} = \text{Circumference} \times \frac{1}{2} \text{Radius}.$

Since $\text{Circumference} = 2 \times \text{Radius} \times \pi$, $\text{Area} = 2 \times \text{Radius} \times \pi \times \frac{1}{2} \text{Radius}$,

or, $\text{Area} = \pi \times \text{Radius}^2$, or πR^2.

Hence,

II. *To find the area of a circle, multiply the square of the radius by* 3.1416.

WRITTEN EXERCISES.

Find the area of a circle whose—

1. Radius is 10 ft.

2. Diameter is 40 rd.

3. Circumference is 18 ch.

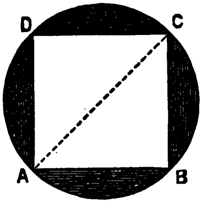
4. Radius is 3 ft. 6 in.

5. A horse is tied to a stake by a rope 18 ft. long; over how many square yards can he graze?

6. A has a garden that is 40 rods square, and B has a circular garden 40 rods in diameter; what is the difference in area?

7. F has a circular fish-pond 30 yards in diameter: he makes a circular gravel walk around it 6 ft. wide; what is the area of the walk?

8. There is a circular farm whose diameter is 30 chains; what is it worth, at $55 an acre?



250. A square is inscribed in a circle when each of its angles is in the circumference.

WRITTEN EXERCISES.

1. Find the side of the largest square that can be cut out of a circular board 16 in. in diameter.

Since A B C is a right triangle,

$$AB^2 + BC^2 = AC^2.$$

But A B = B C.

$$\text{Hence } 2\,AB^2 = AC^2, \text{ or } 256.$$

$$AB^2 = 128,$$

or, $AB = \sqrt{128} = 11.361+$ in., *Ans.*

Hence the

RULE.

To find the side of the largest square that can be cut from a circle, divide the square of the diameter by 2 and extract the square root.

2. Find the side of a square that can be cut from a circle 80 inches in diameter.

3. Find the side of a square field that can be made from a circular field 40 rd. in radius.

4. How large a square can be cut out of a circular yard 100 ft. in circumference?

5. Find the difference between the area of a circular field 60 rd. in circumference and that of the largest square field that can be cut from it.

6. The radius of a circle is 1.5 inches; what is the side of the inscribed square?

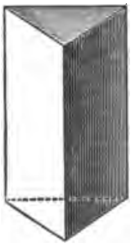
SOLIDS.

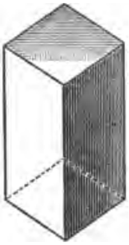
251. A **Solid** is that which has length, breadth, and thickness.

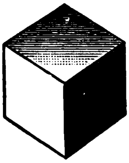
The **Volume** of a solid is the number of times that it contains another solid regarded as the unit of measure.

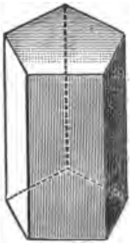
252. The Prism.

A **Prism** is a solid whose ends are equal polygons and whose sides are parallelograms.


Triangular Prism.


Square Prism.


Cube.


Pentangular Prism.

The *bases* are the equal polygons, and the parallelograms form the *convex surface.*

Prisms are named from their bases.

A **Rectangular Parallelopiped,** or **Square Prism,** is a prism whose six faces are all rectangles.

A **Cube** is a rectangular parallelopiped whose six faces are all squares.

A **Right Prism** is a prism whose edges are perpendicular to its bases: the prisms in the cut above are right prisms.

RULE.

The convex surface of a right prism is equal to the perimeter of the base multiplied by the altitude.

WRITTEN EXERCISES.

1. Find the convex surface of a triangular prism the three sides of whose base are 6, 8, and 10 in., respectively, and altitude 40 in.

2. Find the convex surface of a square prism the sides of whose base are each 10 in. and altitude 50 in.

3. Find the convex surface of a pentangular prism, each of the sides of the base being 8 ft. and the altitude 12 ft.

4. Find the surface of a cube whose edge is 6 inches.

5. Find the entire surface of a triangular prism the sides of whose base are 8, 10, and 12 in., respectively, and altitude 20 in.

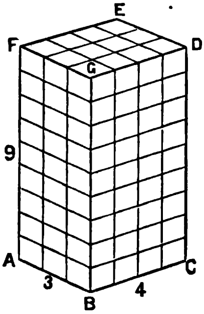
NOTE.—To find the entire surface, add the area of the bases.

6. Find the entire surface of a triangular prism the sides of whose base are each 20 ft. and altitude 40 ft.

253. Let the figure A B C D E F represent a rectangular parallelopiped, or square prism, whose length is 4 in., width 3 in., and altitude 9 in.; what is its volume?

If we divide the length, width, and height into inches and pass planes through these points of division, the prism will be divided into equal cubes.

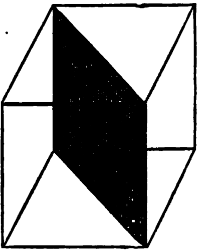
There will be 12 cubic inches in one layer; and since there are 9 such layers, the volume will equal 9 times 12 cubic inches, or 108 cubic inches.



Hence the

RULE.

The volume of a rectangular prism is equal to the area of the base multiplied by the altitude.



A rectangular prism may be divided into two equal triangular prisms, each having the altitude of the rectangular prism and a base equal to half the base of the rectangular prism. Hence the

RULE.

The volume of a triangular prism is equal to the area of the base multiplied by the altitude.

Any prism may be divided into triangular prisms, each of which is equal to the area of the base multiplied by the common altitude, and their sum, which is the volume of the prism, is equal to the sum of their bases or the base of the prism multiplied by the altitude. Hence the general

RULE.

The volume of any prism is equal to the area of the base multiplied by the altitude.

WRITTEN EXERCISES.

1. Find the volume of a square prism whose altitude is 40 ft. and the sides of the base 4 ft.

2. What is the volume of a rectangular prism whose altitude is 60 ft. and the sides of the base 4 ft. and 8 ft., respectively?

3. Find the volume of a triangular prism whose altitude is 100 ft. and the sides of the base each 20 ft.

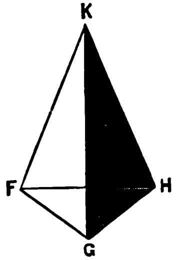
4. Find the volume of a triangular prism whose altitude is 40 ft. and the base a right triangle, with the sides about the right angle 8 ft. and 12 ft., respectively.

5. The water in a stream flows at the rate of 3 miles an hour; if the stream is 4 ft. wide and 3 ft. deep, how many cubic yards of water pass a given point in 24 hours?

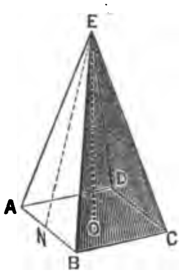
254. The Pyramid.

A **Pyramid** is a solid bounded by a polygon and by several triangles meeting in a point; as, A B C D – E.

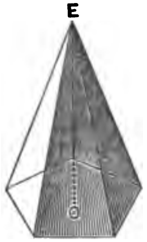
The polygon is called the *base* of the pyramid, and the triangles form the *convex surface.* The *altitude* is the perpendicular distance E O from the vertex to the base. The *slant height* of a pyramid is the altitude of any one of its lateral faces; as, E N.



Triangular Pyramid.



Square Pyramid.



Pentangular Pyramid.

Pyramids are named from their bases.

RULE.

The convex surface of a right pyramid is equal to the perimeter of the base multiplied by half the slant height.

WRITTEN EXERCISES.

1. Find the convex surface of a square pyramid whose slant height is 24 ft. and the sides of the base each 5 ft.

2. Find the convex surface of a triangular pyramid whose slant height is 60 ft. and the sides of the base each 21 ft.

3. Find the convex surface of a pyramid whose slant height is 48 ft. and the base a hexagon whose sides are each 16 ft.

4. Find the entire surface of a square pyramid whose slant height is 50 ft. and each side of the base 20 ft.

5. Find the entire surface of a triangular pyramid whose slant height is 36 in. and the sides of the base are each 8 in.

255. If we make two vessels of tin, the one a pyramid and the other a prism, having equal bases and equal altitudes, the prism will hold just three times as much as the pyramid.

Hence the

RULE.

To find the volume of a pyramid, multiply the area of the base by one-third the altitude.

WRITTEN EXERCISES.

1. Find the volume of a pyramid whose altitude is 12 ft. and the area of the base 15 sq. ft.

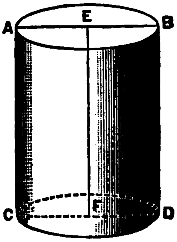
2. Find the volume of a pyramid whose altitude is 45 ft. and the base a square 10 ft. on a side.

3. What is the volume of a rectangular pyramid whose altitude is 60 ft. and the sides of the base 12 and 20 ft., respectively?

4. Find the volume of a pyramid whose altitude is 72 ft. and the base a triangle 24 ft. on each side.

5. Find the volume of a pyramid whose altitude is 24 ft., and having a right triangle for its base whose sides are 8, 15, and 17 ft., respectively.

256. The Cylinder.



Cylinder.

A Cylinder is a round body with equal and parallel circles for its bases and having a uniform diameter.

The *altitude* of a cylinder is the perpendicular distance between its bases.

The *convex surface* of a cylinder is the curved surface which bounds it.

From Art. 145 we have the following

RULE.

To find the convex surface of a cylinder, multiply the circumference of the base by the altitude.

WRITTEN EXERCISES.

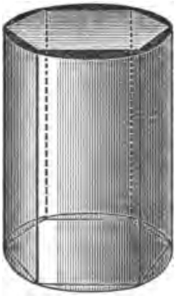
1. Find the convex surface of a cylinder whose altitude is 10 ft. and the radius of the base 4 ft.

2. Find the convex surface of a cylinder whose altitude is 50 ft. and the diameter of the base 6 ft.

3. Find the entire surface of a cylinder whose altitude is 60 ft. and the diameter of the base 5 ft.

4. Find the entire surface of a cylinder whose altitude is 30 ft. and the radius of the base 7 ft.

257. We have found by Art. 253 that *the volume of a prism equals the area of the base multiplied by the altitude.* This will be true when the prism has an infinite number of sides and coincides with the cylinder. Hence the



RULE.

To find the volume of a cylinder, multiply the area of the base by the altitude.

WRITTEN EXERCISES.

1. Required the contents of a cylinder 80 ft. long and 4 ft. in diameter.

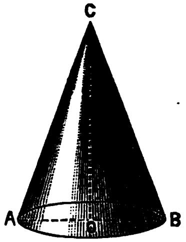
2. Find the contents of a cylinder whose altitude is 4 rd. and the radius of the base 14 in.

3. Required the contents of a log whose length is 15 ft. and the diameter of the base 8⅔ ft.

258. The Cone.

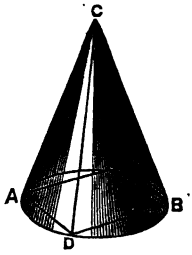
A **Cone** is a round body whose base is a circle, and whose convex surface tapers uniformly to a point called the *vertex.*

The *altitude* of a cone is the perpendicular distance from the vertex to the base; as, C O. The *slant height* is the distance from the vertex to the circumference of the base; as, C A.



Cone.

259. We have found by Art. 254 that *the convex surface of a pyramid equals the perimeter of the base multiplied by one-half the slant height.* This will be true when the pyramid has an infinite number of sides and coincides with the cone. Hence the



RULE.

To find the convex surface of a cone, multiply the circumference of the base by one-half the slant height.

WRITTEN EXERCISES.

1. Find the convex surface of a cone whose slant height is 36 in. and the radius of the base 8 in.

2. Find the convex surface of a cone whose slant height is 60 ft. and the diameter of the base 20 ft.

3. A tent in the form of a cone has a slant height of 18 ft. and a diameter of 30 ft.; how many square yards of muslin are required to make it?

4. Find the entire surface of a cone whose slant height is 66 ft. and the radius of the base 22 ft.

5. Find the entire surface of a cone whose altitude is 15 in. and the radius of the base 8 in.

260. We have found by ART. 255 that *the volume of a pyramid equals the area of the base multiplied by one-third of the altitude.* This will be true when the pyramid has an infinite number of sides and coincides with the cone.

Hence the

RULE.

To find the volume of a cone, multiply the area of the base by one-third of the altitude.

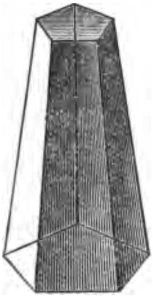
WRITTEN EXERCISES.

1. Find the volume of a cone whose altitude is 12 ft. and the radius of the base 4 ft.

2. Find the volume of a cone whose altitude is 63 ft. and the diameter of the base 20 ft.

3. What is the volume of a cone whose slant height is 17 in. and the diameter of the base 16 in.?

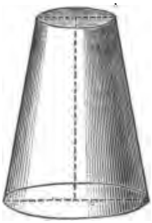
4. Find the volume of a cone whose altitude is 72 ft. and the radius of the base 20 ft.



261. The Frustum of a Pyramid or Cone.

The **Frustum of a Pyramid** is the part of the pyramid which remains after cutting off the top by a plane parallel to the base.

262. The **Frustum of a Cone** is the part of a cone which remains after cutting off the top by a plane parallel to the base.



RULE.

To find the convex surface of a frustum, multiply the sum of the perimeters or circumferences of the bases by one-half the slant height.

WRITTEN EXERCISES.

1. Find the convex surface of the frustum of a square pyramid whose slant height is 24 ft., and the side of the lower base 10 ft. and upper base 6 ft.

2. Find the convex surface of the frustum of a cone whose slant height is 42 ft., and the radii of the bases 12 and 4 ft., respectively.

3. Find the entire surface of the frustum of a cone whose slant height is 50 ft., and the radii of the bases 10 and 5 ft., respectively.

4. Find the entire surface of the frustum of a triangular pyramid whose slant height is 40 in., and the sides of the upper base 4 in. and of the lower base 10 in.

RULE.

263. *To find the volume of a frustum, take the sum of the areas of the two bases, to which add the square root of their product, and multiply this sum by one-third of the altitude.*

WRITTEN EXERCISES.

1. Find the volume of the frustum of a square pyramid the sides of whose bases are 3 ft. and 6 ft. and altitude 12 ft.

The volume $= (3^2 + 6^2 + \sqrt{3^2 \times 6^2})\,\frac{12}{3} = (9 + 36 + 18)\,4 = 252$ cu. ft.

2. Find the contents of the frustum of a cone the radii of whose bases are 3 and 4 ft. and altitude 18 ft.

The contents $= (\pi\,3^2 + \pi\,4^2 + \sqrt{\pi^2 3^2 \times 4^2})\,\frac{18}{3} = (9\pi + 16\pi + 12\pi)\,6$ $= 222\pi = 697.4352$ cu. ft.

3. Find the contents of the frustum of a square pyramid the sides of whose bases are 2 and 8 ft. and altitude 15 ft.

4. Find the contents of a log the radii of whose bases are 2 and 3 ft. and length 60 ft.

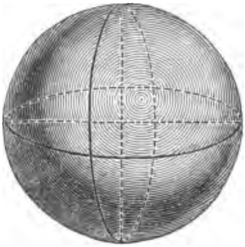
5. How many cubic feet in the frustum of a cone the radii of whose bases are 12 and 8 ft. and altitude 27 ft.?

6. Find the contents of the frustum of a triangular pyramid the sides of whose bases are 4 and 6 ft. and altitude 21 ft.

264. The Sphere.

A **Sphere** is a round body bounded by a curved surface every point of which is equally distant from a point within called the *centre*.

The *diameter* of a sphere is a line passing through the centre and terminating in the surface. The *radius* is the distance from the centre to the surface.



RULE.

The surface of a sphere is equal to the square of the diameter, or four times the square of the radius, multiplied by 3.1416.

WRITTEN EXERCISES.

1. Find the surface of a sphere whose radius is 5 in.

2. Find the surface of a sphere whose radius is 12 in.

3. Assuming the earth to be a sphere 7960 miles in diameter, how many square miles in its surface?

4. What will it cost to plate with gold a sphere 18 in. in diameter at $40.50 a square foot?

5. What will it cost to tin the hemispherical dome of an observatory 40 ft. in diameter, at 10 cents a square foot?

RULE.

265. *The volume of a sphere is equal to one-sixth the cube of its diameter, multiplied by* 3.1416.

WRITTEN EXERCISES.

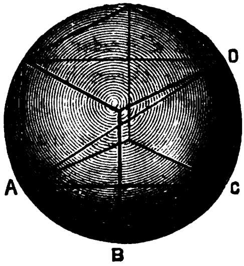
1. Find the volume of a sphere whose radius is 10 in.

2. The outer diameter of a spherical shell is 12 in. and the inner diameter is 8 in.; find the contents.

3. Find the weight of a ball of gold 8 in. in diameter, if a cubic foot weighs 1204 lb.

4. Find the weight of a cannon-ball of cast iron 15 in. in diameter, if a cubic foot weighs 450 lb.

266. A cube is inscribed in a sphere when each of its angles is in the surface of the sphere.



WRITTEN EXERCISES.

1. Find the side of the largest cube that can be cut from a sphere 15 in. in diameter.

It is readily seen that

$$AB^2 + BC^2 + CD^2 = AD^2. \quad \text{But } AB = BC = CD;$$

then

$$3\,AB^2 = AD^2 = 15^2 = 225,$$

$$AB^2 = 75,$$

$$AB = \sqrt{75} = 8.66+ \text{ in.}$$

2. Find the side of the largest cube that can be cut from a sphere 30 in. in diameter.

PROBLEMS IN MENSURATION.

1. How many acres in a triangle whose base is 40 chains and altitude 36 chains?

2. How many square yards in a triangle whose sides are each 50 feet?

3. How many acres in a triangle whose sides are respectively 20, 30, and 40 chains?

4. The hypotenuse of a right triangle is 312 feet and the base is 282 feet; what is the perpendicular?

5. Find the diagonal of a square whose side is unity.

6. Find the diagonal of a cube whose side is unity.

7. How many acres in a parallelogram whose base is 161.5 rods and altitude 106.3 rods?

8. The area of a rectangle is 1872 square rods, and the length is to the width as 4 is to 3; required the length of the sides.

9. The area of a circle is 2 A. 3 R. 15 P.; what is the circumference?

10. Find the side of a square whose area is equal to the area of a circle 40 feet in diameter.

11. Find the length of an arc of 35° in a circle whose radius is 40 rods.

12. Find the area of the largest possible square that can be cut from a circle 62 feet in circumference.

13. Find the diameter of a circle whose area is numerically equal to 10 times its circumference.

14. How many square feet in the bottom and convex surface of a scrap-basket whose lower base is 9 inches in diameter, upper base 12 inches in diameter, and altitude 10 inches?

15. A piece of timber 90 feet long is 6 by 8 inches at the smaller end and 18 by 24 inches at the larger end; how many cubic feet does it contain?

16. Find the diameter of a sphere whose surface and volume are numerically equal.

17. Find the diagonal of a parallelopiped whose base is 5 feet by 7 feet, and altitude 16 feet.

18. Required the entire surface of a square pyramid whose altitude is 36 inches and side of the base 30 inches.

19. Required the dimensions of a cubical bin which will hold 1000 bushels of grain.

20. Find the contents of a cone whose altitude is 60 feet and the radius of the base 20 feet.

21. Find the volume of the frustum of a cone whose altitude is 24 feet, the radius of the lower base being 8 feet and of the upper base 6 feet.

22. If a conical hay-stack 24 feet in diameter and 20 feet high contains 12 tons, what are the dimensions of a similar stack which contains 20 tons?

23. A circular mirror 24 inches in diameter is surrounded by a frame 3 inches wide; what is the cost of the frame, at $2 a square foot?

24. A garden containing 74 square feet less than one-fourth of an acre is planted with hyacinths in squares 6 inches apart; how many plants will be required, if the outer rows are 6 inches from the edge?

25. The largest possible square piece of timber of uniform size is cut from a log 40 feet long, 15 inches in diameter at one end, and 18 inches at the other; how much is wasted?

26. A cylinder is 6 feet in diameter and 8 feet high; what are the dimensions of a similar cylinder whose surface is 64 times as much?

27. How much material will be required to make a hollow sphere whose outside diameter is 15 inches and thickness 1 inch?

28. What is the value of a gold sphere 4 inches in diameter, if a sphere of silver 1 inch in diameter is worth $5, and the value of gold is to the value of silver as 16 to 1?

SECTION XII.

PROGRESSIONS.

ARITHMETICAL PROGRESSION.

267. An **Arithmetical Progression** is a series of numbers which increase or decrease by a **Common Difference.**

Thus, 1, 3, 5, 7, 9 is an *increasing* arithmetical progression in which the common difference is 2.

And 10, 8, 6, 4, 2 is a *decreasing* arithmetical progression in which the common difference is 2.

The numbers composing the series are called the **Terms.**

268. To find the last term of an arithmetical progression.

WRITTEN EXERCISES.

1. Find the 11th term of the series 3, 5, 7, 9, 11, etc.

Process.

1st $= 3$

2d $= 3 + 1 \times 2 = 5$

3d $= 3 + 2 \times 2 = 7$

4th $= 3 + 3 \times 2 = 9$

Hence 11th $= 3 + 10 \times 2 = 23$.

The first term is 3.

The second term equals 3 plus *once* the common difference.

The third term equals 3 plus *twice* the common difference.

Hence the *eleventh* term equals the first term plus *ten* times the common difference, which equals $3 + 10 \times 2 = 23$.

From this problem we derive the

RULE.

The last term equals the first term increased by the common difference multiplied by the number of terms less one.

2. Find the 12th term of the series 1, 3, 5, etc.

3. Find the 31st term of the series 2, 4, 6, etc.

4. Find the 50th term of the series 1, 4, 7, etc.

5. Find the 80th term of the series 10, 15, 20, etc.

6. Find the 103d term of the series 6, $7\frac{1}{3}$, $8\frac{2}{3}$, etc.

7. Find the 21st term of the series 8, 12, 16, etc.

8. Find the 100th term of the series 15, 25, 35, etc.

9. Find the 60th term of the series 216, 214, 212, etc.

10. Find the 12th term of the series $\frac{1}{8}$, $\frac{5}{8}$, $1\frac{1}{8}$, etc.

11. If a body falls $16\frac{1}{12}$ ft. the first second, and in each succeeding second $32\frac{1}{6}$ feet more than in the preceding one, how far will it fall in the 25th second?

269. To find the sum of an arithmetical progression.

WRITTEN EXERCISES.

1. Find the sum of 8 terms of the series 1, 5, 9, etc.

$$\begin{array}{rl}
\text{Sum} & = 1 + 5 + 9 + 13 + 17 + 21 + 25 + 29 \\
\text{Sum} & = 29 + 25 + 21 + 17 + 13 + 9 + 5 + 1 \\
\hline
2\text{ sum} & = 30 + 30 + 30 + 30 + 30 + 30 + 30 + 30 \\
2\text{ sum} & = 30 \times 8 = (1 + 29)\,8
\end{array}$$

$$\text{Sum} = \frac{1 + 29}{2} \times 8 = 120, \textit{ Ans.}$$

The sum of the series is 1 + 5 + 9 + 13 + 17 + 21 + 25 + 29.

Writing the sum of the series in a *reverse order*—that is, 29 + 25 + 21 + 17 + 13 + 9 + 5 + 1—and adding, we have *twice* the sum of the series equal to 30 taken 8 times, or (1 + 29) 8.

Hence the sum of the series equals $\frac{1 + 29}{2} \times 8$, or 120.

From this problem we derive the

RULE.

The sum of an arithmetical progression equals half the sum of the extremes multiplied by the number of terms.

2. Find the sum of 12 terms of the series 6, 9, 12, etc.

PROCESS.—12th term $= 6 + 11 \times 3 = 39$.

$$\text{Sum} = \frac{6 + 39}{2} \times 12 = 270, \textit{Ans.}$$

Find the sum—

3. Of 16 terms of the series 2, 4, 6, etc.
4. Of 20 terms of the series 1, 3, 5, etc.
5. Of 30 terms of the series 2, 6, 10, etc.
6. Of 40 terms of the series 1, 6, 11, etc.
7. Of 24 terms of the series 2, 8, 14, etc.
8. Of 100 terms of the series 1, 4, 7, etc.

9. How many times does a common clock strike in a week?

10. A traveled 15 miles the first day, 18 miles the second, 21 miles the third, and so on for 36 days; how far did he travel?

11. A boy received $5 for January, $10 for February, $15 for March, and so on; how much did he receive for the year?

12. If a body falls $16\frac{1}{12}$ ft. the first second, 3 times as far the second second, 5 times as far the third second, and so on, how far will it fall in 25 seconds?

13. 100 apples are placed in a row 2 yd. apart, the first being 2 yd. from a basket: a boy starts to gather them singly into the basket; how far does he travel?

GEOMETRICAL PROGRESSION.

270. A **Geometrical Progression** is a series of numbers which increase or decrease by a common **Ratio**.

Thus, 1, 3, 9, 27, etc. is an *increasing* geometrical progression, in which the ratio is 3.

And 32, 16, 8, 4, etc. is a *decreasing* geometrical progression, in which the ratio is $\frac{1}{2}$.

The numbers composing the series are called the **Terms.**

271. To find the last term of a geometrical progression.

WRITTEN EXERCISES.

1. Find the 7th term of the series 2, 6, 18, etc.

PROCESS.

1st $= 2$
2d $= 2 \times 3 = 6$
3d $= 2 \times 3^2 = 18$
4th $= 2 \times 3^3 = 54$

Hence 7th $= 2 \times 3^6 = 1458$

The first term is 2.

The second term equals 2 times the *first* power of the ratio.

The third term equals 2 times the *second* power of the ratio.

Hence the *seventh* term equals 2 times the *sixth* power of the ratio, which equals $2 \times 3^6 = 1458$.

From this problem we derive the

RULE.

The last term equals the first term multiplied by the ratio raised to a power denoted by the number of terms less one.

2. Find the 6th term of the series 3, 9, 27, etc.

PROCESS.—Last term $= 3 \times 3^6 = 2187$, *Ans.*

3. Find the 10th term of the series 1, 2, 4, etc.

4. Find the 8th term of the series 5, 15, 45, etc.

5. Find the 7th term of the series 3, 12, 48, etc.

6. Find the 10th term of the series 1, $\frac{1}{2}$, $\frac{1}{4}$, etc.

7. Find the 10th term of the series 1, $\frac{1}{3}$, $\frac{1}{9}$, etc.

8. Find the 9th term of the series $\frac{2}{3}$, $\frac{1}{2}$, $\frac{3}{8}$, etc.

9. A man agreed to labor at the rate of $1 for January, $2 for February, $4 for March, and so on, and to accept for his year's services as much as would be due him under this agreement for December; how much would he receive?

10. A boy agreed to put one cent in his savings bank on New Year's day, and on each succeeding day of the month to double the previous day's deposit; if he had carried out his agreement, what would have been the last day's deposit?

272. To find the sum of a geometrical progression.

WRITTEN EXERCISES.

1. Find the sum of 5 terms of the series 2, 6, 18, etc.

PROCESS.

$$\begin{array}{rcl} \text{Sum} &=& 2 + 6 + 18 + 54 + 162 \\ 3\ \text{sum} &=& \phantom{2 + {}} 6 + 18 + 54 + 162 + 486 \\ \hline (3 - 1)\ \text{sum} &=& 486 - 2 \end{array}$$

$$\text{Sum} = \frac{486 - 2}{3 - 1} = \frac{(162 \times 3) - 2}{3 - 1} = 242, \textit{ Ans.}$$

Multiplying the sum of the series by 3, the ratio, and subtracting the series, we have (3 − 1) times the sum of the series equal to 486 − 2, and the sum of the series equal (486 − 2) divided by (3 − 1).

Observing that 162 is the last term, 3 the ratio, and 2 the first term, we have the

RULE.

The sum of a geometrical progression equals the last term multiplied by the ratio, minus the first term, divided by the ratio less one.

2. Find the sum of 8 terms of the series 3, 6, 12, etc.

PROCESS.—Last term $= 3 \times 2^7 = 384$.

$$\text{Sum} = \frac{384 \times 2 - 3}{2 - 1} = 765, \textit{ Ans.}$$

Find the sum—

3. Of 1, 2, 4, etc. to 10 terms.

4. Of 2, 8, 32, etc. to 9 terms.

5. Of 6, 12, 24, etc. to 12 terms.

6. Of 5, 15, 45, etc. to 14 terms.

7. Of 10, 20, 40, etc. to 10 terms.

8. If the population of a city is doubled every 5 years, what is its population at the end of 50 years, if it begins with 5000 inhabitants?

9. A agreed to labor 21 days on condition that he should receive 1 cent for the first day, 2 cents for the second, 4 cents for the third, and so on; how much did he receive in all?

10. A gentleman, thinking $5000 too much for a farm containing 17 acres, agreed to pay 1 cent for the first acre, 3 cents for the second, 9 cents for the third, and so on; which price was the greater, and how much?

INFINITE SERIES.

273. An **Infinite Series** is a series which has an infinite number of terms.

In a decreasing geometrical progression we *subtract the last term multiplied by the ratio from the first term, and divide by* 1 *minus the ratio.*

In a decreasing geometrical progression of an infinite number of terms the last term becomes so small that it may be regarded as zero; hence from the above we have the following

RULE.

The sum of an infinite series equals the first term divided by 1 *minus the ratio.*

WRITTEN EXERCISES.

1. Find the sum of the infinite series $1 + \frac{1}{2} + \frac{1}{4}$, etc.

PROCESS.—Sum $= \frac{1}{1 - \frac{1}{2}} = \frac{1}{\frac{1}{2}} = 2$, *Ans.*

Find the sum—

2. Of the infinite series $1 + \frac{1}{3} + \frac{1}{9}$, etc.
3. Of the infinite series $1 + \frac{1}{4} + \frac{1}{16}$, etc.
4. Of the infinite series $1 + \frac{1}{10} + \frac{1}{100}$, etc.
5. Of the infinite series $\frac{1}{3} + \frac{1}{6} + \frac{1}{12}$, etc.

SECTION XIII.

MISCELLANEOUS PROBLEMS.

274.—*1.* FIND the greatest common divisor of 1107, 1353, and 1599.

2. Find the least common multiple of 369, 697, and 861.

3. Reduce $\frac{567}{693}$ to its lowest terms.

4. Reduce $42 + \frac{11}{73}$ to an improper fraction.

5. Reduce $.6$ of $4\frac{2}{3}$ of $\dfrac{.8}{\frac{2}{3} \text{ of } 6}$ to a simple fraction.

6. Add $\frac{3}{4} + \frac{4}{5} + \frac{5}{6}$ to $\frac{5}{12} - \frac{3}{10}$.

7. Find the value of $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{7}{8}$ divided by $\frac{5}{6}$ of $\frac{4}{9}$ of $\frac{12}{25}$.

8. What part of $4\frac{4}{5}$ is $2\frac{7}{8}$?

9. What part of $\dfrac{2\frac{2}{3}}{3\frac{1}{3}}$ is $.4\frac{1}{2}$?

10. Divide $16\frac{2}{3}$ by $\dfrac{4\frac{1}{2}}{9}$ of $\dfrac{26}{6\frac{1}{2}}$.

11. Add $37\frac{2}{3} + 18\frac{5}{7} + 42\frac{9}{8} + 100\frac{5}{6}$.

12. Reduce $\dfrac{3\frac{1}{3} \times 3\frac{1}{3} - 1}{3\frac{1}{3} - 1}$ of $\dfrac{15\frac{1}{2}}{12}$.

13. Divide $.0008\frac{1}{7}$ by 57000.

14. Divide 7000 by $.00002\frac{1}{3}$.

15. Reduce $\frac{14}{125}$ and $\frac{23}{50}$ to decimals.

16. Find the value of $(.25 \div 500 + 7.5) \times \dfrac{110}{.005\frac{1}{2}}$.

17. How many posts 6 ft. apart will be required to make 5 miles of fence?

18. How many dry gallons in 80 gallons liquid measure?

19. How many pounds Troy in 50 pounds avoirdupois?

20. What will 5 bu. 3 pk. of berries cost at 8 cents a quart?

21. A railroad 60 ft. wide was run through a county 10 miles in length; how many acres did it cover?

22. If it requires 3 bu. of seed to sow an acre, how many pints will be required to sow a field 80 ft. square?

23. A cylindrical cistern is 6 ft. 6 in. deep, and 6 ft. in diameter; how many hogsheads of water will it hold?

24. The area of a square field is 4 A. 108 sq. rd.; find the perimeter in rods.

25. Simplify $\dfrac{8\frac{3}{4} - 2\frac{2}{3} + 9\frac{5}{6} - 1\frac{1}{2}}{12\frac{1}{2} + 5\frac{2}{3} - 7\frac{1}{2} + 4\frac{1}{3}}$.

26. A bin 8 ft. 6 in. long, 5 ft. 4 in. wide, and 6 ft. 8 in. high is $\frac{3}{4}$ full of wheat; if a bushel is equal to $1\frac{1}{4}$ cu. ft., what are the contents worth at 70 cents a bushel?

27. Find the exact number of days from May 5, 1890, to July 4, 1896.

28. When it is 3.15 A.M., Dec. 25th, at Vienna, longitude 16° 23′ east, what is the time at San Francisco, longitude 122° 23′ west?

29. How deep must a circular cistern 5 ft. in diameter be to hold 50 barrels of $31\frac{1}{2}$ gallons each?

30. How many feet of railing will be required to enclose a square acre? How many to enclose a circular acre?

31. How often will a wheel 5 ft. in diameter revolve in going 10 miles?

32. Multiply $102\frac{117}{211}$ by 45.

33. What is the difference in time between a city whose longitude is 76° 37′ west and a city whose longitude is 13° 23′ east?

34. Divide $426\frac{82}{512}$ by 62.

35. A rectangular field is 40 rd. wide and 15 ch. long; what is it worth, at $120 an acre?

36. What is $37\frac{1}{2}\%$ of $8\frac{2}{3}$?

37. What per cent. of $15\frac{3}{4}$ is $5\frac{1}{4}$?

38. An agent sold goods for $1295, and lost $\frac{12}{47}$ of the cost; how much did the goods cost him?

39. A man lost $475, which was 20% of what he had remaining; how much had he at first?

40. Find the simple interest of $420 for 3 yr. 4 mo. 12 da. at $5\frac{1}{2}\%$.

41. Find the exact interest of $340 from May 8, 1894, to Jan. 11, 1895, at $6\frac{1}{2}\%$.

42. Find the annual interest of $600 from June 10, 1890, to Aug. 15, 1894, at 6%, if the interest for the first two years is paid.

43. A sold a farm for $12460, and gained $\frac{2}{5}$ of the cost; how much did he gain?

44 An agent received $2000 to invest in goods after deducting 2% commission; what is the amount of his commission?

45. An agent sold goods on a commission of 3%; he remitted his employer $1500; what was the amount of goods sold?

46. Find the present worth and true discount of $800 for 1 yr. 6 mo. at 8%.

47. Find the contents of a pan whose altitude is 9 in., the radius of the lower base being 6 in. and of the upper base 8 in.

48. An agent's commission at 2% is $320; what sum did he remit to his employer?

49. What will it cost to paint the dome of a hall which is in the form of a hemisphere 40 ft. in diameter, at 40 cents a square yard?

50. The hypotenuse of a right triangle is 75 ft. and the base is 40 ft.; required the perpendicular.

51. In what time will $4080 at 5% give $668.10 interest?

52. Extract the square root of 430.064644.

53. What sum must be invested in 6% stock at $117\frac{1}{2}$, brokerage $\frac{1}{8}$%, to yield an annual income of $426?

54. What must I pay for 6% bonds to realize $5\frac{1}{2}$% on my investment, brokerage $\frac{1}{8}$%?

55. Find the cube root of $\frac{2}{3}$ to three places of decimals.

56. M sold two houses for $4800 each: on the one he gained 25%, and on the other he lost 25%; did he gain or lose on the entire transaction, and how much?

57. If a bushel equals $1\frac{1}{4}$ cubic feet, how deep must a bin 8 ft. long and 4 ft. wide be to hold 120 bushels?

58. At what rate will $570.05 in 3 yr. 5 mo. 5 da. give $127.11+ interest?

59. Find the 400th term of the series 3, 8, 13, 18, etc.

60. A rectangular box contains 30720 cu. in.; what are its dimensions, if they are to each other as 3, 4, and 5?

61. A 90-day note for $450 was discounted at a bank at 6%; required the proceeds.

62. I sold goods at 5% through an agent who charged me 3%: my commission after paying the agent was $240; required my entire commission.

63. Find the difference between the true discount and the bank discount of $1000 for 1 yr. 3 mo. 15 da. at 7%.

64. How many shares of stock at $113\frac{1}{2}$ can be bought for $7272, brokerage $\frac{1}{8}$%?

65. Find the surface and contents of a sphere 12 in. in diameter.

66. Required the face of a 4-mo. note which when discounted at a bank at 6% yields $240.

67. Find the value of $\dfrac{1}{4+\dfrac{1}{4+\frac{1}{4}}}$.

68. If a sphere 4 in. in diameter weigh 20 lb., what will a sphere 20 in. in diameter weigh?

69. Find the cube root of 45156047481.

70. Find the distance between the lower corner and the opposite upper corner of a room 18 ft. long, 15 ft. wide, and 12 ft. high.

71. What will I realize from the sale of 120 shares of stock sold at $111\frac{1}{4}$, brokerage $\frac{1}{8}\%$?

72. What per cent. will Georgia 6%'s yield bought at $119\frac{7}{8}$, brokerage $\frac{1}{8}\%$?

73. What time after 4 o'clock will the hour- and minute-hands of a watch be opposite each other?

74. A had his house insured for $\frac{5}{8}$ of its value at $1\frac{1}{4}\%$; soon afterwards it was destroyed by fire, and his loss over the insurance was $1700; what was the value of the house?

75. Find the sum of the infinite series $\frac{1}{3} + \frac{1}{12} + \frac{1}{48}$, etc.

76. A has a lot which contains $\frac{9}{10}$ of an acre: he made a walk around it which contained $\frac{47}{576}$ of the entire area; required the width of the walk.

77. What must I ask for goods which cost $120, so that I may deduct $20 from my asking price and still gain 20%?

78. A lost 35% of his sheep; what per cent. must he gain on the remainder to gain 20% on the cost of all?

79. A has a circular garden, and B a square one: each contains an acre; a walk 6 ft. wide is made around each garden; which walk contains the greater area, and how much?

80. How many cubic yards of earth will be required to cover $\frac{1}{4}$ of an acre 10 in. deep?

81. How long will it take two 3-inch pipes to empty a cistern that required three 2-inch pipes 4 hr. to fill?

82. What is the difference between the cube root of the least common multiple of 94106 and 202484 and the square root of their greatest common divisor?

83. A square field containing 10 acres is surrounded by a close board fence 10 ft. high; what did the boards cost, at $13 per M.?

84. The diameter of a circle is 40 ft.; what is the length of 84° 30′ 30″ of the circumference?

85. I sold a horse so that $\frac{2}{3}$ of the gain equaled $\frac{5}{18}$ of the cost; what was the gain per cent.?

86. A bought a horse at 10% less than first cost, and sold him at 20% above first cost, thereby gaining $60; what was the first cost of the horse?

87. After a rainfall of $1\frac{1}{4}$ inches how deep will the water be in a cylindrical cistern 6 ft. in diameter which drains the roof of a house 40 ft. by 30 ft.?

88. A note for $900 was given Apr. 1st at 6% interest, on which a payment of $500 was made Aug. 1st: Nov. 10th the note was purchased at 4% discount on its value at that time; what amount was paid for it?

89. A has a square yard 40 ft. on a side: on each corner is the centre of a circle 20 ft. in radius; find the area within the square and without the circles.

90. B pays a tax of 4% on his income over $2000, and after paying his tax he has remaining $4200; what is his whole income?

91. A horse is tied to the top of a post 6 ft. high by a rope 20 ft. long; over how many square yards can he graze?

92. If stock bought at 10% above par will pay 6% on the investment, what % will it pay if bought at 10% below par?

93. A general formed his army into a solid square, and had 200 men over: he then received a reinforcement of 1000 men, and, increasing each side of the square by 5 men, lacked 25 men to complete the square; how many men were there in the original army?

94. A boy hired to a farmer for 30 days on condition that he should receive $30 and a suit of clothes: at the end of 18 days he concluded to quit work, when it was found that he was entitled to $10 and the suit of clothes; what was the value of the clothes?

95. A sells 5% stock, whose par value is $13400, at 93⅜, and invests the proceeds in 4½% stock at 85⅝, brokerage ⅛% on each transaction; is his income increased or diminished, and what is his surplus?

96. A sold his farm, and gained a certain per cent.: if the farm had cost him 10% less and sold for the same price, he would have gained 16⅔% more; what was his gain per cent.?

97. Four ladies bought a ball of yarn 3 in. in diameter; what part of the diameter must each wind off to share it equally?

98. What is the length of a tape that will wind spirally around a cylinder that is 75 ft. long and 12 ft. in circumference, provided it passes around the cylinder once every 5 ft.?

99. Find the 6th root of 6321363049.

100. Express .0003986 as a common fraction in its lowest terms.

101. How many cannon-balls 2 inches in diameter can be made from a cubic foot of iron?

102. The sum of the interest, amount, present worth, and true discount of a certain sum of money is $800; what is the amount?

103. Find the side of a square whose area is equal to the area of a triangle 16 feet on a side.

104. What principal at 5% compound interest will amount to $259.085 in 5 years?

105. What is the shortest distance from the lower corner of a room 40 ft. long, 30 ft. wide, and 12 ft. high to the opposite upper corner, keeping on the surface of the room?

106. At how many minutes after 3 o'clock will the hour- and minute-hands of a watch be opposite each other?

107. A pays $2 a month dues to a building association; how much has he paid in at the end of 9 years and 2 months?

108. John can do a piece of work in 4 days of 10 hours each, and James in 3 days of 9 hours each; how many hours will they require to finish the job, working together?

109. Find the contents of a cone whose slant height is 10 feet and the radius of the base 6 feet.

110. Find the amount of $946.75 for 6 yr. 4 mo. 17 da. at 7⅓%.

111. How much leather will be required to cover a base-ball 3½ inches in diameter?

112. I sold through a broker 50 shares of bank stock ($100) at ⅞% premium, and invested the proceeds in railroad stock ($50) at ⅞% discount; how many shares did I buy, and what was the surplus, brokerage being ¼% for buying and selling?

113. A has a circular lot 10 rods in diameter, and B has a square one whose diagonal is 12½ rods; which is the larger, and how much?

114. A piece of work can be done in a day of 11½ hours by 2 men, 5 women, or 12 boys; in what time can it be done by 1 man, 1 woman, and 1 boy?

115. A merchant bought a steer weighing 960 pounds for $50, which lost 40% of its weight in dressing; how much per cent. did he gain if he sold the beef at an average price of 10 cents per pound?

116. I buy muslin at 6¼ cents a yard, and mark it at a gain of 20%: finding it damaged, I sell it at 75% of the marked price; what is my gain or loss per cent.?

117. A gave his note for 4 months, which when discounted at a bank at 1% a month gave $500 proceeds; what was the face?

118. A cistern 3 ft. 4 in. long, 2 ft. 9 in. wide, and 7 ft. 6 in. deep has a supply-pipe which pours in 7 gallons of water in 5 minutes, and a waste-pipe which draws out 2 gallons in 6 minutes; how long will it take to fill the cistern if both pipes are open at the same time?

119. On the 1st of Oct., 1893, I borrowed from Mr. Bonn $500, to be paid in 3 months without interest: on July 1, 1894, I paid him $75; on Jan. 1, 1895, I paid him $100; how much was due April 1, 1895?

120. What is the amount of the following bill of lumber at $16 per thousand: 2 pieces 8 × 4 in. 40 ft. long, 3 pieces 8 × 5 in. 20 ft. long, 4 pieces 6 × 4 in. 18 ft. long, 38 pieces 3 × 4 in. 16 ft. long, 80 pieces 2 × 4 in. 18 ft. long, 48 pieces 2 × 8 in. 10 ft. long, 24 pieces 2 × 7 in. 20 ft. long, and 40 pieces $1\frac{1}{2}$ × 6 in. 16 ft. long?

121. A miller bought a ton of wheat at 60 cts. a bushel, which he manufactured into flour; what was his gain per cent. if each bushel yielded $37\frac{1}{2}$ pounds and he received $4.20 a barrel for the flour?

122. My agent sold 5500 lb. of poultry at 8 cts. a pound, and 8040 lb. of butter at $22\frac{1}{2}$ cts. a pound, on $3\frac{1}{2}\%$ commission: he remitted me the proceeds in a 60-day draft, which was discounted at a bank 15 days after date at 10%; what did I realize from the sales?

123. If oranges are bought at the rate of 6 for 5 cts. and sold at the rate of 5 for 6 cts., what is the gain per cent.?

124. A coal-dealer buys a car-load of coal, 33600 lb., at $4.25 a long ton, which he retails at $5.50 a ton; what per cent. does he gain, allowing 3% loss in handling?

125. A vessel is $7\frac{1}{2}$ inches in diameter at the base and $10\frac{1}{2}$ inches at the top; how many gallons of water will it hold if it is $11\frac{1}{2}$ inches high?

126. Find the square root of 2561068449.

127. Find the cube root of 66814683180552.

128. Find the value of $\sqrt{2 - \sqrt{2 + \sqrt{3}}}$.

129. What must be paid for $4\frac{1}{2}\%$ stocks to realize 6% on the investment?

130. If $\frac{5}{18}$ of the price received for an article is loss, what is the loss per cent.?

131. Simplify $\dfrac{(\frac{16}{35} \text{ of } 13\frac{1}{3}) + 9\frac{5}{11} - 2\frac{1}{2}}{8\frac{1}{6} - (3\frac{1}{2} \div 8\frac{1}{5}) + 2\frac{1}{9}}$.

132. At what per cent. below par must $4\frac{1}{2}$% stock be quoted to yield the same per cent. on the investment as a 7% stock at a premium of 30%, brokerage in each case $\frac{1}{8}$%?

133. What principal at $6\frac{3}{4}$% interest will gain $42.80 from May 4, 1895, to Jan. 6, 1896?

134. F has a debt of $5000, which he wishes to meet by the sale of stock at $110\frac{3}{4}$; how many shares must he sell, and what surplus will he have, brokerage $\frac{1}{4}$%?

135. What sum must be invested in U. S. 5's at $112\frac{1}{4}$, brokerage $\frac{1}{4}$%, to secure an annual income of $1200?

136. Simplify $(2.806 + 2\frac{3}{4} \times \dfrac{.45}{.001\frac{1}{2}} + .825) \div 8.75$, and express the result as a decimal.

137. Find the sum of the infinite series $\frac{1}{2} + \frac{1}{3} + \frac{2}{9} +$, etc.

138. What must be the face of a note due 45 days hence which, when discounted at $4\frac{1}{2}$%, will yield $600?

139. Extract the fourth root of 8 to four places of decimals.

140. Extract the sixth root of 10 to four places of decimals.

141. The dial-face of a clock contains 21.65 sq. in.; what is the length of the minute-hand, if it reaches within $\frac{1}{4}$ of an inch of the bounding circle of the dial?

142. If 48 men dig a trench 192 feet long, 20 feet wide, and 12 feet deep in 144 days of 10 hours each, how many days of 11 hours each will it take 40 men to dig a trench 80 feet long, 16 feet wide, and 12 feet deep?

143. If a pipe whose diameter is $2\frac{1}{4}$ in. can empty a tank in 6 hr. 30 min., how long will it take a pipe whose diameter is $5\frac{1}{2}$ in. to empty it?

SECTION XIV.

SUPPLEMENT.

BUSINESS FORMS.

275. Receipt on Account.

THE following form of receipt is used when money is paid on an unsettled account:

Lancaster, May 8, 1895.

Received of E. Oram Lyte one hundred dollars on account.

$100. *Henry F. Bitner.*

276. Receipt in Full.

When an account is paid in full the following form should be used:

Philadelphia, July 3, 1895.

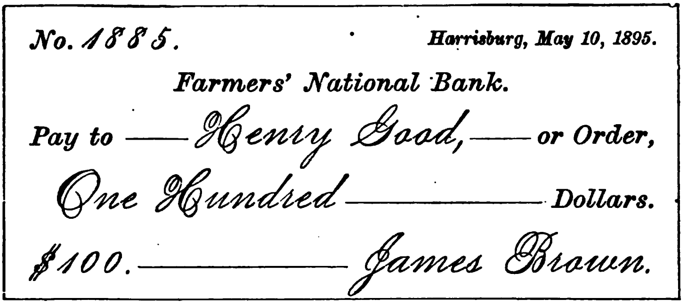
Received of John L. Shroy three hundred and ten $\frac{25}{100}$ dollars, in full of all accounts to date.

310\frac{25}{100}$. *James H. Long.*

277. Bank Check.

A *Check* is a written order addressed to a bank by a person having money deposited, requesting the payment of a certain sum to a specified person or his order.

FORM OF A BANK CHECK.



No. 1885. *Harrisburg, May 10, 1895.*

Farmers' National Bank.

Pay to —— Henry Good, —— *or Order,*

One Hundred —————— *Dollars.*

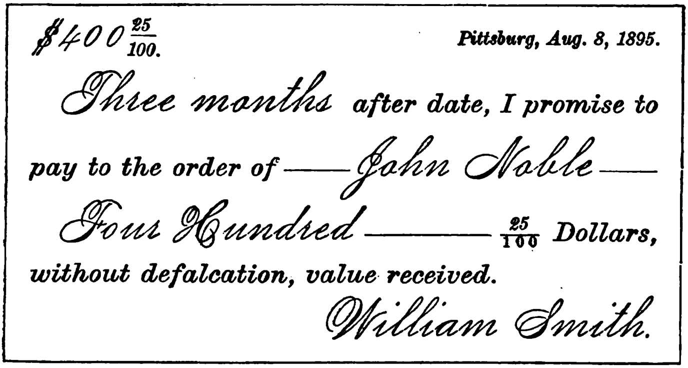
$100. —————— James Brown.

278. Promissory Note.

A *Promissory Note* is a written promise to pay a specified person a specified sum at a specified time.

The following form is that of an individual note, payable after date:

FORM OF A PROMISSORY NOTE.



$400 $\frac{25}{100}$. *Pittsburg, Aug. 8, 1895.*

Three months *after date, I promise to pay to the order of* —— John Noble ——

Four Hundred ———— $\frac{25}{100}$ *Dollars,*

without defalcation, value received.

William Smith.

EXCHANGE.

279. Exchange is the method of making payments in distant places by means of *Drafts* or *Bills of Exchange.*

280. A **Draft** is a written order for the payment of money.

The **Drawer** is the person who signs it.

The **Drawee** is the person requested to pay.

The **Payee** is the person to whom it is payable.

The **Face** is the sum named in the draft.

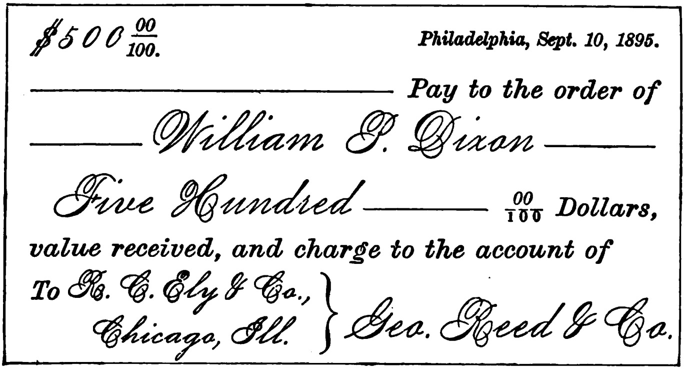
281. There are two kinds of exchange, *Domestic* and *Foreign*. Exchange between places in the same country is called *Domestic Exchange;* exchange between places in different countries is called *Foreign Exchange.*

In domestic exchange a bill is usually called a *Draft;* in foreign exchange a bill is called a *Bill of Exchange.*

282. A **Sight Draft** is one payable on presentation.

In some States, however, three days of grace are allowed. Bankers' drafts omit the words "at sight," and are payable on demand, without grace.

FORM OF A SIGHT DRAFT.



$\$500\frac{00}{100}.$ *Philadelphia, Sept. 10, 1895.*

Pay to the order of

William P. Dixon

Five Hundred $\frac{00}{100}$ *Dollars,*

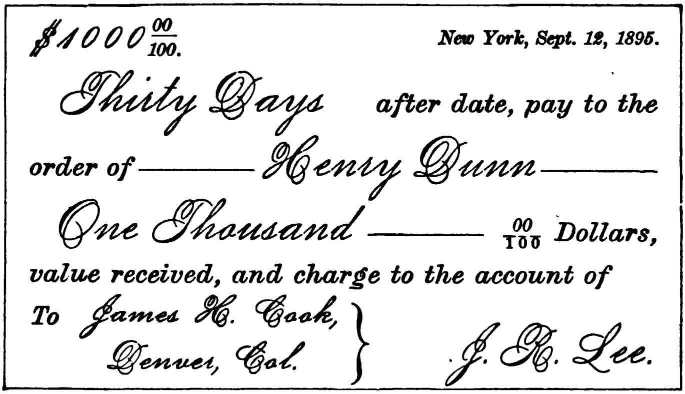
value received, and charge to the account of

To R. C. Ely & Co.,
Chicago, Ill. } *Geo. Reed & Co.*

Suppose William P. Dixon of Philadelphia owes James King of Chicago $500. He procures from the banking-house of Geo. Reed & Co. the above draft, on the back of which he writes "Pay to the order of James King," and signs his name. He then forwards the draft to James King, who presents it to R. C. Ely & Co., bankers, for payment.

283. A **Time Draft** is one payable at a specified time after sight or after the date of the draft.

FORM OF A TIME DRAFT.



\$1000$\frac{00}{100}$. *New York, Sept. 12, 1895.*

Thirty Days *after date, pay to the*

order of ——— Henry Dunn ———

One Thousand ——— $\frac{00}{100}$ *Dollars,*

value received, and charge to the account of

To James H. Cook,
Denver, Col. } J. R. Lee.

284. Acceptance of a time draft is agreeing to pay it when due. This is done by the *drawee* writing "Accepted" across the face, with the date and his name.

285. Exchange is said to be at a **Premium** or **Above Par** when a draft sells for more than its face value.

Exchange is said to be at a **Discount** or **Below Par** when a draft sells for less than its face value.

The rate of exchange between two places depends upon the relative conditions of trade at those places. If the trade between Boston and New Orleans is equal, the exchange is at *par.* If Boston owes New Orleans, the demand in Boston for drafts on New Orleans is greater than the supply, and they are therefore sold at a *premium;* but if New Orleans owes Boston, the supply of drafts on New Orleans in Boston is greater than the demand, and drafts on New Orleans, therefore, are sold at a *discount.*

DOMESTIC EXCHANGE.

286. To find the cost of a draft.

WRITTEN EXERCISES.

1. Find the cost of a sight draft for $800 when exchange is $1\frac{1}{2}\%$ discount.

PROCESS.

$1.000
.015 = Rate of exchange.
.985 = Cost of $1.
800
$788.000, *Ans.*

At a discount of $1\frac{1}{2}\%$ the cost of $1 is $1 − $1\frac{1}{2}$ cents = $.985.

The cost of $800 will be 800 times $.985, which are $788.

2. Find the cost of a draft for $800, due 60 days after date, exchange being $1\frac{1}{2}\%$ premium.

PROCESS.

$1.00
.01 = Dis. for 60 da.
.99 = Cost of $1 at par.
.015 = Premium.
1.005 = Cost of $1.
800
$804.000, *Ans.*

The bank discount of $1 for 60 da. is $.01.

The cost of $1 at par is $1 − $.01 = $.99.

The cost of $1, with the premium, is $.99 + $.015, or $1.005.

The cost of $800 will be 800 times $1.005, which are $804.00.

3. Find the cost of a sight draft for $2000, exchange being $1\frac{1}{4}\%$ premium.

4. Find the cost of a sight draft for $1600 when the exchange is 2% discount.

5. Find the cost of a draft for $5000, due 30 days after date, if exchange is $1\frac{3}{4}\%$ discount.

287. To find the face of a draft.

WRITTEN EXERCISES.

1. What is the face of a sight draft which can be bought for $3045 when exchange is at $1\frac{1}{2}\%$ premium?

PROCESS.

$1.000
.015 = Premium.
$1.015 = Cost of $1.

$1.015)$3045.($3000, *Ans.*
3045

If the premium is 1½%, the cost of $1 will be $1.015.

If the cost of the draft is $3045, its face will be as many dollars as $1.015 is contained times in $3045, which are $3000.

2. Find the face of a 60-day draft, with interest at 6%, which can be bought for $4030 when exchange is at a premium of 1¾%.

PROCESS.

6)60
.01 = Dis. of $1.
$1.00
.01
.99 = Cost of $1 at par.
.0175
1.0075 = Cost of $1.

$1.0075)$4030($4000, *Ans.*
4030

The bank discount of $1 for 60 da. is $.01.

The proceeds of $1 is $.99.

The cost of $1 of exchange is $.99 + $.0175 = $1.0075.

If the cost of the draft is $4030, its face will be $4030 divided by $1.0075, which is $4000.

3. What is the face of a sight draft which can be bought for $484.80 when exchange is at 1% premium?

4. What is the face of a sight draft which can be bought for $5174.40 when exchange is at 2% discount?

5. What is the face of a draft, due 60 days after sight, with interest at 6%, which can be bought for $1173 when exchange is 1¼% discount?

6. What is the face of a draft, due 90 days after sight, with interest at 5%, which can be bought for $1809 when exchange is 1¾% premium?

7. I can purchase a 60-day draft on San Francisco for $7830 when exchange is 1⅛% discount; what is the face?

FOREIGN EXCHANGE.

288. A bill in foreign exchange is usually called a **Bill of Exchange.**

Bills of Exchange are usually made out in sets of three bills, all of the same tenor and date, called the *First*, *Second*, and *Third* of exchange, respectively.

To prevent loss and delay, each bill of a set is sent by a different mail.

If any one of these is paid, the others become void.

289. Exchange on *Great Britain* or *Ireland* is quoted at the value of a pound sterling in United States money.

Thus, sight drafts on a bank or banker in Great Britain or Ireland are quoted, "Bankers' Sterling, sight, $4.86¼." Drafts drawn on merchants, due 60 days after sight, are quoted, "Commercial Bills, 60 days, $4.82½."

Exchange on *France* is quoted at the number of francs in a dollar.

Thus, sight drafts on France are quoted, "Francs, sight, 5.16½," meaning that it takes 5.16½ francs to make a dollar in United States money.

Exchange on *Germany* is quoted at the value of 4 reichsmarks in cents.

Thus, sight drafts on Germany are quoted, "Reichsmarks, sight, 95¾," meaning that 4 reichsmarks are worth $.95¾.

WRITTEN EXERCISES.

1. What will be the cost of a bill of exchange on London for £200 when exchange is quoted at 4.83?

PROCESS.

$4.83
200
$966.00, *Ans.*

Since each pound sterling costs $4.83, £200 will cost 200 times $4.83, which are $966.

2. What will a bill of exchange on Paris cost for 1000 francs, exchange being quoted at $5.16\frac{1}{2}$?

PROCESS.

```
5.165)1000.00($193.61+, Ans.
       5165
      -----
      48350
      46485
      ------
       18650
       15495
       ------
        31550
        30990
        ------
         5600
```

Since $1 will buy 5.165 francs, to buy 1000 francs it will take as many dollars as 5.165 is contained times in 1000, which are $193.61+.

3. What is the cost of a bill of exchange on Berlin for 2000 marks, exchange at 94?

PROCESS.

```
  $.94
   500
-------
$470.00, Ans.
```

If 4 reichsmarks cost 94 cents, 2000 reichsmarks, which is 500 times 4 reichsmarks, will cost 500 times $.94, which are $470.

4. Find the cost of a draft on Liverpool for £500, exchange being $4.85\frac{1}{4}$.

5. Find the cost of a draft for £1000 on London, exchange at 60 days being $4.84\frac{1}{2}$.

6. Find the face of a draft on Glasgow which costs $1556, exchange being $4.86\frac{1}{4}$.

7. What will a draft on Paris for 5200 francs cost when exchange is 5.16?

8. What will a draft on Havre for 840 francs cost when exchange is $5.16\frac{3}{4}$?

9. A merchant paid $420 for a draft on Paris, exchange at $5.18\frac{1}{4}$; what was the face?

10. How much must be paid for a draft on Berlin for 6000 reichsmarks, exchange being 96?

11. Find the cost of a draft on Hamburg for 12600 reichsmarks, exchange at $93\frac{1}{4}$.

DUTIES OR CUSTOMS.

290. Duties, or Customs, are taxes levied on imported goods.

A **Specific Duty** is a fixed tax on goods without regard to their cost.

An **Ad-valorem Duty** is a certain per cent. of the cost of the goods in the country from which they are imported.

Tare is an allowance made for the weight of the box, cask, etc. containing the goods.

Breakage is an allowance for the loss of liquors in bottles. *Leakage* is an allowance for the loss of liquors in casks.

Gross Weight is the weight before any allowances are made. *Net Weight* is the weight after all allowances have been made.

WRITTEN EXERCISES.

1. A merchant imported 400 yards of dress goods, invoiced at 8 francs a yard; what is the duty at 50% ad valorem?

PROCESS.

400
8
3200 francs.
.193
$618 = Cost of goods.
.50
$309.00, *Ans.*

If 1 yard cost 8 francs, 400 yards will cost 3200 francs.

Since 1 franc is worth $.193, 3200 francs are worth $617.6, or $618.

The duty is 50%, or .50 of $618, which is $309.

2. What is the duty at 24 cents a gallon on 300 quart bottles of oil, breakage 5%?

PROCESS.

300
.95
285
.06
$17.10, *Ans.*

Since the breakage is 5%, duty must be paid on 100% − 5%, or 95%, of 300 bottles, which are 285 bottles.

The duty on 1 bottle is 6 cents, and the duty on 285 bottles is $17.10.

3. What is the duty, at 3 cents a pound, on 3750 pounds of tin, tare 10%?

4. What is the duty on 12 boxes of raisins, each weighing 50 pounds, valued at 10 cents a pound, 10% tare, duty 8% ad valorem?

5. What is the duty, at 25% ad valorem, on 50 tons of steel, tare 20 pounds to the cwt., invoiced at 18 cents a pound?

6. What is the duty, at 45 cents a square yard and 40% ad valorem, on 800 yards of carpet, $\frac{3}{4}$ of a yard wide, invoiced at 10 francs a yard?

ANNUITIES.

291. An **Annuity** is a definite sum of money payable at regular intervals of time.

The **Amount,** or **Final Value,** of an annuity is the sum of the amounts of all the payments on interest from the time each is due until the annuity ceases.

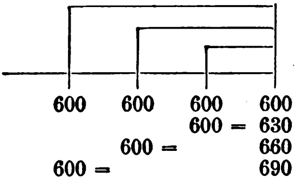
The **Present Value** of an annuity is a sum which, if put on interest for the given time and rate, will amount to the final value.

292. Annuities at Simple Interest.

WRITTEN EXERCISES.

1. What is the final value of an annuity of \$600, payable annually, for 4 yr. at 5%?

ILLUSTRATION.



The last payment will draw no interest; the last but one will draw interest for 1 year; and so on.

It is thus seen that an annuity at simple interest forms an arithmetical progression in which the annuity is the first term, the interest for 1 year is the common difference, the number of years is the number of terms, and the final value is the sum of the series. Therefore, by ART. 269,

The sum of the terms = (\$600 + \$690) $\frac{4}{2}$ = \$2580, *Ans.*

2. Find the present value of an annuity of $800 for 10 yr., at 5%, payable annually.

PROCESS.

$$\text{The last term} = \$800 + 9 \times \$40 = \$1160$$

$$\text{The final value} = (\$800 + \$1160)\tfrac{10}{2} = \$9800$$

$$\text{The present value} = \frac{\$9800}{1.50} = \$6533\tfrac{1}{3},\ \textit{Ans.}$$

RULE.

Take the annuity for the first term, the interest for 1 *year for the common difference, and the time for the number of terms; find the last term, and then the sum of the terms; this sum will be the final value.*

Then find the present value of the final value.

Find the present value—

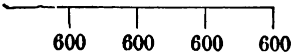
3. Of an annuity of $400 for 5 yr. at 6%.
4. Of an annuity of $600 for 6 yr. at 6%.
5. Of an annuity of $1200 for 10 yr. at 7%.
6. Of an annuity of $1600 for 8 yr. at 4%.
7. Of an annuity of $2000 for 8 yr. at $5\frac{1}{2}$%.
8. Of an annuity of $3000 for 10 yr. at 8%.

293. Annuities at Compound Interest.

WRITTEN EXERCISES.

1. Find the final value of an annuity of $600 for 4 yr. at 5% compound interest.

ILLUSTRATION.



$600\ (1.05)$
$600\ (1.05)^2$
$600\ (1.05)^3$

The last payment will draw no interest; the last but one will draw interest for 1 yr.; the last but two will draw compound interest for 2 yr.; and so on.

It is thus seen that an annuity at compound interest forms a geometrical progression in which the annuity is the first term, 1 plus the rate of interest is the ratio, the number of years is the number of terms, and the final value is the sum of the series. Therefore, by ART. 271,

$$\text{The last term} = \$600\,(1.05)^3.$$

By ART. 272,

$$\text{The sum} = \frac{\$600\,(1.05^4 - 1)}{.05} = \frac{\$600}{.05}(1.05^4 - 1).$$

The number in parenthesis is the compound interest of $1 for 4 yr. at 5%, which by the table on page 208 is .215506. Hence

$$\text{The sum} = \$12000 \times .215506 = \$2586.072,\ \textit{Ans.}$$

2. Find the present value of an annuity of $500 for 6 yr. at 5%.

$$\text{Final value} = \frac{\$500}{.05}(1.05^6 - 1).$$

By using the table on page 208, $(1.05^6 - 1) = .340096$. Hence the

$$\text{Final value} = \$10000 \times .340096 = \$3400.96.$$

We find the present value by dividing by 1.340096, the compound amount of $1 for 6 yr. at 5%. Hence the

$$\text{Present value} = \frac{\$3400.96}{1.340096} = \$2537.85.$$

RULE.

To find the final value, divide the annuity by the rate, and multiply the quotient by the compound interest for the given time and rate.

To find the present value, divide the final value by the compound amount of 1 *dollar for the given time and rate.*

Find the present value—

3. Of an annuity of $800 for 7 yr. at 4%.

4. Of an annuity of $1200 for 8 yr. at 6%.

5. Of an annuity of $1500 for 5 yr. at 5%.

6. Of an annuity of $1600 for 9 yr. at 8%.

7. Of an annuity of $2500 for 10 yr. at 5%.

BUILDING ASSOCIATIONS.

294. Building and Loan Associations, or *Co-operative Banks*, as they are sometimes called, are corporations which receive small deposits at regular periods of time, and invest them in mortgages among their own members or stockholders.

295. The **Members** of an association are the persons who either subscribe for shares as an investment or borrow money of the association. The former are called "*non-borrowers*," and the latter "*borrowers*."

296. The **Shares** are usually $200, and are issued periodically in series. When the sum of the regular payments and profits equals the par value of a share, the series is closed.

297. The **Dues**, or **Installments**, consist of *one dollar a month* on each share held by the members.

298. The **Premium** is the amount bid by a member in excess of simple interest for the purpose of securing the loan.

MODES OF LOANING MONEY.

299. There are three modes of loaning money and fixing the interest used by different associations, known as the *Installment Plan*, the *Gross Plan*, and the *Net Plan*.

By the **Installment Plan** the borrower receives the *par value* of a share, and pays the premium in monthly installments, together with the dues and interest.

ILLUSTRATION.—In borrowing $200 the borrower would receive the full amount and pay 6% interest, or $1 a month, together with the dues and monthly premium.

By the **Gross Plan** the borrower receives only the difference between the face value of the loan and the premium bid, but pays interest on the full amount borrowed.

Ill.—In borrowing $200 at a premium of 25%, the borrower receives $200 less $50, or $150, and pays 6% interest on $200, or $1 a month, together with the monthly dues.

By the **Net Plan** the premium is deducted as in the *gross plan*, but the borrower is charged interest only on the amount he receives.

Ill.—In borrowing $200 at 25% premium the borrower receives $200 less $50, or $150, and pays 6% interest on this amount—that is, $9 a year, or 75 cts. a month—together with the monthly dues.

300. Fines are penalties imposed for non-payment of dues or interest. By the Building Association Laws of Pennsylvania fines cannot exceed 2% a month on all arrears, and no fine can be imposed on unpaid fines.

301. The **Withdrawal Value** of a share is found by adding to the amount paid in on the stock, 6% interest on this sum, and such a share of the profits as is determined by the rules of the association.

CASE I.

302. To find the cost of any number of shares.

1. What is the cost of 8 shares of stock in a building association which has been running 6 yr. 4 mo., payments $1 per month?

Process.	
6 yr. 4 mo. = 76 mo. 76 × $8 = $608, *Ans.*	The dues on 8 shares for 1 mo. will be $8, and for 6 yr. 4 mo., or 76 mo., they will be 76 times $8, or $608.

2. What is the actual cost of 20 shares of stock in a building association which has been running 5 yr. 8 mo., payments $1 per month?

3. I own 10 shares in the 1st series, 16 in the 2d, 20 in the 3d of the West End Building and Loan Association; if the series are 8, 9, and 10 yr. respectively in maturing, how much has my stock cost me, having been fined twice at 2% a month for non-payment of dues?

CASE II.

303. To find the amount of arrears and fines due.

1. A owns 12 shares in a building association, and fails to make his payments for 6 successive months; what is the amount of arrears due, including fines at 2% a month?

PROCESS.

$$\begin{array}{rr} S = (.02 + .12)\,3 = & \$.42 \\ 6 \times \$1 = & 6.00 \\ \hline & \$6.42 \\ & 12 \\ \hline & \$77.04, \textit{ Ans.} \end{array}$$

The fines on 1 share for 6 successive months will be 2 cts. + 4 cts. + 6 cts. + 8 cts. + 10 cts. + 12 cts. It is thus seen to be the sum of an arithmetical progression whose first term is 2 cts., last term 12 cts., and number of terms 6; hence $S = (.02 + .12)\,3 = \$.42$.

The dues on 1 share for 6 mo. are $6; hence the arrears and fines on 1 share are $6.42, and on 12 shares the amount will be $77.04.

2. Mr. Good, having 15 shares in a building association which matured in 10 yr. 4 mo., was fined for 5 successive months for non-payment of dues; how much did he pay into the association, including fines at 2% a month?

3. What is the net profit on 20 shares of stock in a building association which expires in 9 yr. 8 mo., the shares being worth $200 at maturity, and fines 2% a month, the holder having been fined for non-payment of dues for 6 successive months?

4. What is the net profit on 30 shares of stock in a building association which matures in 8 yr. 9 mo., the shares being worth $200 at maturity, if the dues are $1 a month and fines 2% a month, the holder being fined on two occasions for non-payment of dues, the first time for 5 successive months, and the second time for 8 successive months?

CASE III.

304. To find the present value of shares without the profit.

1. What is the present value of 10 shares of stock in an association which has been running for 5 yr. 6 mo., interest at 6%?

PROCESS.

5 yr. 6 mo. = 66 mo.

$$\begin{array}{lr} \text{Int.} = \dfrac{66 \times 67}{4} = & \$11.055 \\ \text{Amt. paid in} \quad = & 66.000 \\ \hline & \$77.055 \\ & 10 \\ \hline & \$770.55, \textit{ Ans.} \end{array}$$

The time is 66 mo.

The first payment on a share, $1, is on interest 66 months, the second payment 65 months, and so on; hence the interest on $1 for the different periods equals the interest on $1 for a number of months represented by an arithmetical progression of which the first term is $1, the last term $66, and the number of terms 66, or ½ (66 × 67). The interest of $1 for 1 mo. at 6% is ½ ct., and for ½ (66 × 67) mo. is $\frac{66 \times 67}{4}$ cts., or $11.055.

The present value of 1 share is $66 plus $11.055, or $77.055, and the value of 10 shares is $770.55.

2. What is the value of 32 shares of stock in the Quaker City Building Association paid up for 8 yr. 7 mo., interest 6%?

3. A owns 12 shares in an association, first series, and 16 shares in the third series; what is his stock worth if the first series is paid up for 7 yr. 6 mo., interest 5%, and the third series is paid up for 5 yr. 8 mo., interest 5%?

CASE IV.

305. To find the actual cost of a loan to a borrower.

1. B bought a loan of 20 shares in a new series of an association, gross plan, at $15 per share premium; what will be the actual cost of the loan if the series matures in 8 years?

PROCESS.

$2 × 20 = $40, Mo. payment.

$$40\,\frac{(96 \times 97)}{4}\text{ cts.} = \$931.20\text{, Int. }6\%.$$

$40 × 96 = $3840, Sum of payments.

$3840 + $931.20 = $4771.20, *Ans.*

The monthly payment on 1 share equals $1 dues and $1 interest, or $2, and on 20 shares the payment is $40.

The first payment is on interest for 96 mo., the second 95 mo., and so on; hence the interest on the payments will be $931.20.

The sum of the payments equals $40 × 96 = $3840, and the loan costs $931.20 + $3840, or $4771.20.

2. A bought a loan of 30 shares, new series, in an association at $75 premium, net plan; if the series matures in 9 yr., what is the actual cost of his loan?

3. B bought a loan of 20 shares, new series, in an association on the installment plan at 50 cents a month premium; if the series matures in 9 yr., what is the actual cost of the loan?

4. M bought a loan of 30 shares, new series, in an association at $75 premium, net plan; if the series matures in 8 years, what is the actual cost of his loan?

CASE V.

306. To find the rate of interest received by a non-borrower.

1. D owns 8 shares in an association, dues $1 a share; if the series matures in 8 yr. 6 mo., what rate of interest does he receive?

PROCESS.

8 yr. 6 mo. = 102 mo.

$200 − $102 = $98, Profit.

$$\frac{102 \times 103}{24} = \$4.37\tfrac{3}{4}\text{, Int. }1\%.$$

$$\$98 \div \$4.37\tfrac{3}{4} = 22.38+$$

22.38%, *Ans.*

The sum of the payments on 1 share for 8 yr. 6 mo. is $102; hence the gain is $200 − $102, or $98.

The interest on $1 a month for 102 months at 1% is $4.37¾. The rate is therefore $98 divided by $4.37¾, or 22.38+%.

2. What is the gain % on the stock of an association which matures in 10 yr.?

3. What is the gain % on the stock of an association which matures in 12 yr.?

4. By the annual report of the West End Building Association, made at the end of the 7th year, the present value of the first series is $160.40; what was the rate of gain at that time?

5. N owns 10 shares of stock in a building association which matures in 8 yr. 8 mo.; what rate of interest does he receive on his investment?

CASE VI.

307. To find the interest paid by a borrower.

1. A buys a loan of 10 shares, 60 cents premium a month, on the installment plan; what rate of interest will he pay if the series matures in $8\frac{1}{3}$ yr.?

PROCESS.

$\$2.60 \times \frac{100 \times 101}{4} = \$65.65$, Int. on 1 share at 6%.

$\$2.60 \times 100 = \$260$, Amount of payments on 1 share.

$\$260 + \$65.65 = \$325.65$, Actual cost of 1 share.

$\$325.65 - \$200 = \$125.65$, Int. on 1 share for $8\frac{1}{3}$ yr.

$\$125.65 \div 8\frac{1}{3} = \$15.07$, Average int. for 1 yr.

$\$15.07 \div \$200 = .075+$, or $7\frac{1}{2}\%$, Rate of interest.

2. B buys a loan of $3000 at 55 cts. premium a month on the installment plan; what are his monthly payments, and what average rate will he pay if the series matures in 9 yr. 2 mo.?

3. D buys a loan of 10 shares on the gross plan at $60 premium, new series; what rate of interest does he pay if the series matures in $8\frac{1}{3}$ yr.?

4. E buys a loan of 12 shares, net plan, at the beginning of a series, at $50 premium per share; what rate of interest does he pay if the loan matures in 10 years?

5. F buys a loan of 20 shares in an association, at 65 cts. a month premium, at the beginning of the 4th year; what rate of interest does he pay if the series matures in 9 yr.?

THE METRIC SYSTEM

OF WEIGHTS AND MEASURES.

308. THE **Metric System** of weights and measures originated in France in 1795, and was adopted by the leading countries of Europe and South America, in many of which it was made compulsory. In 1864, Great Britain by act of Parliament permitted its use throughout the empire, and similar action was taken by the United States Congress in 1866. Although the system is very simple, it has not been generally adopted in this country, its use being confined principally to the sciences and to the operations of the Coast Survey department.

309. The *Metric System* is based upon the decimal system of notation.

The *lower denominations* are expressed by writing before the name of the unit

Milli—, to denote *one-thousandth* of the unit.
Centi—, to denote *one-hundredth* of the unit.
Deci—, to denote *one-tenth* of the unit.

The *higher denominations* are expressed by writing before the names of the unit

Deca—, to denote *ten* times the unit.
Hecto—, to denote *one hundred* times the unit.
Kilo—, to denote *one thousand* times the unit.
Myria—, to denote *ten thousand* times the unit.

The denominations printed in bold type in each table are in most frequent use.

310. Measures of Length.

The **Meter** is the *unit of length.*

NOTE.—The meter was intended to be, and is very nearly, the ten-millionth part of the distance from the Equator to the North Pole, as ascertained by actual measurement of the arc of a meridian.

TABLE.

10 **millimeters** ($^{mm.}$) = 1 **centimeter**, $^{cm.}$
10 centimeters = 1 decimeter, $^{dm.}$
10 decimeters = 1 **meter**, $^{m.}$
10 meters = 1 decameter, $^{Dm.}$
10 decameters = 1 hectometer, $^{Hm.}$
10 hectometers = 1 **kilometer**, $^{Km.}$
10 kilometers = 1 myriameter, $^{Mm.}$

EQUIVALENTS.

1 meter = 39.37 in.
1 kilometer = .6214 mi.

1 yard = .9144$^{m.}$
1 mile = 1.6093$^{Km.}$

Cloth, etc. are measured by the *meter*, long distances by the *kilometer*, and very small distances by the *millimeter.*

ORAL EXERCISES.

How many—

1. Meters in a kilometer? Hectometer? Myriameter?

2. Millimeters in a meter? Decameter? Decimeter?

3. Centimeters in a meter? Hectometer? Kilometer?

4. Decimeters in a meter? Decameter? Hectometer?

5. What is measured by the meter? Kilometer? Millimeter?

Approximately—

6. How many inches in a meter?

7. What part of a mile is a kilometer?

311. Measures of Area.

The **Square Meter** is the *unit of area.*

TABLE.

100 **square millimeters** (sq. mm.) = 1 **square centimeter,** sq. cm.
100 square centimeters = 1 square decimeter, sq. dm.
100 square decimeters = 1 **square meter,** sq. m.
100 square meters = 1 square decameter, sq. Dm.
100 square decameters = 1 square hectometer, sq. Hm.
100 square hectometers = 1 **square kilometer,** sq. Km.
100 square kilometers = 1 square myriameter, sq. Mm.

The square decameter is called an **are** (a.).

100 ares = 1 **hectare,** Ha.

EQUIVALENTS.

1 square meter = 1.196 sq. yd.
1 sq. kilometer = .3861 sq. mi.
1 are = .0247 A.

1 square yard = .8361 sq. m.
1 square mile = 2.59 sq. Km.
1 acre = 40.47 a.

Surfaces of ordinary objects are measured by the *square meter,* land as in farms is measured by the *are* and *hectare,* while larger areas are measured by the *square kilometer.*

ORAL EXERCISES.

How many—

1. Square meters in a square hectometer?
2. Square centimeters in a square meter?
3. Square meters in a square kilometer?
4. Square millimeters in a square meter?
5. Square miles in 10 square kilometers?
6. Ares are equivalent to an acre?
7. Acres in 10 ares?
8. Ares in 100 acres?

312. Measures of Volume.

The **Cubic Meter**, sometimes called the *stere* ($^{\text{st.}}$), is the *unit of volume.*

TABLE.

1000 cubic millimeters ($^{\text{cu. mm.}}$) = 1 **cubic centimeter,** $^{\text{cu. cm.}}$
1000 cubic centimeters = 1 cubic decimeter, $^{\text{cu. dm.}}$
1000 cubic decimeters = 1 **cubic meter,** $^{\text{cu. m.}}$

EQUIVALENTS.

1 cubic meter = 1.308 cu. yd.
1 stere = .2759 cord.
1 cubic yard = .7645 $^{\text{cu. m.}}$
1 cord = 3.624 $^{\text{st.}}$

The contents of ordinary solids are measured by the *cubic meter;* wood is measured by the *stere.*

313. Measures of Capacity.

The **Liter** is the *unit of capacity.* It equals a *cubic decimeter.*

TABLE.

10 milliliters ($^{\text{ml.}}$) = 1 **centiliter,** $^{\text{cl.}}$
10 centiliters = 1 deciliter, $^{\text{dl.}}$
10 deciliters = 1 **liter,** $^{\text{l.}}$
10 liters = 1 decaliter, $^{\text{Dl.}}$
10 decaliters = 1 **hectoliter,** $^{\text{Hl.}}$
10 hectoliters = 1 kiloliter, $^{\text{Kl.}}$
10 kiloliters = 1 myrialiter, $^{\text{Ml.}}$

EQUIVALENTS.

1 liter = 1.0567 liq. qt.
1 liter = .9081 dry qt.
1 hectoliter = 2.8376 bu.
1 hectoliter = 26.417 gal.
1 liquid qt. = .9463 $^{\text{l.}}$
1 dry qt. = 1.101 $^{\text{l.}}$
1 bushel = .3524 $^{\text{Hl.}}$
1 liter = 61.022 cu. in.

Liquids are measured by the *liter*, and grains, etc. are measured by the *hectoliter.*

ORAL EXERCISES.

How many—

1. Liters in a hectoliter? In a kiloliter?
2. Deciliters in a liter? In a decaliter?
3. Liters in a cubic meter? In a stere?
4. Centiliters in a liter? In a hectoliter?
5. Milliliters in a liter? In a kiloliter?

Approximately—

6. What part of a hectoliter is a bushel?
7. How many liters in a gallon? In a bushel?

314. Measures of Weight.

The **Gram** is the *unit of weight.* It is the weight of a cubic centimeter of distilled water at the temperature of 39.2° Fahrenheit.

TABLE.

10 milligrams ($^{\text{mg.}}$) = 1 centigram, $^{\text{cg.}}$
10 centigrams = 1 decigram, $^{\text{dg.}}$
10 decigrams = 1 gram, $^{\text{g.}}$
10 grams = 1 decagram, $^{\text{Dg.}}$
10 decagrams = 1 hectogram, $^{\text{Hg.}}$
10 hectograms = 1 kilogram, $^{\text{Kg.}}$
10 kilograms = 1 myriagram, $^{\text{Mg.}}$
10 myriagrams = 1 quintal, $^{\text{q.}}$
10 quintals = 1 metric ton, $^{\text{T.}}$

EQUIVALENTS.

1 gram = 15.432 gr.
1 kilogram = 2.2046 lb. av.
1 metric ton = 1.1023 T.

1 grain = .0648 $^{\text{g.}}$
1 lb. av. = .4536 $^{\text{Kg.}}$
1 T. = .9072 $^{\text{T.}}$

The *gram* is used in weighing light articles and in mixing medicines, the *kilogram* in weighing ordinary articles, and the *metric ton* in weighing heavy articles.

ORAL EXERCISES.

How many—

1. Grams in a kilogram? In a hectogram?
2. Milligrams in a gram? In a kilogram?
3. Decigrams in a decagram? In a hectogram?
4. Centigrams in a hectogram? In a kilogram?

NUMERATION AND NOTATION.

315. A denominate number expressed in the metric system is called a **Metric Number.**

A *metric number* when expressed in a single denomination may be read in either of two ways, as follows: $64.25^{\text{m.}}$ may be read 64 and 25 hundredths meters, or 64 meters and 25 centimeters.

A *metric number* may be expressed in terms of any denomination. Thus, $6^{\text{Dm.}}$, $4^{\text{m.}}$, $2^{\text{dm.}}$, $5^{\text{cm.}}$, may be expressed as $6.425^{\text{Dm.}}$, $64.25^{\text{m.}}$, $642.5^{\text{dm.}}$, or $6425^{\text{cm.}}$

EXERCISES IN NUMERATION.

Read the following:

1. $45.26^{\text{Dm.}}$
2. $816.35^{\text{m.}}$
3. $416.5^{\text{cl.}}$
4. $42.167^{\text{Hg.}}$
5. $481.64^{\text{g.}}$
6. $23.25^{\text{sq. cm.}}$
7. $43.14^{\text{cu. m.}}$
8. $1432^{\text{cu. dm.}}$
9. $.534^{\text{Dm.}}$
10. $.000378^{\text{cg.}}$
11. $.02134^{\text{Mg.}}$
12. $1478^{\text{ml.}}$
13. $43.18^{\text{Kl.}}$
14. $.0482^{\text{sq. Hm.}}$
15. $.0164^{\text{cu. cm.}}$

EXERCISES IN NOTATION.

1. Write 8 meters and 7 centimeters in terms of meters. In terms of decimeters. In terms of centimeters.

PROCESS.
$8.07^{\text{m.}}$
$80.7^{\text{dm.}}$
$807^{\text{cm.}}$

Write the number beginning with the unit of measure of each table, and then place the decimal point so that the required denomination is in the units' place.

2. Write 18 decameters, 4 meters, 6 decimeters, and 7 centimeters in terms of decameters. In terms of meters. In terms of decimeters. In terms of centimeters.

3. Write 16 hectometers, 6 decameters, and 5 centimeters in terms of meters. In terms of decameters. In terms of decimeters.

4. Write $8^{\text{Dl.}}$, $5^{\text{l.}}$, $4^{\text{cl.}}$, $3^{\text{ml.}}$ in terms of liters. In terms of decaliters. In terms of milliliters.

5. Write $6^{\text{Kg.}}$, $4^{\text{Dg.}}$, $6^{\text{g.}}$, $7^{\text{cg.}}$, and $5^{\text{mg.}}$ in terms of grams. In terms of decigrams. In terms of decagrams.

6. Write $64^{\text{sq. Dm.}}$, $45^{\text{sq. m.}}$, and $18^{\text{sq. cm.}}$ in terms of square meters. In terms of square decimeters.

7. Write $8^{\text{cu. Dm.}}$, $7^{\text{cu. m.}}$, and $6^{\text{cu. dm.}}$ in terms of cubic meters. In terms of cubic decameters.

8. Write $14^{\text{cu. Dm.}}$, $137^{\text{cu. m.}}$, and $375^{\text{cu. cm.}}$ in terms of cubic meters. In terms of cubic decimeters.

REDUCTION OF METRIC NUMBERS.

316. The reduction of *metric numbers* may be effected by the following principles:

Prin. 1.—*A metric number may be reduced to a lower denomination by moving the decimal point one place to the right for each decrease in the denomination.*

Thus, $64.416^{\text{Dl.}} = 644.16^{\text{l.}} = 6441.6^{\text{dl.}} = 64416^{\text{cl.}}$, etc.

Prin. 2.—*A metric number may be reduced to a higher denomination by moving the decimal point one place to the left for each increase in the denomination.*

Thus, $6345^{\text{cm.}} = 634.5^{\text{dm.}} = 63.45^{\text{m.}} = 6.345^{\text{Dm.}}$, etc.

These principles do not apply to the tables of square and cubic measure. *In these tables the decimal point must be moved*

two *places for each change in the denomination of square measure, and* **three** *places for each change in the denomination of cubic measure.*

Thus, 45.6747$^{\text{sq. Dm.}}$ = 4567.47$^{\text{sq. m.}}$ = 456747$^{\text{sq. dm.}}$, etc.;

148$^{\text{cu. mm.}}$ = .148$^{\text{cu. cm.}}$ = .000148$^{\text{cu. dm.}}$, etc.

WRITTEN EXERCISES.

Reduce—

1. 7.321$^{\text{mg.}}$ to centigrams.
2. 4567$^{\text{Dg.}}$ to decigrams.
3. 86754$^{\text{cm.}}$ to kilometers.
4. 47654$^{\text{mm.}}$ to decameters.
5. 47375$^{\text{cl.}}$ to hectoliters.
6. 6.4765$^{\text{Kl.}}$ to centiliters.
7. 6756$^{\text{cm.}}$ to meters.
8. 6.476$^{\text{Hl.}}$ to liters.
9. 93456$^{\text{cg.}}$ to kilograms.
10. .478654$^{\text{Mg.}}$ to centigrams.
11. 8476543$^{\text{sq. m.}}$ to square kilometers.
12. 4.786953$^{\text{cu. m.}}$ to cubic centimeters.
13. 95432$^{\text{cu. mm.}}$ to cubic decimeters.
14. .00579$^{\text{Kl.}}$ to liters.
15. 48.567$^{\text{Mg.}}$ to metric tons.

16. What is the weight in decagrams of a liter of distilled water at 39.2° Fah.?

17. What is the weight in kilograms of a stere of distilled water?

18. Reduce 45.16$^{\text{cu. cm.}}$ to centiliters.

19. Reduce 42.87$^{\text{Ha.}}$ to ares.

20. What is the weight of 16.51$^{\text{cu. m.}}$ of water in myriagrams?

21. How many centiliters of water will weigh 184$^{\text{mg.}}$?

22. How many decaliters of water will weigh 16.84$^{\text{Kg.}}$?

ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION OF METRIC NUMBERS.

317. If metric numbers be reduced to decimals of the same denomination, they may be added, subtracted, multiplied, and divided as ordinary denominate decimals.

WRITTEN EXERCISES.

1. Add 46$^{\text{dm.}}$, 2.737$^{\text{Dm.}}$, and 1.82453$^{\text{Hm.}}$

Reducing to the decimal of a meter and adding, we have

$$\begin{array}{rcl} 46^{\text{dm.}} & = & 4.6^{\text{m.}} \\ 2.737^{\text{Dm.}} & = & 27.37 \\ 1.82453^{\text{Hm.}} & = & \underline{182.453} \\ & & 214.423^{\text{m.}},\ \textit{Ans.} \end{array}$$

2. Subtract 277.94$^{\text{dg.}}$ from 14.5786$^{\text{Dg.}}$

Reducing to the decimal of a gram and subtracting, we have

$$\begin{array}{rcl} 14.5786^{\text{Dg.}} & = & 145.786^{\text{g.}} \\ 277.94^{\text{dg.}} & = & \underline{27.794} \\ & & 117.992^{\text{g.}},\ \textit{Ans.} \end{array}$$

3. Multiply 432.15$^{\text{dl.}}$ by 4, and express the result in decaliters.

Reducing to the decimal of a liter and multiplying, we have

$$\begin{array}{r} 43.215^{\text{l.}} \\ \underline{4} \\ 172.860^{\text{l.}} = 17.286^{\text{Dl.}},\ \textit{Ans.} \end{array}$$

4. Divide 1562.5$^{\text{cm.}}$ by 1.25, and express the result in meters.

Reducing to the decimal of a meter and dividing, we have

$$\begin{array}{l} 1.25)1562.5^{\text{cm.}}(1250^{\text{cm.}} = 12.5^{\text{m.}},\ \textit{Ans.} \\ \quad\quad\ \ \underline{125} \\ \quad\quad\ \ \ 312 \\ \quad\quad\ \ \ \underline{250} \\ \quad\quad\quad\ 625 \\ \quad\quad\quad\ \underline{625} \end{array}$$

5. Divide $320.84^{\text{dg.}}$ by $.26^{\text{Dg.}}$

Reducing to the decimal part of a gram and dividing, we have

$$2.6^{\text{g.}})32.084^{\text{g.}}(12.38,\ \textit{Ans.}$$

$$\begin{array}{r}\underline{26}\\ 60\\ \underline{52}\\ 88\\ \underline{78}\\ 104\\ \underline{104}\end{array}$$

Add—

6. $432.14^{\text{dm.}}$, $10.2325^{\text{Dm.}}$, $62.8^{\text{m.}}$, and $7512^{\text{cm.}}$

7. $8214^{\text{cl.}}$, $3278^{\text{Hl.}}$, $.6213^{\text{Kl.}}$, and $4156^{\text{ml.}}$

8. $46.352^{\text{Dg.}}$, $12.34^{\text{dg.}}$, $40400^{\text{cg.}}$, and $213^{\text{mg.}}$

9. $2.48213^{\text{sq. Dm.}}$, $12467.8^{\text{sq. dm.}}$, and $356.789^{\text{sq. m.}}$

10. $.621543^{\text{cu. Dm}}$, $321.767^{\text{cu. m.}}$, and $143254^{\text{cu. dm.}}$

Subtract—

11. $456.7^{\text{dl.}}$ from $11.6214^{\text{Hl.}}$

12. $.12371^{\text{Km.}}$ from $48564^{\text{cm.}}$

13. $97004^{\text{mg.}}$ from $1.25782^{\text{Hg.}}$

14. $678^{\text{cu. dm.}}$ from $.432^{\text{cu. Dm.}}$

15. $10830.2^{\text{sq. dm.}}$ from $8.64321^{\text{sq. Dm.}}$

Multiply—

16. $4.62^{\text{dg.}}$ by 1.23, and express the result in decaliters.

17. $4.3216^{\text{Hm.}}$ by 2.4, and express the result in decimeters.

18. $3216^{\text{cl.}}$ by 23.6, and express the result in kiloliters.

19. $1.3578^{\text{sq. Dm.}}$ by 6.3, and express the result in square millimeters.

20. $.432^{\text{cu. Dm.}}$ by .71, and express the result in cubic centimeters.

Divide—

21. $86.751^{\text{Kl.}}$ by $9^{\text{Dl.}}$

22. $31.629^{\text{Dg.}}$ by $39^{\text{mg.}}$

23. $.9672^{\text{Km.}}$ by $26^{\text{dm.}}$

24. $8.11332^{\text{Km.}}$ by $372^{\text{mm.}}$

25. $1788.57^{\text{cg.}}$ by 6.3, and express the result in kilograms.

26. $1.05156^{\text{Dm.}}$ by .56, and express the result in centimeters.

318. To reduce a metric number to a common denominate number, or a common denominate number to a metric number.

WRITTEN EXERCISES.

1. Reduce 4.228$^{\text{Dm.}}$ to feet.

PROCESS.

4.228$^{\text{Dm.}}$ = 42.28$^{\text{m.}}$

42.28
3.28 ft.
33824
8456
12684
138.6784 ft., *Ans.*

4.228$^{\text{Dm.}}$ are equal to 42.28$^{\text{m.}}$ In 1 meter there are 3.28 ft., and in 42.28$^{\text{m.}}$ there are 42.28 times 3.28 ft., or 138.6784 feet.

2. Reduce 4 rd. and 6 ft. to meters.

PROCESS.

4 rd. 6 ft. = 72 ft.

1 ft. = .3048$^{\text{m.}}$
72
6096
21336
21.9456$^{\text{m.}}$

4 rd. and 6 ft. equal 72 ft. In 1 foot there are .3048$^{\text{m.}}$, and in 72 feet there are 72 times .3048$^{\text{m.}}$, or 21.9456$^{\text{m.}}$

Measures of Length.

Reduce—

3. 8.05$^{\text{Dm.}}$ to feet.
4. 2.046$^{\text{Km.}}$ to yards.
5. 80$^{\text{Km.}}$ to miles.
6. 320 yards to decameters.
7. 5 miles to hectometers.
8. 2000 feet to kilometers.

Measures of Surface.

Reduce—

9. 600$^{\text{a.}}$ to acres.
10. 4.21$^{\text{a.}}$ to square yards.
11. .635$^{\text{sq. Mm.}}$ to sq. rods.
12. 1260 sq. yd. to sq. Dm.
13. 2 A. and 25 P. to ares.
14. 640 sq. mi. to sq. kilometers.

Measures of Volume.

Reduce—

15. 120$^{\text{st.}}$ to cubic feet.
16. 315600$^{\text{cu. dm.}}$ to cubic yards.
17. 256 cu. ft. to steres.
18. 48 cu. yd. and 15 cu. ft. to steres.

Measures of Capacity.

Reduce—

19. 12.05$^{\text{Dl.}}$ to gallons.
20. 72.56$^{\text{Hl.}}$ to bushels.
21. 362.5$^{\text{dl.}}$ to liquid quarts.
22. 63 gal. to decaliters.
23. 50 bushels to hectoliters.
24. 2 gallons to centiliters.

Measures of Weight.

Reduce—

25. 50$^{\text{g.}}$ to grains.
26. 87.654$^{\text{Dg.}}$ to lb. av.
27. 12.8624$^{\text{Kl.}}$ to lb. av.
28. 8625 grains to hectograms.
29. 15 lb. av. to kilograms.
30. 83$^{\text{T.}}$ to hundredweights.

PROBLEMS.

1. What will 16.25$^{\text{Dm.}}$ of cloth cost at $2.50 a meter?

2. What will 567.8$^{\text{Hg.}}$ of beef cost at 30 cents a kilogram?

3. If 10$^{\text{st.}}$ of wood cost $25, what will 36.5$^{\text{st.}}$ cost?

4. If 12$^{\text{l.}}$ of coal oil cost 75 cents, what will 5 hectoliters cost?

5. If a man travels 75$^{\text{Hm.}}$ in an hour, how long will it take him to travel 3$^{\text{Mm.}}$?

6. If cloth is bought at \$4 a meter and sold at \$5 a yard, how much is gained on 60 meters?

7. If wood is bought at \$2 a stere and sold at \$8 a cord, how much is gained on 1200 steres?

8. If molasses is bought at 50 cents a gallon and sold at 20 cents a liter, how much is gained on the sale of a hogshead containing 63 gallons?

9. If wheat is bought at 60 cents a bushel and sold at \$2 a hectoliter, how much is gained on the sale of 100 bushels?

10. If land is bought at \$200 an acre and sold at \$10 an are, how much is gained on the sale of 150 acres?

11. A tailor buys cloth at \$4.50 a yard; at what price per meter must he sell it to gain 50 cents a yard?

12. A merchant buys sugar at 5 cents a pound; at what price per kilogram must he sell it to gain 2 cents a pound?

13. A barrel of flour weighs 196 pounds; what metric number expresses its weight?

14. Express in metric numbers the rate of a train running 30 miles an hour.

15. A farm contains 150 acres; what is its area in ares?

16. A car weighs 56000 pounds; what is its weight in metric tons?

17. If the longitudes of two places on the equator differ by 10° 40′, what is the distance between the two places in kilometers, a degree being $69\frac{1}{6}$ miles?

18. If the distance to the moon is 240000 miles, how many kilometers is it from the earth?

19. If 40 meters of cloth cost \$240, what will 60 yards cost?

20. If a bar of metal weighs 24 lb. to the foot, how many kilograms does it weigh to the decameter?

ANSWERS TO WRITTEN EXERCISES.

Page 19.—*Ex. 3.* 157; *4.* 142; *5.* 182; *6.* 189; *7.* 180; *8.* 193; *9.* 206; *10.* 156; *11.* 174; *12.* 112; *13.* 133; *14.* 124; *15.* 226; *16.* 173; *17.* 142; *18.* 203; *19.* 199; *20.* 249; *21.* $1270; *22.* $733; *23.* $1647; *24.* $2085; *25.* $17.26; *26.* $10.10; *27.* $17.41; *28.* $12.88.

Page 20.—*Ex. 29.* 88; *30.* 223; *31.* 277; *32.* 170; *33.* 107; *34.* 126; *35.* 185; *36.* 1675; *37.* 2506; *38.* 1946; *39.* 1203; *40.* 2123; *41.* 1313; *42.* 1488; *43.* 168; *44.* 180; *45.* 158; *46.* 75; *47.* 161; *48.* 89; *49.* 181; *50.* 3191; *51.* 3238; *52.* 3191; *53.* 2886; *54.* 3092; *55.* 2350; *56.* 2583; *57.* 3172; *58.* 2174; *59.* 1482; *60.* 2360; *61.* 969; *62.* 851; *63.* 1763; *64.* 992; *65.* 1441; *66.* 2545; *67.* 2255; *68.* 2176; *69.* 2280; *70.* 2162; *71.* 2452; *72.* 2126; *73.* 1045; *74.* 1471; *75.* 2469; *76.* 20061; *77.* 30074; *78.* 22529; *79.* 31662; *80.* 33576; *81.* 31370; *82.* 104367.

Page 21.—*Ex. 83.* 2564; *84.* 3448; *85.* 2856; *86.* 3117; *87.* 1948; *88.* 2000; *89.* 1814; *90.* 2272; *91.* 2552; *92.* 1399; *93.* 3279; *94.* 3652; *95.* 2431; *96.* 4018; *97.* 1598; *98.* 2257; *99.* 1944; *100.* 1612; *101.* 2858; *102.* 3111; *103.* 226155.———*1.* $1051; *2.* 706 miles; *3.* 20694 lb.; *4.* 2999 eggs; *5.* $89.88; *6.* 264 flowers, $19.35; *7.* 489.

Page 22.—*Ex. 8.* $38.50; *9.* $599.15; *10.* $104.44; *11.* 1,343,374; *12.* $166,450; *13.* $8775; *14.* $8360; *15.* 5,760,585.

Page 23.—*Ex. 16.* $1,432,420,266; *17.* $688,965,549,176.65; *18.* 1230; *19.* 4560; *20.* 7890; *21.* 51762; *22.* 58434.

Page 24.—*Ex. 23.* 8325; *24.* 7665; *25.* 91804; *26.* 91337; *27.* $87951.51.

Page 25.—*Ex. 28.* 912880.

Page 28.—*Ex. 2.* 164; *3.* 441; *4.* 511; *5.* 253; *6.* 653; *7.* 624; *8.* 2223; *9.* 7721; *10.* 1331; *11.* 1412; *12.* 6350; *13.* $7722.02; *14.* $56320.42; *15.* $42431.02; *16.* 35502; *17.* 50223; *18.* 40432; *19.* 60000.———*1.* $160; *2.* $1410; *3.* 4345 men; *4.* $5504; *5.* 1520 bu.; *6.* $5530; *7.* $1120; *8.* $2130.

Page 29.—*Ex. 4.* 425; *5.* 207; *6.* 176; *7.* 738; *8.* 257; *9.* 187; *10.* 368; *11.* 178; *12.* 286; *13.* 265.

Page 30.—*Ex. 14.* $106.08; *15.* $598.91; *16.* $360.16; *17.* $682.50; *18.* $243.66; *19.* 37139; *20.* 56288; *21.* 31346; *22.* 44328; *23.* 63; *24.* 448; *25.* 126; *26.* 240; *27.* 648; *28.* 523; *29.* 322; *30.* 197; *31.* 330; *32.* 522; *33.* 1143; *34.* 754; *35.* 1031; *36.* 354; *37.* 652; *38.* 1054; *39.* 312; *40.* 588; *41.* 780; *42.* 134; *43.* 322; *44.* 712; *45.* 1108; *46.* 230; *47.* 3324; *48.* 724; *49.* 27747; *50.* 460702; *51.* 493889; *52.* 810299; *53.* 383807; *54.* 132790; *55.* 403322; *56.* 530865; *57.* 5,741,182; *58.* 8,727,606; *59.* 1,731,502; *60.* 6,999,767; *61.* $58,388,316; *62.* $35,132,192; *63.* $1,326,249.67; *64.* $1,228,611; *65.* 51; *66.* 186; *67.* 392; *68.* 132; *69.* 253; *70.* 631; *71.* 202; *72.* 733; *73.* 880; *74.* 1596; *75.* 2326; *76.*

5629; *77.* 7389; *78.* 135346; *79.* 325; *80.* 3697; *81.* 3333; *82.* 6728; *83.* 5939; *84.* 642; *85.* 3205; *86.* 2193; *87.* 2907; *88.* 2214; *89.* 999; *90.* 3758.

Page 31.—*Ex. 91.* 1894; *92.* 13416; *93.* 6245; *94.* 2090; *95.* 97,991; *96.* $36.26.——*1.* $4470; *2.* 2860; *3.* 3812; *4.* 21636; *5.* $7832; *6.* 4455; *7.* $7617.25; *8.* $300; *9.* $824; *10.* $339; *11.* $2373.

Page 32.—*Ex. 12.* $4,465,497.65; *13.* $812665.89; *14.* 50141; *15.* $40,084,066.14.

Page 35.—*Ex. 3.* 69; *4.* 180; *5.* 305; *6.* 615; *7.* 4936; *8.* 9648; *9.* $25.38; *10.* $37.24; *11.* $49.12; *12.* $47.88; *13.* $9.72; *14.* $257.30; **[4]** *15.* 8544; *16.* 12568; *17.* 24568; *18.* 32864; *19.* 21728; *20.* 6472; *21.* 17,264; *22.* 34836; *23.* 21968; *24.* 24688; *25.* 20024; *26.* 12480; *27.* 24200; *28.* 28356; *29.* 20568; *30.* 16864; *31.* 31452; *32.* 18268; **[5]** *33.* 10680; *34.* 15710; *35.* 30710; *36.* 41080; *37.* 27160; *38.* 8090; *39.* 21580; *40.* 43545; *41.* 27460; *42.* 30860; *43.* 25030; *44.* 15600; *45.* 30250; *46.* 35445; *47.* 25710; *48.* 21080; *49.* 39315; *50.* 22835; **[6]** *51.* 12816; *52.* 18852; *53.* 36852; *54.* 49296; *55.* 32592; *56.* 9708; *57.* 25896; *58.* 52254; *59.* 32952; *60.* 37032; *61.* 30036; *62.* 18720; *63.* 36300; *64.* 42534; *65.* 30852; *66.* 25296; *67.* 47178; *68.* 27402; **[7]** *69.* 14952; *70.* 21994; *71.* 42994; *72.* 57512; *73.* 38024; *74.* 11326; *75.* 30212; *76.* 60963; *77.* 38444; *78.* 43204; *79.* 35042; *80.* 21840; *81.* 42350; *82.* 49623; *83.* 35994; *84.* 29512; *85.* 55041; *86.* 31969; **[8]** *87.* 17088; *88.* 25136; *89.* 49136; *90.* 65728; *91.* 43456; *92.* 12944; *93.* 34528; *94.* 69672; *95.* 43936; *96.* 49376; *97.* 40048; *98.* 24960; *99.* 48400; *100.* 56712; *101.* 41136; *102.* 33728; *103.* 62904; *104.* 26536; **[9]** *105.* 19224; *106.* 28278; *107.* 55278; *108.* 73944; *109.* 48888; *110.* 14562; *111.* 38844; *112.* 78381; *113.* 49428; *114.* 55548; *115.* 45054; *116.* 28080; *117.* 54450; *118.* 63801; *119.* 46278; *120.* 37944; *121.* 70767; *122.* 41103.

Page 36.—*Ex. 123.* 1230; *124.* 2945; *125.* 2736; *126.* 4734; *127.* 3934; *128.* 4746; *129.* 2856; *130.* 3312; *131.* 12030; *132.* 18270; *133.* 29470; *134.* 9728; *135.* 170247; *136.* 1,730,496; *137.* 28,459,548; *138.* 200,138,105; *139.* 1,111,111,101; *140.* 8,888,888,889.——*1.* $1485.90; *2.* 10080 ft.; *3.* 1,490,640 mi.; *4.* 36960 ft.; *5.* $130.50; *6.* $2837.25; *7.* $25734.40; *8.* 1,680,000 mi.; *9.* 63400 mi.; *10.* 15840 yd.; *11.* $42.75.

Page 37.—*Ex. 3.* 3024; *4.* 5580; *5.* 16192; *6.* 13650; *7.* $416.50; *8.* $703.50; *9.* $1031.22; *10.* $2004.66; *11.* 10624; *12.* 23604; *13.* 22875; *14.* 40664.

Page 38.—*Ex. 15.* 4878; *16.* 6552; *17.* 12768; *18.* 10998; *19.* 34782; *20.* 21216; *21.* 31476; *22.* 28560; *23.* 56826; *24.* 39042; *25.* 40824; *26.* 44030; *27.* 49950; *28.* 39641; *29.* 38552; *30.* 67068; **[16]** *31.* 3456; *32.* 4832; *33.* 6656; *34.* 6688; *35.* 5056; *36.* 8384; *37.* 9888; *38.* 14528; *39.* 13792; *40.* 6912; *41.* 9072; *42.* 14288; *43.* 6736; *44.* 7712; *45.* 14736; *46.* 10224; *47.* 6640; *48.* 12624; *49.* 3744; *50.* 11280; *51.* 15696; *52.* 7824; *53.* 9952; *54.* 14880; **[56]** *55.* 12096; *56.* 16912; *57.* 23296; *58.* 23408; *59.* 17696; *60.* 29344; *61.* 34608; *62.* 50848; *63.* 48272; *64.* 24192; *65.* 31752; *66.* 50008; *67.* 23576; *68.* 26992; *69.* 51576; *70.* 35784; *71.* 23240; *72.* 44184; *73.* 13104; *74.* 39480; *75.* 54936; *76.* 27384; *77.* 34832; *78.* 52080; **[84]** *79.*

18144; *80.* 25368; *81.* 34944; *82.* 35112; *83.* 26544; *84.* 44016; *85.* 51912; *86.* 76272; *87.* 72408; *88.* 36288; *89.* 47628; *90.* 75012; *91.* 35364; *92.* 40488; *93.* 77364; *94.* 53676; *95.* 34860; *96.* 66276; *97.* 19656; *98.* 59220; *99.* 82404; *100.* 41076; *101.* 52248; *102.* 78120; **[92]** *103.* 19872; *104.* 27784; *105.* 38272; *106.* 38456; *107.* 29072; *108.* 48208; *109.* 56856; *110.* 83536; *111.* 79304; *112.* 39744; *113.* 52164; *114.* 82156; *115.* 38732; *116.* 44344; *117.* 84732; *118.* 58788; *119.* 38180; *120.* 72588; *121.* 21528; *122.* 64860; *123.* 90252; *124.* 44988; *125.* 57224; *126.* 85560; **[123]** *127.* 26568; *128.* 37146; *129.* 51168; *130.* 51414; *131.* 38868; *132.* 64452; *133.* 76014; *134.* 111684; *135.* 106026; *136.* 53136; *137.* 69741; *138.* 109839; *139.* 51783; *140.* 59286; *141.* 113283; *142.* 78597; *143.* 51045; *144.* 97047; *145.* 28782; *146.* 86715; *147.* 120663; *148.* 60147; *149.* 76506; *150.* 114390; **[234]** *151.* 50544; *152.* 70668; *153.* 97344; *154.* 97812; *155.* 73944; *156.* 122616; *157.* 144612; *158.* 212472; *159.* 201708; *160.* 101088; *161.* 132678; *162.* 208962; *163.* 98514; *164.* 112788; *165.* 215514; *166.* 149526; *167.* 97110; *168.* 184626; *169.* 54756; *170.* 164970; *171.* 229554; *172.* 114426; *173.* 145548; *174.* 217620.——*178.* 430848; *179.* 286875; *180.* 66,636; *181.* 458763; *182.* 1,259,268; *183.* 504944; *184.* 2,560,272; *185.* 757442; *186.* 1,126,248; *187.* 464370; *188.* 1,966,707; *189.* 4,122,841; *190.* 6,440,772; *191.* 26,685,165; *192.* 16,148,945; *193.* 6,526,083; *194.* 9,084,096; *195.* 14,903,838; *196.* 75,762,742; *197.* 54,361,883; *198.* 19,387,500; *199.* 297,892,224; *200.* 293,365,653; *201.* 129,030,680.

Page 39.—*Ex. 1.* $16524; *2.* $4320; *3.* $28,575; *4.* $1680.75; *5.* 27,454 gal.; *6.* 1,173,840 lb.; *7.* $3997.50; *8.* $1716.75; *9.* 353275 men; *10.* $21091.84; *11.* $11014.12; *12.* 2,694,580 lb.; *13.* 157680 hr.; *14.* 13570 mi.; *15.* $3304.35.

Page 40.—*Ex. 16.* $2754.72; *17.* $92,954,802.12; *18.* 67,715,672; *19.* 62,982,234; *20.* $8,467,313.19; *21.* 37,944,731; *22.* $44,186,296; *23.* $944,733,510; *24.* 795041; *25.* 1,555,515; *26.* 4,390,009; *27.* 6,982,534; *28.* 103,770,134; *29.* 139,063,041; *30.* 2,597,716; *31.* 3,232,042; *32.* 9,333,654; *33.* 120,884,412; *34.* 15,223,824; *35.* 30,417,442; *36.* 1,111,111,101; *37.* 2,222,222,202; *38.* 3,333,333,303; *39.* 4,444,444,404; *40.* 5,555,555,505; *41.* 8,888,888,889; *42.* 17,777,777,778; *43.* 26,666,666,667; *44.* 35,555,555,556; *45.* 44,444,444,445; *46.* 90,810,081; *47.* 10,079,918,991; *48.* 90,900,891,081; *49.* 1,019,090,909; *50.* 3,087,542,754.

Page 41.—*Ex. 6.* 129,000; *7.* 884,000; *8.* 16,064,000; *9.* 3,604,200,000; *10.* 603,203,200.——*1.* 1,056,000 ft.; *2.* 840000 gr.; *3.* 3,097,600 sq. yd.

Page 42.—*Ex. 4.* 920000 lb.; *5.* $30000; *6.* 132,000,000 ft.; *7.* 24000 sq. rd.——*1.* $112; *2.* $1377; *3.* $2,220,000; *4.* 86400 sec.; *5.* 13440 bu.; *6.* 864360 lb.; *7.* $887500; *8.* 28086.

Page 45.—*Ex. 3.* 288; *4.* 227; *5.* 321; *6.* 375; *7.* 344$\frac{2}{5}$; *8.* $324; *9.* $428; *10.* $759; *11.* $7.11; *12.* $20.04.

Page 46.—*Ex. 13.* 1189; *14.* 1659; *15.* 671; *16.* 1156; *17.* 432; *18.* 456; *19.* 208; *20.* 540$\frac{1}{8}$; *21.* 957$\frac{8}{9}$; *22.* 439$\frac{2}{7}$; *23.* 507$\frac{4}{8}$; *24.* 1646; *25.* 847$\frac{3}{5}$; *26.* 996$\frac{6}{9}$; *27.* 162; *28.* 143; *29.* 134$\frac{1}{5}$; *30.* 141; *31.* 112$\frac{2}{7}$;

32. $122\frac{4}{8}$; *33.* $142\frac{8}{9}$; *34.* $400\frac{6}{8}$; *35.* 660; *36.* $603\frac{3}{6}$; *37.* 801; *38.* 2227; *39.* $20085\frac{1}{3}$; *40.* $20541\frac{3}{4}$; *41.* $16017\frac{1}{5}$; *42.* $20104\frac{2}{6}$; *43.* 60123; *44.* $124999\frac{7}{8}$; *45.* 123606; *46.* $80906\frac{2}{9}$; *47.* 160800; *48.* 5761; *49.* 12235; *50.* 8189; *51.* 10304; *52.* 44670; *53.* 231646; *54.* 961639; *55.* 1,125,956; *56.* 526959.——*1.* 209 yd.; *2.* $52\frac{1}{7}$ wk.; *3.* 1210 pk.; *4.* 312 tons; *5.* 256 wk.

Page 47.—*Ex. 6.* 196 sheep; *7.* $3312.50; *8.* 1120 ft.; *9.* 186330 mi.; *10.* $400\frac{4}{12}$ yr.——*2.* 41; *3.* 44; *4.* 48; *5.* 52; *6.* 57; *7.* 61; *8.* 52; *9.* 39; *10.* 42; *11.* 34; *12.* 35; *13.* 36; *14.* 41; *15.* 27; *16.* 38; *17.* 32; *18.* 31; *19.* 33.

Page 48.—*Ex. 20.* 52; *21.* 48; *22.* 45; *23.* 52; *24.* 35; *25.* 61; *26.* 72; *27.* 32; *28.* 123; *29.* 182; *30.* 92; *31.* 88; *32.* 217; *33.* 136; *36.* 105; *37.* 204; *38.* 302; *39.* 405; *40.* 85; *41.* 45; *42.* 372; *43.* 417; *44.* 274; *45.* 217; *46.* 61; *47.* 67; *48.* 62; *49.* 216; *50.* 321; *51.* 315; *52.* 1993; *53.* 312; *54.* 562; *55.* 321.

Page 49.—*Ex. 56.* 3201; *57.* 2296; *58.* $3246\frac{141}{246}$; *59.* 4445; *60.* 2341; *61.* 3060; *62.* 630; *63.* 4006; *64.* $1512\frac{268}{312}$; *65.* 1280; *66.* $454\frac{248}{512}$; *67.* 402; *68.* 846; *69.* $1969\frac{137}{521}$; *70.* $620\frac{116}{848}$; *71.* 3241; *72.* 8531; *73.* $927\frac{1408}{8462}$; *74.* $2226\frac{3143}{3784}$; *75.* 3431; *76.* 5136; *77.* $$19721\frac{2499}{4216}$; *78.* $$837.97\frac{6931}{11521}$; *79.* $12.76.——*1.* 48 bbl.; *2.* 123 bbl.; *3.* $214; *4.* 605 gal.; *5.* 576 bu.; *6.* 378 sq. ft.; *7.* 316 acres.

Page 50.—*Ex. 8.* 31680 rev.; *9.* 62; *10.* $$136\frac{360}{365}$; *11.* 234 cu. ft.; *12.* 66 horses; *13.* 63 gal.; *14.* 3200 hr.

Page 51.—*Ex. 39.* 28 – 2 rem.; *40.* 28 – 6 rem.; *41.* 14 – 15 rem.; *42.* 13 – 30 rem.; *43.* 13 – 9 rem.; *44.* 12 – 22 rem.; *45.* 12 – 3 rem.; *46.* 10 – 85 rem.; *47.* 23 – 25 rem.; *48.* 20 – 200 rem.; *49.* 23; *50.* 17 – 230 rem.; *51.* 1 – 162 rem.; *52.* 1 – 241 rem.; *53.* 1 – 184 rem.; *54.* 1 – 62 rem.; *55.* 2 – 216 rem.; *56.* 2 – 2645 rem.; *57.* 2 – 1321 rem.; *58.* 1 – 4026 rem.; *59.* 1 – 2285 rem.; *60.* 1 – 2005 rem.; *61.* 1 – 1260 rem.; *62.* 1 – 462 rem.

Page 52.—*Ex. 64.* 39 – 747 rem.; *65.* 13 – 3014 rem.; *66.* 16 – 384 rem.; *67.* 23 – 2754 rem.; *68.* 10 – 4432 rem.; *69.* 3512 – 120 rem.; *70.* 44 – 385 rem.; *71.* 30 – 5321 rem.; *72.* 14 – 10248 rem.; *73.* 62 – 127 rem.; *74.* 31 – 137 rem.; *75.* 16 – 1326 rem.——*1.* 876 d. – 5 cts.; *2.* 52 mi. – 48 ch.; *3.* 26 acres – 52 perches; *4.* 432 boxes; *5.* 431 tons – 1125 lb.; *6.* 16 lb. – 6125 gr.; *7.* 120 lb.; *8.* 82 mi.; *9.* 3 hr. – 2150 sec.

Page 53.—*Ex. 2.* 225; *3.* 920; *4.* 42; *5.* 15060; *6.* 37284; *7.* 231; *8.* 11; *9.* 30; *10.* 922.——*1.* 687655; *2.* 114086; *3.* 110667; *4.* 146; *5.* 4,132,360; *6.* 216.

Page 54.—*Ex. 7.* 61; *8.* 1,056,566; *9.* 522776; *10.* 48126; *11.* 262144; *12.* 119210; *13.* $19868; *14.* $19272; *15.* 35, 42, 30; *16.* 41 – 9915 rem.; *17.* $42; *18.* 743 mi.; *19.* $342; *20.* $9.

Page 55.—*Ex. 2.* $132; *3.* $1000; *4.* $61.20; *5.* 70 days; *6.* 40 days; *7.* $1995; *8.* $17472; *9.* $18750; *10.* 10 days; *11.* 116 acres; *12.* $3010; *13.* $2552.

Page 59.—*Ex. 3.* 3 × 5 × 7; *4.* 3 × 7 × 11; *5.* 3 × 11 × 13; *6.* 2 × 5 × 5 × 11; *7.* 2 × 5 × 7 × 11; *8.* 2 × 5 × 7 × 13; *9.* 5 × 5 × 7 × 7; *10.* 2 × 3 × 5 × 7 × 11; *11.* 7 × 7 × 11 × 13; *12.* 3 × 5 × 7 ×

11; *13.* 2 × 3 × 5 × 5 × 7; *14.* 3 × 3 × 3 × 5 × 11; *15.* 2 × 2 × 2 × 7 × 11; *16.* 2 × 2 × 2 × 2 × 5 × 13; *17.* 2 × 2 × 3 × 3 × 5 × 7; *18.* 2 × 2 × 2 × 2 × 2 × 2 × 3 × 3 × 3; *19.* 2 × 3 × 3 × 3 × 5 × 7; *20.* 2 × 2 × 2 × 2 × 3 × 3 × 11; *21.* 2 × 2 × 2 × 3 × 3 × 5 × 5; *22.* 3 × 3 × 3 × 7 × 11; *23.* 2 × 3 × 3 × 3 × 3 × 3 × 5; *24.* 3 × 3 × 3 × 3 × 3 × 3 × 5; *25.* 7 × 7 × 11 × 11; *26.* 5 × 5 × 5 × 7 × 7; *27.* 2 × 2 × 3 × 3 × 5 × 5 × 7; *28.* 2 × 3 × 3 × 3 × 3 × 3 × 3 × 5; *29.* 2 × 2 × 2 × 2 × 2 × 2 × 2 × 5 × 13; *30.* 3 × 5 × 7 × 11 × 11; *31.* 5 × 5 × 5 × 11 × 11; *32.* 5 × 5 × 5 × 13 × 17; *33.* 5 × 5 × 5 × 5 × 5 × 11; *34.* 2 × 2 × 3 × 11 × 17 × 17.

Page 61.—*Ex. 7.* 2; *8.* 32; *9.* 14; *10.* 20; *11.* 15 cts.; *12.* 24 cts.

Page 62.—*Ex. 13.* 9 doz.; *14.* 60 lb.; *15.* 29 boxes; *16.* 25 cts.

Page 64.—*Ex. 3.* 10; *4.* 12; *5.* 15; *6.* 22; *7.* 24; *8.* 30; *9.* 35; *10.* 2; *11.* 84; *12.* 105; *13.* 210; *14.* 156; *15.* 42; *16.* 32; *17.* 36; *18.* 30; *19.* 24; *20.* 64; *21.* 72; *22.* 35; *23.* 56; *24.* 45; *25.* 96; *26.* 105; *27.* 120; *28.* 240; *29.* 210; *30.* 15 acres; *31.* 16 ft.; *32.* 35 bu.; *33.* 16 acres, 123 fields.

Page 66.—*Ex. 3.* 23; *4.* 31; *5.* 41; *6.* 47; *7.* 53; *8.* 61; *9.* 67; *10.* 83; *11.* 1; *12.* 31; *13.* 203; *14.* 112; *15.* 123; *16.* 141; *17.* 61; *18.* 41; *19.* 101; *20.* 71; *21.* 103; *22.* 41; *23.* 97; *24.* 61; *25.* 147; *26.* 639; *27.* 1; *28.* 513; *29.* 4; *30.* 41; *31.* 83.

Page 68.—*Ex. 3.* 120; *4.* 180; *5.* 120; *6.* 720; *7.* 2520; *8.* 792; *9.* 1008; *10.* 660; *11.* 12096; *12.* 1560; *13.* 1320; *14.* 3600; *15.* 2240; *16.* 17640; *17.* 1920; *18.* 3780; *19.* 3000; *20.* 3600.

Page 69.—*Ex. 21.* $3872; *22.* 240 acres; *23.* $54; *24.* 2016 gal.; *25.* 360 min.

Page 70.—*Ex. 3.* 2871; *4.* 10013; *5.* 19803; *6.* 13889; *7.* 15466; *8.* 16399; *9.* 16983; *10.* 10065; *11.* 6699; *12.* 2130; *13.* 7704; *14.* 27025; *15.* 14443; *16.* 58835; *17.* 110837; *18.* 13419; *19.* 24255; *20.* 485377.——*1.* 2 × 2 × 3 × 3 × 5 × 5 × 7 × 7; *2.* $23, A 11 cows, B 13 cows; *3.* 720 rd., A 80 da., B 60 da., C 48 da., D 45 da.

Page 71.—*Ex. 4.* 59; *5.* 195; *6.* 3500; *7.* 17 ft.; *8.* $858; *9.* 40170; *10.* 15000; *11.* 4 qt., 48 times; *12.* 120 da.; A 15, B 12, C 10, D 8 times; *13.* 15 ft.; *14.* High, 5; grammar, 12, primary, 17.

Page 75.—*Ex. 8.* $26\frac{3}{8}$; *9.* $28\frac{4}{11}$; *10.* $25\frac{5}{12}$; *11.* $24\frac{5}{16}$; *12.* $40\frac{7}{20}$; *13.* $29\frac{1}{32}$; *14.* $56\frac{25}{33}$; *15.* $40\frac{5}{57}$; *16.* $38\frac{25}{84}$; *17.* $43\frac{35}{96}$; *18.* $66\frac{44}{73}$; *19.* $99\frac{83}{91}$; *20.* $47\frac{15}{22}$; *21.* $39\frac{7}{31}$; *22.* $47\frac{9}{33}$; *23.* $48\frac{2}{35}$; *24.* $48\frac{31}{42}$; *25.* $50\frac{17}{43}$; *26.* $65\frac{5}{44}$; *27.* $69\frac{32}{45}$; *28.* $43\frac{72}{92}$; *29.* $64\frac{5}{123}$; *30.* $47\frac{128}{131}$; *31.* $34\frac{167}{241}$; *32.* $91\frac{49}{74}$; *33.* $97\frac{41}{75}$; *34.* $61\frac{50}{111}$; *35.* $31\frac{119}{126}$; *36.* 60; *37.* $26\frac{106}{315}$; *38.* $24\frac{112}{412}$; *39.* $50\frac{166}{245}$.

Page 76.—*Ex. 7.* $\frac{1310}{21}$; *8.* $\frac{2132}{25}$; *9.* $\frac{1507}{15}$; *10.* $\frac{247}{8}$; *11.* $\frac{1546}{37}$; *12.* $\frac{2015}{32}$; *13.* $\frac{5441}{72}$; *14.* $\frac{1808}{17}$; *15.* $\frac{2584}{5}$; *16.* $\frac{271}{4}$; *17.* $\frac{680}{9}$; *18.* $\frac{629}{10}$; *19.* $\frac{811}{11}$; *20.* $\frac{903}{31}$; *21.* $\frac{2106}{25}$; *22.* $\frac{2562}{25}$; *23.* $\frac{6374}{31}$; *24.* $\frac{16707}{53}$; *25.* $\frac{4007}{10}$; *26.* $\frac{10301}{101}$; *27.* $\frac{1979}{116}$; *28.* $\frac{3699}{129}$; *29.* $\frac{4853}{47}$; *30.* $\frac{1227}{10}$; *31.* $\frac{1916}{9}$; *32.* $\frac{7732}{11}$; *33.* $\frac{3078}{5}$; *34.* $\frac{1831}{15}$; *35.* $\frac{5065}{21}$.

Page 78.—*Ex. 9.* $\frac{5}{6}$, $\frac{7}{9}$; *10.* $\frac{2}{9}$, $\frac{4}{9}$; *11.* $\frac{16}{17}$, $\frac{32}{33}$; *12.* $\frac{12}{13}$, $\frac{2}{3}$; *13.* $\frac{8}{11}$, $\frac{8}{11}$; *14.* $\frac{7}{9}$, $\frac{9}{11}$; *15.* $\frac{5}{9}$, $\frac{5}{7}$; *16.* $\frac{9}{11}$, $\frac{4}{5}$; *17.* $\frac{4}{7}$, $\frac{7}{11}$; *18.* $\frac{6}{7}$, $\frac{7}{8}$; *19.* $\frac{6}{11}$, $\frac{17}{26}$; *20.* $\frac{4}{11}$; $\frac{6}{7}$; *21.* $\frac{2}{3}$; *22.* $\frac{2}{3}$; *23.* $\frac{2}{3}$; *24.* $\frac{5}{6}$; *25.* $\frac{7}{8}$; *26.* $\frac{7}{8}$; *27.* $\frac{3}{4}$; *28.* $\frac{5}{11}$; *29.* $\frac{5}{17}$; *30.* $\frac{10}{11}$; *31.* $\frac{6}{7}$; *32.* $\frac{2}{11}$; *33.* $\frac{11}{12}$; *34.* $\frac{2}{5}$; *35.* $\frac{3}{4}$; *36.* $\frac{8}{13}$.

Page 79.—*Ex. 17.* $\frac{16}{24}, \frac{21}{24}, \frac{10}{24}$; *18.* $\frac{30}{40}, \frac{35}{40}, \frac{36}{40}$; *19.* $\frac{30}{70}, \frac{25}{70}, \frac{49}{70}$; *20.* $\frac{5}{14}, \frac{12}{14}, \frac{3}{14}$; *21.* $2\frac{5}{15}, \frac{10}{15}, \frac{12}{15}$; *22.* $5\frac{24}{72}, \frac{60}{72}, \frac{17}{72}$; *23.* $1\frac{98}{294}, \frac{126}{294}, \frac{189}{294}$ $\frac{30}{294}$; *24.* $\frac{100}{120}, \frac{105}{120}, \frac{108}{120}, \frac{110}{120}$; *25.* $\frac{48}{136}, \frac{28}{136}, \frac{16}{136}, \frac{9}{136}$; *26.* $\frac{126}{210}, \frac{98}{210}, \frac{120}{210}, \frac{27}{210}$; *27.* $\frac{15}{39}, \frac{27}{39}, \frac{7}{39}$; *28.* $\frac{9}{15}, \frac{8}{15}, \frac{8}{15}$; *29.* $\frac{378}{420}, \frac{385}{420}, \frac{390}{420}$; *30.* $\frac{189}{315}, \frac{245}{315}, \frac{75}{315}$; *31.* $\frac{84}{144}, \frac{99}{144}$ $\frac{76}{144}$; *32.* $\frac{64}{160}, \frac{56}{160}, \frac{25}{160}$; *33.* $\frac{21}{70}, \frac{65}{70}, \frac{18}{70}$; *34.* $\frac{77}{84}, \frac{45}{84}, \frac{34}{84}$.

Page 80.—*Ex. 22.* 2; *23.* $2\frac{61}{120}$; *24.* $1\frac{1}{35}$; *25.* $2\frac{5}{56}$.

Page 81.—*Ex. 26.* $2\frac{11}{84}$; *27.* $1\frac{43}{120}$; *28.* $2\frac{25}{252}$; *29.* $1\frac{17}{24}$; *30.* $2\frac{3}{10}$; *31.* $2\frac{3}{28}$; *32.* $2\frac{137}{360}$; *33.* $2\frac{23}{42}$; *34.* $1\frac{131}{135}$; *35.* $2\frac{3}{14}$; *36.* $3\frac{11}{20}$; *37.* $2\frac{131}{168}$; *38.* $2\frac{113}{280}$; *39.* $4\frac{3}{40}$; *41.* $34\frac{1}{12}$; *42.* $398\frac{1}{8}$; *43.* $160\frac{25}{84}$; *44.* $338\frac{1}{4}$; *45.* $722\frac{10}{33}$.

Page 82.—*Ex. 22.* $\frac{11}{42}$; *23.* $\frac{3}{10}$; *24.* $\frac{1}{143}$; *25.* $\frac{13}{126}$; *26.* $\frac{1}{132}$; *27.* $\frac{1}{42}$; *28.* $\frac{22}{779}$; *29.* $\frac{1}{24}$; *30.* $\frac{11}{30}$; *31.* $\frac{52}{63}$; *32.* $\frac{19}{70}$; *33.* $\frac{15}{26}$; *34.* $\frac{91}{180}$; *35.* $\frac{41}{84}$; *36.* $\frac{67}{210}$; *37.* $\frac{61}{285}$; *38.* $\frac{109}{132}$; *39.* $\frac{1}{136}$; *41.* 6; *42.* $5\frac{1}{6}$; *43.* $8\frac{13}{24}$; *44.* $13\frac{7}{30}$; *45.* $12\frac{2}{21}$; *46.* $45\frac{2}{3}$; *47.* $49\frac{7}{8}$; *48.* $55\frac{19}{24}$; *49.* $64\frac{1}{7}$; *50.* $28\frac{2}{3}$; *51.* $89\frac{71}{72}$; *52.* $374\frac{23}{30}$.

Page 83.—*Ex. 1.* $25\frac{1}{2}$ tons, $7\frac{5}{6}$ tons; *2.* $434\frac{1}{18}$ acres; *3.* \$$83\frac{33}{40}$; *4.* \$$784\frac{3}{4}$; *5.* \$$1766\frac{57}{68}$; *6.* $433\frac{27}{70}$ bu.; *7.* $110\frac{7}{72}$ bu.; *8.* \$$1712\frac{1}{4}$; *9.* $267\frac{31}{84}$ rods; *10.* \$$264\frac{1}{2}$; *11.* \$$11\frac{1}{4}$.

Page 85.—*Ex. 25.* $8\frac{1}{3}$; *26.* $16\frac{7}{8}$; *27.* 5; *28.* $10\frac{47}{58}$; *29.* $7\frac{2}{3}$; *30.* $5\frac{15}{41}$; *31.* 82; *32.* 384; *33.* $223\frac{1}{2}$; *34.* 812; *35.* $616\frac{2}{7}$; *36.* $1207\frac{1}{5}$; *37.* $4\frac{8}{10}$; *38.* $6\frac{1}{9}$; *39.* $24\frac{3}{4}$; *40.* $16\frac{7}{8}$; *41.* 21; *42.* 550; *43.* 1359; *44.* $684\frac{8}{15}$; *45.* $1066\frac{2}{3}$; *46.* $994\frac{2}{7}$; *47.* 600; *48.* $682\frac{1}{2}$; *49.* $143\frac{3}{4}$; *50.* 105; *51.* 4275; *52.* $98\frac{2}{5}$; *53.* 500; *54.* $1742\frac{1}{2}$; *55.* 2026; *56.* $3241\frac{7}{8}$; *57.* $255\frac{1}{2}$; *58.* 6266; *59.* $297\frac{1}{2}$; *60.* $255\frac{1}{9}$; *61.* 2310; *62.* $851\frac{1}{2}$; *63.* $260\frac{1}{3}$; *64.* $195\frac{6}{11}$; *65.* 167; *66.* 360.

Page 87.—*Ex. 5.* $\frac{8}{15}$; *6.* $\frac{55}{153}$; *7.* $\frac{7}{20}$; *8.* $\frac{15}{28}$; *9.* $\frac{825}{1792}$; *10.* $\frac{14}{45}$; *11.* $\frac{5}{27}$; *12.* $\frac{3}{5}$; *13.* $\frac{7}{25}$; *14.* $\frac{130}{783}$; *15.* $\frac{55}{126}$; *16.* $\frac{19}{21}$; *17.* $\frac{343}{380}$; *18.* $\frac{81}{275}$; *19.* $44\frac{19}{64}$; *20.* 12; *21.* 117; *22.* $95\frac{5}{6}$; *23.* $88\frac{2}{21}$; *24.* 260; *25.* $\frac{15}{32}$; *26.* $\frac{1}{32}$; *27.* $\frac{1}{9}$; *28.* $\frac{2}{7}$; *29.* $\frac{12}{25}$; *30.* $\frac{23}{33}$; *31.* $\frac{28}{37}$; *32.* $\frac{13}{35}$; *33.* $\frac{1}{12}$; *34.* $17\frac{3}{5}$; *35.* $\frac{84}{125}$; *36.* $\frac{27}{1375}$; *37.* $\frac{1024}{19383}$; *38.* \$46; *39.* \$$35\frac{7}{16}$; *40.* \$$3\frac{1}{2}$; *41.* \$$32\frac{41}{54}$; *42.* $478\frac{1}{6}$ cts.; *43.* \$635; *44.* \$4215; *45.* \$$173\frac{1}{3}$; *46.* 912 cts.; *47.* \$$619\frac{1}{5}$; *48.* \$$90\frac{3}{4}$.

Page 88.—*Ex. 49.* $130\frac{1}{2}$ cu. ft.; *50.* \$$1338\frac{8}{9}$; *51.* \$$9027\frac{3}{64}$; *52.* \$$18.14\frac{1}{8}$.

Page 89.—*Ex. 25.* $\frac{2}{25}$; *26.* $\frac{1}{7}$; *27.* $\frac{11}{135}$; *28.* $\frac{14}{201}$; *29.* $\frac{8}{45}$; *30.* $\frac{17}{209}$; *31.* $4\frac{5}{18}$; *32.* $7\frac{7}{24}$; *33.* $2\frac{47}{126}$; *34.* $3\frac{7}{51}$; *35.* $2\frac{1}{2}$; *36.* $2\frac{1}{2}$; *37.* $\frac{3}{50}$; *38.* $\frac{13}{21}$; *39.* $\frac{27}{437}$; *40.* $\frac{49}{375}$; *41.* $\frac{1}{100}$; *42.* $\frac{13}{804}$; *43.* $\frac{4}{45}$; *44.* $\frac{9}{1111}$; *45.* $172\frac{13}{25}$; *46.* $133\frac{43}{56}$; *47.* $91\frac{5}{77}$; *48.* $321\frac{2}{9}$; *49.* $72\frac{2}{7}$; *50.* $95\frac{3}{13}$; *51.* $136\frac{62}{165}$; *52.* $20\frac{39}{140}$ oz.; *53.* $30\frac{7}{16}$ da.

Page 90.—*Ex. 5.* 18; *6.* 90; *7.* $225\frac{1}{3}$; *8.* $93\frac{3}{14}$; *9.* 240; *10.* 80; *11.* $681\frac{3}{4}$; *12.* 462; *13.* 140.

Page 91.—*Ex. 14.* $\frac{32}{55}$; *15.* $2\frac{4}{13}$; *16.* $1\frac{28}{49}$; *17.* $\frac{7}{8}$; *18.* $2\frac{58}{99}$; *19.* $\frac{50}{77}$; *20.* $1\frac{178}{303}$; *21.* $\frac{50}{121}$; *22.* $1\frac{2}{7}$; *23.* $11\frac{5}{11}$; *24.* $4\frac{2}{3}$; *25.* $1619\frac{2}{3}$; *26.* $1371\frac{1}{9}$; *27.* $11\frac{1}{4}$; *28.* $1\frac{35}{43}$; *29.* $5\frac{5}{6}$; *30.* $1\frac{16}{49}$; *31.* $1\frac{85}{112}$; *32.* $2\frac{7}{624}$; *33.* $2\frac{62}{65}$; *34.* $7\frac{1}{2}$; *35.* $\frac{369}{1525}$; *36.* $3\frac{489}{539}$; *37.* $19\frac{47}{52}$; *38.* $\frac{35}{48}$; *39.* $1\frac{2}{9}$; *40.* $\frac{8}{567}$; *41.* $6\frac{33}{35}$; *42.* $6\frac{1}{4}$ cts.; *43.* \$6; *44.* \$$11\frac{3}{5}$; *45.* \$$6\frac{3}{4}$; *46.* $3\frac{1}{5}$ cts.; *47.* \$$4\frac{1}{5}$; *48.* \$$6\frac{7}{8}$; *49.* \$$1\frac{3}{5}$; *50.* $4\frac{4}{5}$ cts.; *51.* $4\frac{3}{8}$ cts.; *52.* \$6; *53.* $2\frac{1}{2}$ cts.; *54.* $\frac{4}{27}$; *55.* $\frac{1}{18}$; *56.* 1; *57.* \$$\frac{5}{8}$, \$125.

Page 92.—*Ex. 58.* 30 wk.; *59.* \$$1535\frac{1}{5}$; *60.* $14\frac{5}{68}, 9\frac{3}{106}, 5\frac{1}{2}$.

Page 93.—*Ex. 3.* $\frac{16}{21}, 2\frac{8}{21}$; *4.* $1\frac{2}{5}, \frac{3}{4}$; *5.* $1\frac{1}{2}$; *6.* $\frac{72}{85}$; *7.* $\frac{1755}{4096}$; *8.* $1\frac{44}{75}$; *9.* $3\frac{1}{3}$; *10.* $13\frac{43}{189}$; *11.* $\frac{94}{113}, \frac{171}{338}$; *12.* $3\frac{2}{9}$; *13.* $\frac{14}{19}$; *14.* $\frac{7}{8}$.

Page 94.—*Ex.* *2.* $\frac{1}{6}$; *3.* $\frac{1}{5}$; *4.* $\frac{11}{12}$; *5.* $\frac{1}{5}$; *6.* $\frac{3}{4}$; *7.* $\frac{2}{5}$; *8.* $\frac{7}{8}$; *9.* $\frac{2}{11}$; *11.* $\frac{3}{32}$; *12.* $\frac{2}{27}$; *13.* $\frac{1}{14}$; *14.* $\frac{1}{6}$; *15.* $\frac{5}{8}$; *16.* $\frac{5}{6}$; *17.* $\frac{1}{10}$; *18.* $\frac{1}{192}$; *20.* $\frac{5}{8}$; *21.* $\frac{7}{22}$; *22.* $\frac{3}{5}$; *23.* 1$\frac{1}{7}$; *24.* $\frac{5}{6}$; *25.* $\frac{5}{8}$; *26.* $\frac{1}{15}$; *27.* $\frac{51}{202}$.

Page 95.—*Ex.* *2.* 600; *3.* 1200; *4.* 744; *5.* 100; *6.* 450; *7.* 16$\frac{2}{3}$; *8.* 90; *9.* 150; *10* 19$\frac{1}{8}$; *11.* 115$\frac{3}{7}$; *12.* $\frac{48}{55}$; *13.* 6$\frac{3}{4}$; *14.* 19$\frac{1}{5}$; *15.* 100; *16.* 100; *17.* 100.——*2.* \$7$\frac{1}{3}$; *3.* \$1.45$\frac{1}{5}$; *4.* \$51; *5.* \$480$\frac{3}{10}$; *6.* \$56$\frac{4}{7}$.

Page 96.—*Ex.* *7.* \$325; *8.* \$5.60; *9.* \$8.80; *10.* \$1800; *11.* \$1680; *12.* \$211$\frac{1}{5}$; *13.* \$64; *14.* \$7728; *15.* 8 cts.; *16.* \$4$\frac{1}{2}$; *17.* \$3.12$\frac{1}{2}$; *18.* \$127$\frac{1}{5}$; *19.* \$134$\frac{2}{9}$; *20.* \$46$\frac{2}{3}$; *21.* 53$\frac{19}{32}$ cts.; *22.* 581 acres; *23.* \$14560; *24.* 230 cattle.

Page 97.—*Ex.* *1.* 77$\frac{17}{45}$; *2.* $\frac{38322}{91}$; *3.* $\frac{59}{61}$; *4.* $\frac{7}{9}$, $\frac{11}{14}$, $\frac{5}{6}$; *5.* 5$\frac{29}{35}$; *6.* $\frac{7}{33}$; *7.* 2$\frac{5}{231}$; *8.* 165$\frac{3}{56}$; *9.* 63$\frac{8}{9}$; *10.* $\frac{8}{9}$; *11.* $\frac{15}{143}$; *12.* 51$\frac{2}{21}$; *13.* 39$\frac{3}{8}$; *14.* 1$\frac{173}{183}$; *15.* 992$\frac{1}{84}$; *16.* 2$\frac{1}{4}$; *17.* $\frac{35335}{101}$; *18.* $\frac{127}{128}$; *19.* $\frac{43879}{7}$; *20.* $\frac{1037}{1098}$; *21,* 2$\frac{94}{125}$; *22.* 3$\frac{5}{21}$; *23.* $\frac{17}{27}$; *24.* $\frac{6}{7}$; *25.* 9$\frac{3}{16}$; *26.* $\frac{8853}{22}$; *27.* 3$\frac{15}{16}$; *28.* $\frac{312}{468}$, $\frac{416}{468}$, $\frac{429}{468}$, $\frac{432}{468}$, $\frac{90}{468}$.

Page 98.—*Ex.* *29.* 1$\frac{5}{134}$; *30.* 12$\frac{2}{5}$; *31.* 10$\frac{38}{77}$; *32.* 384; *33.* 121$\frac{33}{47}$; *34.* 3$\frac{1}{20}$; *35.* 84$\frac{14}{15}$; *36.* 1327$\frac{9}{10}$; *37.* $\frac{2}{5}$; *38.* 140; *39.* $\frac{8981}{11}$; *40.* 23$\frac{151}{180}$; *41.* 4$\frac{11}{40}$; *42.* 144$\frac{311}{492}$; *43.* 130553$\frac{13}{54}$; *44.* 14$\frac{673}{988}$; *45.* $\frac{1965}{786}$; *46.* 5$\frac{1}{3}$; *47.* 2$\frac{1}{12}$; *48.* 3$\frac{45}{47}$; *49.* 6$\frac{6}{17}$; *50.* 1$\frac{141}{763}$; *51.* 5$\frac{1}{2}$.

Page 99.—*Ex.* *1.* \$90$\frac{3}{4}$; *2.* 212$\frac{2}{3}$ lb.; *3.* \$2862$\frac{1}{2}$; *4.* 1124 ft.; *5.* \$51.90$\frac{1}{2}$; *6.* \$6$\frac{5}{6}$; *7.* \$15.43$\frac{3}{4}$; *8.* 20$\frac{90}{133}$ tons; *9.* \$79$\frac{1}{3}$; *10.* 6$\frac{12}{13}$ lb.; *11.* \$4375; *12.* \$7200; *13.* $\frac{1}{4}$; *14.* $\frac{1}{5}$; *15.* 502$\frac{6}{7}$, 684$\frac{4}{9}$; *16.* 30 days; *17.* 8$\frac{4}{7}$ hr.

Page 100.—*Ex.* *18.* \$11700; *19.* House \$5265$\frac{5}{8}$, barn \$3159$\frac{3}{8}$; *20.* 1000 bu.; *21.* 26$\frac{12}{13}$ ft.; *22.* 2$\frac{92}{129}$; *23.* \$54000; *24.* 1200 bu.; *25.* 18 mo.; *26.* \$1600; *27.* 1088 sheep.

Page 107.—*Ex.* *2.* $\frac{1}{4}$; *3.* $\frac{17}{20}$; *4.* $\frac{16}{25}$; *5.* $\frac{3}{8}$; *6.* $\frac{5}{8}$; *7.* $\frac{7}{8}$; *8.* 10$\frac{1}{10}$; *9.* 4$\frac{5}{16}$; *10.* 12$\frac{15}{16}$; *11.* 40$\frac{3}{8}$; *12.* 2$\frac{1}{800}$; *13.* 4$\frac{1}{80}$; *15.* $\frac{1}{3}$; *16.* $\frac{2}{3}$; *17.* $\frac{3}{16}$; *18.* 4$\frac{53}{150}$; *19.* 5$\frac{7}{120}$; *20.* 6$\frac{2}{15}$; *21.* 22$\frac{1}{30}$; *22.* 41$\frac{5}{12}$; *23.* 48$\frac{7}{9}$; *24.* 25$\frac{5}{6}$; *25.* 81$\frac{7}{15}$; *26.* 42$\frac{1}{11}$; *27.* 14$\frac{13}{20}$; *28.* $\frac{1}{2000}$; *29.* 1$\frac{49}{200}$; *30.* $\frac{29}{400}$; *31.* $\frac{9}{400}$; *32.* $\frac{469}{750}$; *33.* 4$\frac{3}{8}$; *34.* 4$\frac{5}{6}$; *35.* $\frac{19}{500}$; *36.* 4$\frac{1}{150}$; *37.* 17$\frac{1}{125}$; *38.* 8$\frac{2}{3}$; *39.* 12$\frac{1}{24}$; *40.* 26$\frac{1}{30}$; *41.* 75$\frac{3}{4}$; *42.* 400$\frac{7}{8}$; *43.* 375$\frac{3}{8}$; *44.* 112$\frac{1}{16}$; *45.* 2$\frac{1}{400}$; *46.* 45$\frac{11}{24}$.

Page 108.—*Ex.* *2.* .5; *3.* .25; *4.* .75; *5.* .125; *6.* .375; *7.* .625; *8.* .875; *9.* .3125; *10.* .48; *11.* .9375; *12.* .076; *13.* .1875; *14.* .64; *15.* .3$\frac{1}{3}$; *16.* .4; *17.* .24; *18.* .4375; *19.* .16$\frac{2}{3}$; *20.* .53125; *21.* 5.25; *22.* 9.75; *23.* 7.875; *24.* 18.6; *25.* 9.36; *26.* 4.4$\frac{2}{7}$; *27.* 5.6$\frac{2}{3}$; *28.* 7.7; *29.* 8.125; *30.* 9.1$\frac{1}{9}$; *31.* 75.1875; *32.* 4.421875; *33.* 17.136; *34.* 42.65625; *35.* 105.0112; *36.* .43359375; *37.* 16.015625; *38.* 4.016; *39.* 1.327$\frac{1}{7}$; *40.* 6.03875; *41.* 2.024; *42.* 4.0625; *43.* 8.7075; *44.* 1.005; *45.* 40.034.——*2.* 132.461; *3.* 410.7449; *4.* 4485.3212; *5.* 570.5437; *6.* 142.443; *7.* 7994.2494; *8.* 323.73; *9.* 202.39; *10.* 792.239; *11.* 530.115; *12.* 221.487; *13.* 219.18; *14.* 832.984; *15.* 574.823; *16.* 1848.474.

Page 109.—*Ex.* *17.* 125017.7112; *18.* 308.2463717; *19.* \$555.52; *20.* \$1933.22.——*2.* 81.68; *3.* 67.98; *4.* 114.183; *5.* 208.1093; *6.* 18.7934; *7.* 10.24166; *8.* 15.9933; *9.* 394.929191; *10.* 713.885; *11.* 751.099; *12.* 83.2522; *13.* .882346; *14.* .231255; *15.* 1.000999.

Page 110.—*Ex.* *16.* 10.90317; *17.* 9.03798; *18.* .001482; *19.* .0001;

20. 1.277214; *21.* 32.0573; *22.* 37.9959; *23.* 215.678138; *24.* .159984; *25.* 10.3199983; *26.* 399.6; *27.* 110.499 acres; *28.* 200.452 tons.

Page 111.—*Ex. 2.* 13.268; *3.* 11.928; *4.* 197.9775; *5.* 2.279624; *6.* 1.085562; *7.* .00009072; *8.* .25; *9.* .0544; *10.* .8091; *11.* .0273; *12.* .0474; *13.* .109616; *14.* .000324; *15.* .03584; *16.* .006862; *17.* .049704; *18.* .2706672; *19.* .0167805; *20.* .0017263; *21.* .0005796; *22.* .0000266; *23.* 3.474894432; *24.* .0000015984; *25.* 6.1625; *26.* .014162; *27.* 40.12; *28.* .0001403; *29.* .001302, .009424, .0003875, .0025606; *30.* .0117459, .00007533, .00006789, .000007533; *31.* 86.213, 862.13, 8621.3, 86213.

Page 113.—*Ex. 3.* 5.16; *4.* 43.2; *5.* 82160; *6.* 12345; *7.* 356; *8.* 21.24; *9.* .1257; *10.* .00023; *11.* .00022; *12.* 1.001; *13.* .223; *14.* .082; *15.* .054; *16.* .123; *17.* .082; *18.* .37; *19.* 7.1; *20.* 40.02; *21.* .67; *22.* $.123\frac{47}{59}$; *23.* .00004; *24.* 40000; *25.* .03; *26.* 20000; *27.* 16; *28.* 54321000; *29.* 80, $2.6\frac{2}{3}$; *30.* .0023, .00023, .000023; *31.* 40,000,000; *32.* .00000004; *33.* 2.4, 810, 11.1; *34.* 2.22, .302, 17.1; *35.* 10000.0001.

Page 114.—*Ex. 2.* 256; *3.* 86400; *4.* 30.864; *5.* 24210.6; *6.* 308.21; *8.* 79; *9.* 98300; *10.* 1026.6; *11.* 1234.57; *12.* 53.1.

Page 115.—*Ex. 14.* 321200; *15.* 2010.15; *16.* 45237; *17.* 1851.85; *18.* 4115.2.———*2.* 3470.08; *3.* 86.44; *4.* 30.4936; *5.* 8.55; *6.* 1285; *7.* 3999.

Page 116.—*Ex. 9.* 370; *10.* 27.61728; *11.* 142.72; *12.* 100.48; *13.* 38; *15.* 141; *16.* 360; *17.* .6486; *18.* 3.855; *19.* 26862.———*1.* $9; *2.* $2; *3.* $50; *4.* $32; *5.* $37.375; *6.* $3600; *7.* $14300; *8.* $210; *9.* $43.

Page 117.—*Ex. 10.* $7.875; *11.* $5.75; *12* $11.25.———*2.* $24.70; *3.* $44.6875; *4.* $118.32; *5.* $53.28; *6.* $1623.60; *7.* $232.80; *8.* $\$70.31\frac{1}{4}$; *9.* $\$6.41\frac{1}{4}$; *10.* $26.22; *11.* $27.82; *12.* $\$52.54\frac{1}{2}$; *13.* $\$172.456\frac{1}{4}$; *14.* $1679.40; *15.* $48.15; *16.* $482.40.

Page 118.—*Ex. 1.* $\frac{97}{1500}$; *2.* .244140625; *3.* .0998; *4.* 1.01135; *5.* 135.92; *6.* 20726; *7.* 2.713, *8.* 6.8972; *9.* $1\frac{1}{600}$; *10.* 25.3465; *11.* 1.9390575; *12.* 16754.32; *13.* 817,145,600; *14.* 60.07; *15.* 3.06168; *16.* 12.35546875; *17.* .0682154; *18.* 5,000,000,000; *19.* .000000005; *20.* 27152.1; *21.* .0293992; *22.* $\frac{1}{7}$; *23.* $\frac{1}{900}$; *24.* 117.060875; *25.* .406752; *26.* .20703125; *27.* 11.1013; *28.* $11.154\frac{2}{3}$; *29.* 3.333; *30.* 1.000001; *31.* .99; *32.* 4, .25.

Page 119.—*Ex. 33.* 358.4 acres; *34.* 436 bu.; *35.* 56278.8 lb.; *36.* 302.17 rd.; *37.* 5745.5 gal.; *38.* $85.25; *39.* $\$148.43\frac{3}{4}$; *40.* $298.39; *41.* $\$212.98\frac{3}{4}$; *42.* $423.8493; *43.* $416.20; *44.* $90.675; *45.* 50 lb.; *46.* $152.76.

Page 123.—*Ex. 5.* $25.58; *6.* $23.68; *7.* $204.16; *8.* $551.05; *9.* $4619.12.

Page 135.—*Ex. 2.* 64922 oz.; *3.* 38688 gr.; *4.* 25660 gr.; *5.* 28800 gr.; *6.* 49000 gr.; *7.* 10761 ft.; *8.* 47520 in.; *9.* 79200 ft.; *10.* 6,195,200 sq. yd.; *11.* 3,162,240 sq. in.; *12.* 1,399,680 cu. in.; *13.* 20480 cu. ft.; *14.* 1188 cu. ft.; *15.* 3546 gi.; *16.* 376 pt.; *17.* 2772 cu. in.; *18.* 2688 cu. in.; *19.* 43008.4 cu. in.; *20.* 368640 ♏.; *21.* 2,678,400 sec.; *22.* 56430″; *23.* 864000″; *24.* 4448 far.; *25.* 7920 in.; *26.* 8,764,800 in.; *27.* 3014 sq. rd.

Page 136.—*Ex. 2.* 13 cwt. 33 lb. $5\frac{1}{3}$ oz.; *3.* 10 oz. 10 pwt.; *4.* 4 yd. $1\frac{1}{5}$ in.; *5.* 228 rd. 3 yd. $5\frac{1}{7}$ in.; *6.* 117 sq. rd. 10 sq. yd. 108 sq. in.; *7.* 2 qt. 1 pt. $1\frac{1}{3}$ gi.; *8.* 1 pk. $3\frac{3}{7}$ qt.; *9.* 21° 25′ $42\frac{6}{7}$″.———*2.* 8 cwt. 5 lb.; 3. 1

oz. 7 pwt. 18 gr.; *4.* 9℥ 4ʒ 2℈ 8 gr.; *5.* 212 rd.; *6.* 101 sq. rd. 18 sq. yd. 1 sq. ft. 50.4 sq. in.; *7.* 2 qt. 1 pt. 3.84 gi.; *8.* 3 pk. 7 qt. 1.04 pt.; *9.* 42′ 54″.

Page 137.—*Ex. 2.* 10 cwt. 13 lb. 1 oz.; *3.* 2 lb. 2 oz. 15 pwt. 5 gr.; *4.* 3 mi. 26 rd. 5 yd.; *5.* 17 cd. 104 cu. ft.; *6.* 12 A. 21 sq. rd. 19 sq. yd. 5 sq. ft. 108 sq. in.; *7.* 48 A. 80 sq. rd.; *8.* 15 gal. 2 qt. 1 gi.; *9.* 992 bu. 1 pk. 4 qt. 1.69+ pt.; *10.* 11 gal.; *11.* 5 gal. 1 qt. 1$\frac{3}{7}$ pt.; *12.* 1 Cong. 3 O. 2 fʒ 21♏; *13.* 1 da. 4 hr. 26 min. 56 sec.; *14.* 9 S. 13° 25′; *15.* £2 6 s. 6 d. 3 far.; *16.* 4 lb. 9 oz. 2 pwt. 4 gr.; *17.* £1 5 s. 2 d.; *18.* 3 T. 18 cwt. 55 lb. 2 oz.; *19.* 3 mi. 92 rd. 1 yd. 10 in.

Page 138.—*Ex. 20.* 8 mi. 165 rd. 3 yd. 2 ft. 2.64 in.; *21.* 3 cd. 93 cu. ft. 429 cu. in.; *22.* 250 gal.; *23.* 3387 bu. 3 pk. 2.4+ qt.; *24.* 24° 2′ 4″; *25.* 7 A. 38 P. 23 sq. yd. 5 sq. ft. 72 sq. in.; *26.* 63 ch. 3$\frac{1}{33}$ li.; *27.* 4 mi. 63 rd. 6 in.; *28.* £13 12 s. 11 d.; *29.* 4 da. 18 hr. 16 min. 45 sec.; *30.* 2592 score; *31.* 18 cu. yd. 24 cu. ft. 766 cu. in.———*2.* $\frac{5}{8}$; *3.* $\frac{101}{768}$; *4.* $\frac{108}{175}$; *5.* $\frac{1}{25}$; *6.* $\frac{11}{20}$; *7.* $\frac{1}{3}$; *8.* $\frac{121}{640}$; *9.* $\frac{5}{48}$; *10.* $\frac{61}{168}$; *11.* $\frac{29}{432}$; *12.* $\frac{61}{720}$; *13.* $\frac{1}{6}$.

Page 139.—*Ex. 2.* .953125 bu; *3.* .6465$\frac{5}{63}$ wk.; *4.* .180$\frac{1}{3}$ S.; *5.* .0130208$\frac{1}{3}$ mi.; *6.* .381955 + A.; *7.* .58$\frac{1}{3}$ yd.; *8.* .11875 ton; *9.* £14.8625; *10.* .2325 ton; *11.* .48125 A.; *12.* .855 lb.———*1.* $47.04; *2.* $42; *3.* $6.72; *4.* 284 da.

Page 140.—*Ex. 5.* $102.40; *6.* 31,622,400 sec.; *7.* $435.60; *8.* 12672; *9.* $157.50; *10.* 6225 mi.; *11.* 9142$\frac{6}{7}$ lb.; *12.* $209.64—; *13.* 45.82\frac{1}{12}$; *14.* $36; *15.* 2 bu. 7 qt., 30 boxes; *16.* 188,133,788,160,000 mi.; *17.* 422550 reams; *18.* 91$\frac{7}{11}$ kegs.

Page 141.—*Ex. 19.* 47$\frac{1}{49}$ A.; *20.* 14 wk. 3 da. 9 hr. 20 min.; *21.* $2282.75; *22.* 90 lots; *23.* 1.08\frac{1}{3}$; *24.* 15,076,800 sec.; *25.* 1.4 gr.; *26.* 50 E.; *27.* 208$\frac{1}{3}$ reams; *28.* 3 wk.; *29.* 6 oz.; *30.* 1672 steps; *31.* 20 min. past 1 o'clock, A. M., Feb. 26, 1895.

Page 142.—*Ex. 3.* 247 mi. 245 rd. 2 yd. 1 ft.; *4.* 429 A. 72 P. 17 sq. yd. 4 sq ft. 72 sq. in.

Page 143.—*Ex. 5.* 225 mi. 276 rd. 8 in.; *6.* 115 lb. 3 oz. 9 pwt. 4 gr.———*2.* 33° 6′ 54″; *3.* 17 mi. 279 rd. 4 yd. 2 ft. 5 in.; *4.* 17 A. 129 P. 25 sq. yd. 1 sq. ft. 28 sq. in.; *5.* 13 gal. 1 qt. 1 pt. 2 gi.

Page 144.—*Ex. 3.* 84 yr. 3 mo. 11 da.; *4.* 3 yr. 11 mo. 28 da.; *5.* 63 yr. 2 mo. 26 da.; *6.* 56 yr. 2 mo. 3 da.; *7.* 49 yr. 10 mo.; *8.* 228 da.; *9.* 235 da.; *10.* 146 da.; *11.* 183 da.; *12.* 883 da.; *13.* 2545 da.; *14.* 3812 da.

Page 145.—*Ex. 2.* 128 da. 4 min.; *3.* 305° 5′; *4.* 9 T. 3 cwt. 86 lb. 4 oz.; *5.* 215 gal. 1 qt.; *6.* £301 10 s.———*2.* 13 lb. 1 oz. 9 pwt. 5 gr.; *3.* 15° 42′ min. 13$\frac{1}{2}$″; *4.* 84 A. 36 P.; *5.* £25 2 s. 8 d.

Page 146.—*Ex. 7.* 4$\frac{1}{6}$; *8.* 8$\frac{1}{3}$; *9.* 5 kegs.

Page 147.—*Ex. 3.* 70°; *4.* 25°; *5.* 30°; *6.* 100° 40′; *7.* 101° 1′ 10″; *8.* 24° 49′ 50″; *9.* 76° 23′ 22″; *10.* 51° 18′ 51″; *11.* 120° 34′ 16″; *12.* 108° 4′ 7″.

Page 149.—*Ex. 3.* 5 hr. 5 min. 33$\frac{7}{15}$ sec.; *4.* 53 min. 17$\frac{1}{15}$ sec.; *5.* 5 hr. 37 min. 52$\frac{2}{15}$ sec.; *6.* 7 hr. 56 min. 30$\frac{8}{15}$ sec.; *7.* 9 hr. 17 min. 33$\frac{7}{15}$ sec. A. M., same day.

Page 150.—*Ex. 8.* 2 hr. 46 min. 40 sec. P. M.; *9.* 77° 35′; *10.* 108° 53′; *11.* 65° 32′ 10″; *12.* 130° 1′ 27″; *13.* 121° 28′ E.; *14.* 9′ 5″ W.; *15.* Set back 5 hr. 17 min. $5\frac{7}{15}$ sec.; *16.* 8 hr. 1 min. 40 sec. P. M.; *17.* Set ahead 6 hr. 50 min. $49\frac{1}{15}$ sec.; *18.* May 4, 2 hr. 14 min. 36 sec. P. M.———*1.* 41 cwt. 25 lb. 1 oz.; *2.* £29 10 s. $11\frac{2}{3}$ d.; *3.* 235 da.

Page 151.—*Ex. 4.* $61.88; *5.* 4 mi. 6 rd. 2 yd. 1 ft. 4.76 in.; *6.* £31 16 s. $7\frac{1}{2}$ d.; *7.* 3,982,417.92 cu. in.; *8.* 15 mirrors; *9.* 2 hr. 48 min. $57\frac{1}{2}$ sec.; *10.* 108° 33′ 41″; *11.* 207 da.; *12.* $2452.50; *13.* $$16.91\frac{1}{4}$; *14.* 2225.22 ft.; *15.* $45.15; *16.* 14 min. $31\frac{1}{9}$ sec.; *17.* 16 times; *18.* 1st west, 18° 45 min.; 2d east, 53°; *19.* $\frac{3}{5}$; *20.* East, 26° 15′.

Page 152.—*Ex. 21.* $148.50; *22.* $4763\frac{41}{48}$ mi.; *23.* 300°; *24.* A 171 bu. 3 pk., B 229 bu.; *25.* 16 ft., 1535 rails; *26.* 332 T. 12 cwt. 80 lb.; *27.* £3 16 s. .36 d.; *28.* 170.86 − marks; *29.* 106 lb. $3\frac{1}{2}$ oz.; *30.* .822 +; *31.* 152 lb.; *32.* 6.957 + oz.; *33.* 184 ft.

Page 153.—*Ex. 34.* 487 da.; *35.* 1 hr. 54 min. $8\frac{8}{9}$ sec.; *36.* $162.40; *37.* 86.15 + francs; *38.* 8.8 times; *39.* 43° 22′ 24.5″; *40.* .45514 + mi.; *41.* $37.46 +; *42.* $50.75; *43.* 21 min. $21\frac{1}{3}$ sec. past 1 o'clock; *44.* 62 mi. 103 rd. $4\frac{31}{50}$ yd.; *45.* $$1.636\frac{1}{4}$; *46.* $24.66 +.

Page 155.—*Ex. 1.* 600 sq. ft.; *2.* 88 sq. yd.; *3.* 9600 sq. rd.; *4.* 180 A.; *5.* 12 A.; *6.* 24 rd.; *7.* $23\frac{2}{9}$ sq. yd.; *8.* $117\frac{1}{3}$ sq. yd.; *9.* 15 A.

Page 156.—*Ex. 10.* 66 ch.; *11.* $94.50; *12.* $$23.46\frac{2}{3}$; *13.* $32.

Page 157.—*Ex. 1.* 350 sq. in.; *2.* 80 sq. yd.; *3.* 15 A.; *4.* 32 rd.; *5.* 60 ft.; *6.* 630 A.; *7.* $45\frac{3}{8}$ sq. ft.; *8.* $9\frac{7}{27}$ sq. yd.; *9.* $\frac{1}{8}$A.

Page 158.—*Ex. 1.* 31.416 in.; *2.* 62.832 in.; *3.* 97.3896 ft.; *4.* 314.16 rd.; *5.* 3.183 ft.; *6.* 12.732 in.; *7.* 25.464 rd.; *8.* 63.66 ch.; *9.* 314.16 sq. in.; *10.* 1256.64 sq. ft.; *11.* 795.75 sq. rd.; *12.* 1017.8784 sq. ft.; *13.* 1256.64 sq. ft.; *14.* 9.4248 ft.; *15.* 1520.5344 sq. ft.; *16.* 23.26 + sq. rd.; *17.* 2010.624 A.; *18.* 62.832 rd.; *19.* 10 mi. 132 rd.

Page 159.—*Ex. 1.* $$46\frac{2}{3}$; *2.* $$266\frac{2}{3}$; *3.* $311.04; *4.* $259.20; *5.* $23.04; *6.* $86.25; *7.* $12.10.

Page 160.—*Ex. 8.* $302.40; *9.* $12.25; *10.* Matting, $$1.48\frac{4}{27}$; *11.* $$33.07\frac{1}{3}$; *12.* $23\frac{8}{9}$ rolls; *13.* $$6.02\frac{2}{9}$; *14.* $$10.88\frac{2}{3}$; *15.* $257.40; *16.* 260 yd.

Page 161.—*Ex. 1.* 192 cu. ft.; *2.* 176 cu. yd.; *3.* $614\frac{1}{8}$ cu. ft.; *4.* 48000 cu. ft., 1500 T.

Page 162.—*Ex. 5.* 54 cd.; *6.* $393.75; *7.* $$5484\frac{3}{8}$; *8.* $320.

Page 163.—*Ex. 1.* 376.992 sq. ft.; *2.* 226.1952 cu. ft.; *3.* 1256.64 sq. ft.; *4.* 471.24 sq. ft., 589.05 cu. ft.; *5.* 30.5568 cu. ft.; *6.* 2150.42 cu. in.; *7.* $26.18; *8.* 13.09 cu. ft.; *9.* Cylinder, by 24.588 cu. ft.; *10.* Alt. 8 ft., 502.656 sq. ft.

Page 164.—*Ex. 1.* 128 bu.; *2.* 144 bu.; *3.* 3072 bu.; *4.* 3072 bu.; *5.* 96.42 + bu.; *6.* 77.14 + bu.; *7.* 1446.4 + bu.; *8.* 385.7 + bu.; *9.* .213 + bu.; *10.* 2.56 + bu.

Page 165—*Ex. 11.* $1.82; *12.* 262.38 + bbl.; *13.* 1692.0576 gal.; *14.* 44.763 bbl.; *15.* 699.42 + hhd.; *16.* 184.078 + tons; *17.* 13.403 + ft.; *18.* 21.39 + in.

Page 166.—*Ex. 1.* 12 ft.; *2.* 27 ft.; *3.* 20 ft.; *4.* $37\frac{1}{2}$ ft.; *5.* 48

ft.; *6.* 12 ft.; *7.* 42 ft.; *8.* 75 ft.; *9.* 30 ft.; *10.* 180 ft.; *11.* 54 ft.; *12.* \$8; *13.* \$24; *14.* \$105; *15.* \$60.48; *16.* \$201.60.

Page 168.—*Ex. 1.* $38\frac{26}{33}$ perches; *2.* 10080 bricks; *3.* 22883 bricks; *4.* 46634 bricks; *5.* \$1195.32.

Page 169.—*Ex. 6.* $31\frac{11}{15}$ perches; *7.* \$288.32$\frac{32}{99}$.——*1.* \$5.28; *2.* \$115.50; *3.* \$7.33$\frac{1}{3}$; *4.* \$29.93+; *5.* 122.87+ sq. yd.; *6.* 30 A.; *7.* 528 rd.; *8.* \$720; *9.* $292\frac{1}{2}$ boards.

Page 170.—*Ex. 10.* $285\frac{5}{27}$ cu. yd.; *11.* 11.91+ ft.; *12.* $842\frac{3}{16}$ cu. ft.; *13.* \$96.25; *14.* $212\frac{97}{144}$ lb.; *15.* \$13.16$\frac{1}{4}$; *16.* 64.45+ bu.; *17.* 2 pk. 6 qt. 1.91+ pt.; *18.* 4.78+ ft.; *19.* \$52.27−; *20.* \$16.27$\frac{1}{2}$; *21.* 1695.68+ perches; *22.* 3 T. 18 cwt. $95\frac{65}{128}$ lb.; *23.* 4.09+ cu. ft.

Page 171.—*Ex. 24.* 5177.3568 sq. ft.; *25.* 203.712 A.; *26.* 70532 bricks; *27.* \$281.015; *28.* M 5 A. 144 P., N 8 A. 136 P.; *29.* \$6466.80−; *30.* 8 hr. 7 min. 36 sec. A. M.; *31.* $9953\frac{19}{37}$; *32.* $28\frac{4}{5}$ ft.; *33.* \$754.27+; *34.* 485.415+ A.; *35.* 3.08+ ft.

Page 173.—*Ex. 1.* $\frac{3}{20}$, $\frac{2}{5}$; *2.* $\frac{1}{5}$, $\frac{1}{2}$; *3.* $\frac{1}{4}$, $\frac{3}{4}$; *4.* $\frac{3}{10}$, $\frac{4}{5}$; *5.* $\frac{9}{100}$, $\frac{4}{25}$; *6.* $\frac{3}{25}$, $\frac{9}{50}$; *7.* $\frac{1}{8}$, $\frac{1}{6}$; *8.* $\frac{6}{25}$, $\frac{1}{3}$; *9.* $\frac{1}{16}$, $\frac{1}{9}$; *10.* $\frac{3}{15}$, $\frac{7}{25}$; *11.* $\frac{9}{10}$, $\frac{5}{4}$; *12.* $\frac{2}{3}$, $\frac{3}{2}$; *13.* $\frac{1}{12}$, $\frac{3}{8}$; *14.* $\frac{5}{8}$, $\frac{7}{8}$; *15.* $\frac{1}{200}$, $\frac{1}{400}$.——*2.* 50%, $16\frac{2}{3}$%, 75%; *3.* $33\frac{1}{3}$%, $12\frac{1}{2}$%, $57\frac{1}{7}$%; *4.* 25%, 10%, $58\frac{1}{3}$%; *5.* 20%, $8\frac{1}{3}$%, $11\frac{1}{9}$%; *6.* $6\frac{1}{4}$%, 60%, 40%; *7.* 30%, $62\frac{1}{2}$%, $37\frac{1}{2}$%; *8.* $66\frac{2}{3}$%, $66\frac{2}{3}$%, $66\frac{2}{3}$%; *9.* $\frac{4}{5}$%, $\frac{1}{2}$%; *10.* $\frac{1}{4}$%, 250%.

Page 175.—*Ex. 23.* 112.5 A.; *24.* \$114.40.

Page 176.—*Ex. 25.* 856 bu.; *26.* \$6550; *27.* \$8400; *28.* \$3159; *29.* \$5.01$\frac{1}{2}$; *30.* \$3150; *31.* \$6300; *32.* \$6981.05; *33.* \$210.

Page 177.—*Ex. 22.* \$2125; *23.* 1680 A.; *24.* \$2000; *25.* \$735; *26.* \$700.

Page 178.—*Ex. 27.* 55400 men; *28.* \$4200; *29.* \$153; *30.* \$940; *31.* \$3640; *32.* \$7500; *33.* 12600; *34.* \$1500.

Page 179.—*Ex. 22.* 20%; *23.* $7\frac{11}{27}$%; *24.* 5%; *25.* $16\frac{2}{3}$%; *26.* $41\frac{3}{17}$%; *27.* 500%; *28.* 6%; *29.* $6\frac{1}{4}$%.

Page 180.—*Ex. 30.* $57\frac{1}{7}$%; *31.* $16\frac{2}{3}$%; *32.* \$95.——*1.* £85 14 s.; *2.* 32 A. 40 P.; *3.* $29\frac{1}{6}$%; *4.* $19\frac{4}{9}$; *5.* $193\frac{37}{51}$; *6.* $23\frac{2}{11}$%; *7.* 50%; *8.* 20%; *9.* \$2454; *10.* \$400; *11.* \$2.70; *12.* \$20000; *13.* $4\frac{16}{21}$%; *14.* \$9000.

Page 181.—*Ex. 15.* $88\frac{1}{3}$%; *16.* $6\frac{2}{3}$%; *17.* \$2300; *18.* $133\frac{1}{3}$%; *19.* \$2.59$\frac{2}{7}$; *20.* 40%; *21.* 50%; *22.* \$4840; *23.* $133\frac{1}{3}$%; *24.* \$11000; *25.* \$5.10; *26.* $25\frac{1}{3}$%; *27.* \$3$\frac{1}{3}$ loss.

Page 182.—*Ex. 28.* 27 yr.; *29.* \$60000; *30.* \$3.12$\frac{1}{2}$ loss.

Page 183.—*Ex. 4.* \$91; *5.* \$7200; *6.* $3\frac{1}{2}$%; *7.* \$1000; *8.* \$310.05; *9.* \$170.73+; *10.* 3%; *11.* \$4217.35.

Page 184.—*Ex. 12.* \$3921.56+; *13.* \$266.01−; *14.* \$6437.50; *15.* 92235.29+ lb.; *16.* \$1720.71$\frac{7}{8}$; *17.* \$2812; *18.* \$439.33−; *19.* \$6292.84; *20.* Entire com. \$240, agent's com. \$100, paid employer \$7760; *21.* \$1177.75; *22.* \$96.50.

Page 185.—*Ex. 23.* 2997.57+ bu.; *24.* \$2050; *25.* $2\frac{1}{2}$%.

Page 186.—*Ex. 2.* .002$\frac{1}{2}$; *3.* .001$\frac{1}{2}$; *4.* .001$\frac{3}{4}$; *5.* .006; *6.* \$14.60; *7.* \$196.

Page 187.—*Ex. 8.* $6580; *9.* Ass'd $9125, proceeds $8942.50; *10.* .003; *11.* $2008.60; *12.* .003; *13.* $55566; *14.* .004; *15.* $7250.

Page 188.—*Ex. 3.* $9, $33; *4.* $54, $81, $108, $94.50, $121.50; *5.* $65; *6.* $180; *7.* 1½%; *8.* $3820; *9.* $12000; *10.* 1⅕%; *11.* $2551.02+.

Page 189.—*Ex. 12.* $3840; *13.* $9045; *14.* $1870; *15.* $6860; *16.* $200; *17.* $212; *18.* $451.50.

Page 191.—*Ex. 12.* $84; *13.* $115; *14.* $207; *15.* $313.60; *16* $336; *17.* $56.70; *18.* $220.13⅓; *19.* $851.136; *20.* $429.33⅓; *21.* $6495; *22.* $8244; *23.* $769.06⅔; *24.* $1167.60; *25.* $44.73⅓; *26.* $78.70; *27.* $412.66⅔; *28.* $240; *29.* $110.50; *30.* $109.08⅓; *31.* $123.75; *32.* $138.60; *33.* $276.53⅓; *34.* $8.88; *35.* $13.10.

Page 192.—*Ex. 36.* $29.63⅓; *37.* $45.33¾; *38.* $84.32; *39.* $7.74–.———*2.* $90.45; *3.* $79.80; *4.* $124.80.

Page 193.—*Ex. 5.* $14.125; *6.* $5.488; *7.* $17.50; *8.* $85.21–; *9.* $588; *10.* $1636.52; *11.* $129.80; *12.* $.4048; *13.* $3.15; *14.* $44.461; *15.* $356.93–; *16.* $1119.92–; *17.* $660.48+; *18.* $3964; *20.* $93.16⅔; *21.* $884.35⅔; *22.* $2.66+; *23.* $543.06–; *24.* $212.06¼; *25.* $75.45–; *26.* $141.78–.

Page 194.—*Ex. 2.* $202.50; *3.* $105.60; *4.* $231; *5.* $284.66⅔; *6.* $1118.40; *7.* $469.875; *8.* $130.325; *9.* $821.10; *10.* $264.

Page 195.—*Ex. 2.* $45.27+; *3.* $136.43–; *4.* $635.18–; *5.* $91.98–; *6.* $389.11–; *7.* $40.94–; *8.* $989.82–.

Page 196.—*Ex. 2.* $480; *3.* $42; *4.* $1100; *5.* $3400; *6.* $365; *7.* $720; *8.* $480; *9.* $1702.63+; *10.* $215.34–; *11.* $1296; *12.* $23.75–.

Page 197.—*Ex. 2.* 7 yr. 8 mo.; *3.* 2 yr. 6 mo.; *4.* 1 yr. 8 mo.; *5.* 4 yr. 5 mo.; *6.* 2 yr. 7 mo.; *7.* 1 yr. 2 mo. 3 da.; *8.* 3 yr. 3 mo. 3 da.; *9.* 10 yr. 10 mo. 10 da.; *10.* 3 mo. 24 da.; *11.* 6 mo. 6 da.; *12.* 24 da.; *13.* 20 yr., 12½ yr., 25 yr.; *14.* 33⅓ yr., 28 4/7 yr., 36 4/11 yr.

Page 198.—*Ex. 2.* 6%; *3.* 5%; *4.* 5%; *5.* 5½%; *6.* 7%; *7.* 4.68–%; *8.* 6%; *9.* 5¾+%; *10.* 6%; *11.* 6%; *12.* 4½%; *13.* 12%; *14.* 3¼%; *15.* 25%, 16⅔%, 14 2/7%; *16.* 20%, 6⅔%, 40%.

Page 200.—*Ex. 1.* $451.80; *2.* $242.00; *3.* $660.004–.

Page 201.—*Ex. 4.* $858.05+; *5.* $1015; *6.* $966.40; *7.* $2080.80; *8.* $.40.

Page 203.—*Ex. 2.* $433.36⅔; *3.* $748.86⅔; *4.* $646.33⅓; *5.* $257.43⅓; *6.* $244.92–; *7.* $678.25.

Page 204.—*Ex. 2.* $220.77–; *3.* $562.71+; *4.* $1789.61+; *5.* $1963.79–.

Page 205.—*Ex. 6.* $1871.52–; *7.* $1273.10+.

Page 206.—*Ex. 2.* $654.30+; *3.* $184.0125; *4.* $1057.78–; *5.* $264.775; *6.* $1024.79+; *7.* $1117.0816.

Page 207.—*Ex. 2.* $37.08; *3.* $63.05; *4.* $310.796; *5.* $249.728; *6.* 6.3672%.

Page 209.—*Ex. 8.* $632.6784; *9.* $861.8232; *10.* $7189.946; *11,* $138.2496; *12.* $125.18832; *13.* $13.10+; *14.* $54.75, $54.60, $57.378, $57.417+.———*2.* $494.70.

Page 210.—*Ex.* *3.* \$70; *4.* \$228; *5.* \$157.50; *6.* $32\frac{1}{2}\%$; *7.* $31\frac{3}{5}\%$; *8.* \$168; *9.* \$55; *10.* 5%; *11.* 50%; *12.* $2\frac{1}{2}\%$ loss; *13.* \$325.584; *14.* 75%; *15.* $27\frac{7}{9}\%$; *16.* $\$258.82\frac{6}{17}$.

Page 211.—*Ex.* *2.* \$85.72 – ; *3.* \$72.13 + ; *4.* \$44.67 + ; *5.* \$317.65 – ; *6.* \$19.07 –.

Page 212.—*Ex.* *7.* \$57.62 +; *8.* \$168.50 –; *9.* \$3.26 +; *10.* Cash \$13.04 +.

Page 213.*—*Ex.* *2.* \$2.00; *3.* \$5.00; *4.* \$1.41 + ; *5.* \$3.12; *6.* \$12.00; *7.* \$16.63 + ; *8.* \$1249.50; *9.* \$20; *10.* \$2352; *11.* \$5005.21 + ; *12.* \$14.58 + ; *13.* \$1206.16 –; *14.* \$8.10; *15.* $\$3.06\frac{1}{4}$; *16.* \$892.50.

Page 214.—*Ex.* *2.* \$2030.45 + : *3.* \$243.04 –; *4.* \$1212.12 + ; *5.* $\$2666.66\frac{2}{3}$; *6.* \$2828.28 + ; *7.* \$1855.67 +.

Page 215.—*Ex.* *8.* \$1530.61 + ; *9.* \$730.96 + ; *10.* \$1224.49 –; *11.* \$998.43 –; *12.* \$1997.87 +.

Page 217.—*Ex.* *2.* \$10175; *3.* $\$10933\frac{1}{8}$; *4.* $\$2480\frac{5}{8}$; *5.* $\$6007\frac{1}{2}$; *6.* \$11230; *7.* \$18060; *8.* \$11750: *9.* 50 shares; *10.* 125 shares; *11.* $\$13015\frac{5}{8}$; *12.* 64 shares; *13.* 34 shares; *14.* 37 shares; *15.* 84 shares; *16.* 301 shares, value \$31605; *17.* $104\frac{3}{8}$.

Page 218.—*Ex.* *18.* $113\frac{1}{2}$; *19.* 37 shares; *20.* $20\frac{1}{2}\%$; *21.* $2\frac{3}{4}\%$; *22.* \$1920; *23.* Profit \$416, com. \$16; *25.* \$320; *26.* \$288; *27.* \$6000; *28.* \$594.

Page 219.—*Ex.* *30.* \$10971; *31.* \$3525; *32.* \$7840; *33.* \$3732; *34.* $\$28531\frac{1}{4}$; *36.* 5%; *37.* 5%; *38.* $83\frac{1}{3}$; *39.* 5.31 + %.

Page 220.—*Ex.* *40.* \$264; *41.* \$8925; *42.* 7% stock, by .04 + %. *43.* 150; *44.* 5%; *45.* $\$3422\frac{1}{2}$.——*1.* $12\frac{1}{2}\%$; *2.* \$24; *3.* 40%; *4.* \$3140.70 + ; *5.* \$3831.68 + ; *6.* \$69.21 –; *7.* \$1873.09 +.

Page 221.—*Ex.* *8.* \$9600; *9.* \$98.7111; *10.* \$33.21 –; *11.* \$86.41 –; *12.* \$627.70 + ; *13.* 4 yr. 9 mo. 8 + da.; *14.* $5\frac{1}{2}\%$; *15.* \$453.70 + ; *16.* \$16.11 + ; *17.* \$165.41 + ; *18.* \$238; *19.* \$599.37 + ; *20.* $85\frac{33}{56}$; *21.* $83\frac{1}{16}$.

Page 222.—*Ex.* *1.* 3; *2.* $\frac{1}{10}$; *3.* 2; *4.* $\frac{4}{5}$; *5.* $4\frac{2}{5}$; *6.* $\frac{2}{9}$; *7.* $\frac{1}{7}$; *8.* $\frac{4}{5}$; *9.* 3; *10.* $\frac{1}{6}$; *11.* 32; *12.* $\frac{14}{15}$.

Page 223.—*Ex.* *13.* 4; *14.* $1\frac{1}{16}$; *15.* The latter; *16.* $\frac{1}{2}$; *17.* 578; *18.* 64.

Page 224.—*Ex.* *1.* 12; *2.* 27; *3.* 16; *4.* 2; *5.* 2; *6.* 27; *7.* $13\frac{1}{5}$; *8.* 6; *9.* $3\frac{3}{5}$; *10.* 4; *11.* 192; *12.* 18; *13.* $3\frac{13}{15}$; *14.* 48; *15.* 375.

Page 226.—*Ex.* *3.* \$255; *4.* $28\frac{10}{11}$ days; *5.* \$12000; *6.* $\$30\frac{20}{21}$; *7.* 680 days; *8.* \$72; *9.* 24 tons; *10.* $89\frac{1}{4}$ tons; *11.* $56\frac{4}{5}$ days; *12.* 84 ft.; *13.* $\$516.09\frac{3}{8}$; *14.* $99\frac{13}{63}$ days; *15.* \$1175.23.

Page 227.—*Ex.* *16.* 32 days; *17.* $26\frac{2}{3}$ min.; *18.* $70\frac{10}{57}$ days; *19.* 1120 men; *20.* \$1360; *21.* 44.8 days; *22.* $107\frac{3}{23}$ days; *23.* $193\frac{1}{3}$ days; *24.* \$800; *25.* $212\frac{1}{2}$ days; *26.* $110\frac{1}{4}$ days; *27.* \$1.80; *28.* 84 ft.; *29.* 4 hr. 17 min. $9\frac{1}{6}$ sec.

Page 228.—*Ex.* *2.* \$2232; *3.* 36 rd.

Page 229.—*Ex.* *4.* 54 men; *5.* \$280; *6.* 480 mi.; *7.* $9\frac{1}{3}$ hr.; *8.* $\$37\frac{1}{2}$; *9.* $18\frac{3}{4}$ days; *10.* $1828\frac{4}{7}$ lb.; *11.* 7 hr.; *12.* 80 days; *13.* 150 cows; *14.* 60 days.

* For additional answers to examples in Bank Discount, see page 353.

Page 230.—*Ex. 2.* A \$1008, B \$816; *3.* A \$2040, B \$1920, C \$1240; *4.* L \$650, M \$780, N \$910, O \$1040.

Page 231.—*Ex. 5.* A \$1195, B \$239; *6.* A \$74.95+, B \$110.68−, C \$164.62−; *7.* M \$1040, N \$520, P \$260; *8.* A \$760, B \$1140, C \$1710; *9.* A daughter $\$4166\frac{2}{3}$, a son $\$3333\frac{1}{3}$, widow $\$6666\frac{2}{3}$; *10.* A \$6480, B \$8856, C \$9664; *11.* A $202\frac{26}{37}$ bu., B $162\frac{6}{37}$ bu., C $135\frac{5}{37}$ bu.; *12.* A \$400, B \$560, a cow \$40; *13.* M. $\$382\frac{1}{2}$, N \$255, O $\$382\frac{1}{2}$.

Page 232.—*Ex. 2.* M \$570.40, N \$694.40; *3.* R \$297, S \$429; *4.* A \$195, B \$351, C \$252; *5.* R. \$24, S \$33.60, T \$57.60; *6.* A \$1248, B \$1040, C \$624.

Page 233.—*Ex. 7.* M \$2607, N \$2112; *8.* A \$3380, B \$4992; *9.* A \$20, B \$15.75, C \$29.75; *10.* Each \$1550.20; *11.* A \$2760, B \$2840, C \$2876.

Page 235.—*Ex. 4.* 6 mo. 4 da.; *5.* July 20.

Page 236.—*Ex. 6.* April 23; *7.* In 14 mo.; *8.* 5 mo.−; *9.* July 6; *10.* 16 mo.; *11.* 20 mo.; *12.* March 22; *13.* 2 mo. 24 da. later; *14.* Aug. 17; *15.* 27 days; *16.* 18 mo. 10 da.

Page 238.—*Ex. 3.* June 24.

Page 239.—*Ex. 4.* Nov. 18, 1890; *5.* Nov. 10, 1891; *6.* July 24, 1892.

Page 242.—*Ex. 1.* 225; *2.* 2025; *3.* 6889; *4.* 9801; *5.* 93025; *6.* 1728; *7.* 15625; *8.* 614125; *9.* 998001; *10.* 970299; *11.* 300763; *12.* 15.625; *13.* 4225; *14.* 39304; *15.* $\frac{3375}{10648}$; *16.* $6\frac{1}{4}$; *17.* $44\frac{4}{9}$; *18.* $37\frac{1}{27}$; *19.* .9801; *20.* .000729; *21.* .003375; *22.* .000000001; *23.* .997002999; *24.* .000001; *25.* .998001; *26.* 5625; *27.* 15625; *28.* .000000000001; *29.* 47524; *30.* 131044; *31.* 36.6025; *32.* 1772.41; *33.* 5.6169; *34.* 1,002,001; *35.* 100.2001; *36.* 1.002001.

Page 246.—*Ex. 2.* 21; *3.* 14; *4.* 35; *5.* 21; *6.* 81; *7.* 99; *8.* 15; *9.* 6; *10.* 12; *11.* 25; *12.* 7; *13.* 9; *14.* 6; *15.* 16; *16.* 8.

Page 249.—*Ex. 4.* 16; *5.* 21; *6.* 25; *7.* 45; *8.* 52; *9.* 67; *10.* 84; *11.* 94; *12.* 123; *13.* 111; *14.* 112; *15.* 204; *16.* 324; *17.* 512; *18.* 607; *19.* 876; *20.* 321; *21.* 425; *22.* 562; *23.* 972; *24.* 527; *25.* 721; *26.* 999; *27.* 1051; *28.* 2004; *29.* 2345; *30.* 2504.

Page 250.—*Ex. 32.* .17; *33.* .65; *34.* .81; *35.* .92; *36.* 4.6; *37.* 23.4; *38.* .063; *39.* 1.296; *40.* 14.31; *41.* .0099; *42.* .001; *43.* 1.01; *45.* 1.4142+; *46.* 1.7320+; *47.* 2.2360+; *48.* 2.4494+; *49.* 2.6457+; *50.* 2.8284+.

Page 251.—*Ex. 53.* .522+; *54.* .326+; *55.* .8291+; *56.* .6928+; *57.* .6123+; *58.* .6454+; *59.* .6324+; *60.* .9486+.——*1.* 80 rd.; *2.* \$192; *3.* $16\frac{2}{3}$ ft.; *4.* $62\frac{1}{4}$ ft.

Page 252.—*Ex. 5.* 80 rd. wide, 160 rd. long; *6.* \$28.80; *7.* 65 men; *8.* 41000 men; *9.* $2375\frac{1}{9}$ sq. yd.

Page 253.—*Ex. 3.* 40 ft.; *4.* 38.73− ft.; *5.* 49.24+ rd.; *6.* 169.70+ rd.; *7.* 54 ft.; *8.* 27.71+ ft.; *9.* 48.67+ ft.; *10.* 17.677+ ft.; *11.* 64 ft.; *12.* 45.25+ rd.

Page 255.—*Ex. 2.* 96 rd.; *3.* 25 times; *4.* 44 ft.; *5.* 50 rd. wide, 60 rd. long; *6.* \$540; *7.* 25.225 rd.

Page 259.—*Ex. 4.* 32; *5.* 43; *6.* 54; *7.* 65; *8.* 76; *9.* 85;

10. 93; *11.* 123; *12.* 231; *13.* 507; *14.* 325; *15.* 526; *16.* 642; *17.* 708; *18.* 1234.

Page 260.—*Ex. 21.* 342; *22.* 561; *23.* 723; *24.* 566; *25.* 652; *26.* 743; *27.* 2345; *28.* 5432; *29.* 7539.

Page 261.—*Ex. 31.* .43; *32.* .55; *33.* .67; *34.* .86; *35.* .125; *36.* .436; *37.* .036; *38.* .099; *39.* 3.33; *41.* 1.4422+; *42.* 1.5873+; *43.* 2.9240+; *44.* .6463+; *45.* 4.0207+; *46.* 4.4814+.

Page 262.—*Ex. 49.* $\frac{12}{25}$; *50.* $\frac{27}{55}$; *51.* .69336+; *52.* 1.07721+; *53.* .8735+; *54.* .9085+.——*1.* 60 in.; *2.* 3750 sq. in.; *3.* 3½ ft.; *4.* in. nearly; *5.* 2.919+ ft.

Page 263.—*Ex. 6.* 5.113+ ft.; *7.* \$10.88$\frac{8}{9}$.

Page 264.—*Ex. 2.* 7½ in.; *3.* 16 ft.; *4.* \$13500; *5* 1:512; *6.* 20 ft.; *7.* 22½ ft., 15 ft., 8 ft.; *8.* 4 ft., 2⅔ ft., 1⅓ ft.; *9.* 8 to 27; *10.* 25 to 16; *11.* 5.2− ft.

Page 268.—*Ex. 1.* 320 sq. ft.; *2.* 6 A.; *3.* 80 sq. yd.; *4.* 3 A.; *5.* 36 A.; *6.* 72¼ sq. ft.; *7.* 64 rd.

Page 269.—*Ex. 8.* 624 sq. ft.; *9.* \$4200; *10.* \$900; *11.* 80 A.; *12.* 10 A.; *13.* 122.26− ft.; *14.* 50 rd.

Page 270.—*Ex. 1.* 800 sq. in.; *2.* 11¼ A.; *3.* 90 A.; *4.* 270 A. ——*1.* 3600 sq. ft.; *2.* 15 A.; *3.* 60 A.; *4.* 242 ft.

Page 271.—*Ex. 5.* 24 sq ft; *6.* 3507.40+ sq. ft.——*1.* 800 sq. in.; *2.* 16¼ A.; *3.* 112 A.; *4.* 30 in.

Page 273.—*Ex. 1.* 31.416 in.; *2.* 125.664 rd.; *3.* 958.188 ch.; *4.* 1432.5696 ft.; *5.* 15.915 in.; *6.* 31.83 ft.; *7.* 100 rd.; *8.* 2000 ch.; *9.* 25007.136 mi.; *10.* 267036 mi.; *11.* 16806.72+; *12.* 1.91− ft.; *13.* 14.16 yd.; *14.* 47.124 ft.; *15.* 94.248 ft.

Page 274.—*Ex. 1.* 314.16 sq. ft.; *2.* 1256.64 sq. rd.; *3.* 25.7823 sq. ch.; *4.* 38.4846 sq. ft.; *5.* 113.0976 sq. yd.; *6.* 343.36 sq. rd.; *7.* 201.0624 sq. yd.; *8.* \$3887.73.

Page 275.—*Ex. 2.* 21.21+ in.; *3.* 56.56+ rd.; *4.* 22.51− ft.; *5.* 104.1+ sq. rd.; *6.* 2.12+ in.

Page 277.—*Ex. 1.* 960 sq. in.; *2.* 2000 sq. in.; *3.* 480 sq. ft.; *4.* 216 sq. in.; *5.* 679.37+ sq. in.; *6.* 2746.41+ sq. ft.

Page 278.—*Ex. 1.* 640 cu. ft.; *2.* 1920 cu. ft.; *3.* 17320.5+ cu. ft.; *4.* 1920 cu. ft.; *5.* 168960 cu. yd.

Page 279.—*Ex. 1.* 240 sq. ft.; *2.* 1890 sq. ft.; *3.* 2304 sq. ft.; *4.* 2400 sq. ft.

Page 280.—*Ex. 5.* 459.71+ sq. in.——*1.* 60 cu. ft.; *2.* 1500 cu. ft.; *3.* 4800 cu. ft.; *4.* 5985.96+ cu. ft.; *5.* 480 cu. ft.

Page 281.—*Ex. 1.* 251.328 sq. ft.; *2.* 942.48 sq. ft.; *3.* 981.75 sq. ft.; *4.* 1627.3488 sq. ft.——*1.* 1005.312 cu. ft.

Page 282.—*Ex. 2.* 282.22+ cu. ft.; *3.* 884.884 cu. ft.——*1.* 904.78+ sq. in.; *2.* 1884.96 sq. ft.; *3.* 94.248 sq. yd.

Page 283.—*Ex. 4.* 6082.1376 sq. ft.; *5.* 628.32 sq. in.——*1.* 201.0624 cu. ft.; *2.* 6597.36 cu. ft.; *3.* 1005.312 cu. in.; *4.* 30159.36 cu. ft.

Page 284.—*Ex. 1.* 768 sq. ft.; *2.* 2111.15+ sq. ft.; *3.* 2748.9 sq. ft.; *4.* 890.23−. sq. in.

Page 285.—*Ex. 3.* 420 cu. ft.; *4.* 1193.808 cu. ft.; *5.* 8595.41+ cu. ft.; *6.* 230.36+ cu. ft.——*1.* 314.16 sq. in.; *2.* 1809.56+ sq. in.; *3.* 199,056,802.56 sq. mi.; *4.* \$286.27+; *5.* \$251.32+.

Page 286.—*Ex. 1.* 4188.8 cu. in.; *2.* 636.69+ cu. in.; *3.* 186.79− lb.; *4.* 460.19+ lb.———*2.* 17.32+ in.

Page 287.—*Ex. 1.* 72 A.; *2.* 120.28+ sq. yd.; *3.* 29 A. 8− P.; *4.* 133.49+ ft.; *5.* 1.414+; *6.* 1.732+; *7.* 107 A. 47.45 P.; *8.* Length 49.96− rd., width 37.47− rd.; *9.* 75.61+ rd.; *10.* 35.45− ft.; *11.* 24.43+ rd.; *12.* 194.72+ sq. ft.; *13.* 40; *14.* 2.76− sq. ft.; *15.* 130 cu. ft.; *16.* 6.

Page 288.—*Ex. 17.* 18.16+ ft.; *18.* 3240 sq. in.; *19.* 10.75+ ft.; *20.* 25132.8 cu. ft.; *21.* 3719.65+ cu. ft.; *22.* D 28.45+ ft., A 23.71+ ft.; *23.* \$3.53+; *24.* 42849; *25.* 28.31− cu. ft.; *26.* D 48 ft., A 64 ft.: *27.* 616.80+ cu. in.; *28.* \$5120.

Page 290.—*Ex. 2.* 23; *3.* 62; *4.* 148; *5.* 405; *6.* 142; *7.* 88; *8.* 1005; *9.* 98; *10.* $5\frac{5}{8}$; *11.* $788\frac{1}{12}$ ft.

Page 291.—*Ex. 3.* 272; *4.* 400; *5.* 1800; *6.* 3940; *7.* 1704; *8.* 14950; *9.* 1092 times; *10.* 2430 mi.; *11.* \$390; *12.* $10052\frac{1}{12}$ ft.; *13.* 11 mi. 152 rd. 4 yd.

Page 292.—*Ex. 3.* 512; *4.* 10935; *5.* 12288; *6.* $\frac{1}{512}$; *7.* $\frac{1}{19683}$; *8.* $\frac{2187}{32768}$; *9.* \$2048; *10.* \$10,737,418.24.

Page 293.—*Ex. 3.* 1023; *4.* 174762; *5.* 24570; *6.* 11,957,420; *7.* 10230; *8.* 5,120,000.

Page 294.—*Ex. 9.* \$20971.51; *10.* \$640700.81.———*2.* $1\frac{1}{2}$; *3.* $1\frac{1}{3}$; *4.* $1\frac{1}{9}$; *5.* $\frac{2}{3}$.

Page 295.—*Ex. 1.* 123; *2.* 43911; *3.* $\frac{9}{11}$; *4.* $\frac{8077}{73}$; *5.* $\frac{14}{25}$; *6.* $2\frac{1}{2}$; *7.* $2\frac{3}{16}$; *8.* $\frac{115}{192}$; *9.* $\frac{9}{16}$; *10.* $8\frac{1}{3}$; *11.* $200\frac{19}{56}$; *12.* $5\frac{43}{72}$; *13.* $.0000000\frac{1}{7}$; *14.* 300,000,000; *15.* .112, .46; *16.* 150010.

Page 296.—*Ex. 17.* 4401; *18.* 68.75+ gal.; *19.* 60.76+ lb. *20.* \$14.72; *21.* 72 A. 116 P. 11 sq. yd.; *22.* $28\frac{76}{363}$ pt.; *23.* 21.82+ hhd.; *24.* 109.4− rd.; *25.* $\frac{173}{180}$; *26.* $\$126.93\frac{1}{3}$; *27.* 2252 da.; *28.* 59 min. 56 sec. past 5 o'clock P. M. Dec. 24th; *29.* 10.72+ ft.; *30.* 834.85− ft., 739.86− ft.; *31.* 3361.34+; *32.* $4614\frac{201}{211}$; *33.* Difference 6 hr.; *34.* $6\frac{13865}{15872}$.

Page 297.—*Ex. 35.* \$1800; *36.* $3\frac{1}{4}$; *37.* $33\frac{1}{3}\%$; *38.* \$1739; *39.* \$2850; *40.* \$77.77; *41.* \$15.01+; *42.* \$81.552; *43.* \$3560; *44.* \$39.21+; *45.* \$1546.39+; *46.* P. W. \$714.28+, T. D. \$85.71+; *47.* 1394.87+ cu. in.; *48.* \$15680; *49.* \$111.70+; *50.* 63.44+ft.

Page 298.—*Ex. 51.* 3 yr. 3 mo. 9 da.; *52.* 20.738; *53.* $\$8351\frac{3}{8}$; *54.* $108\frac{85}{88}$; *55.* .873+; *56.* \$640 loss; *57.* $4\frac{1}{16}$ ft.; *58.* $6\frac{1}{2}\%$; *59.* 1998; *60.* 24 in., 32 in., and 40 in.; *61.* \$443.25; *62.* \$600; *63.* \$7.498+; *64.* 64 shares; *65.* Surface 452.39+ sq. in., volume 904.78+ cu. in.; *66.* \$244.89+; *67.* $\frac{17}{72}$; *68.* 2500 lb.

Page 299.—*Ex. 69.* 3561; *70.* 26.32+ ft.; *71.* \$13335; *72.* 5%; *73.* $54\frac{6}{11}$ min. past 4; *74.* \$9600; *75.* $\frac{1}{9}$; *76.* 4 ft. $1\frac{1}{2}$ in.; *77.* \$164; *78.* $84\frac{8}{13}\%$; *79.* Square walk by 537.67+ sq. ft.; *80.* $336\frac{1}{9}$ cu. yd.; *81.* $2\frac{2}{3}$ hr.; *82.* 328.47+; *83.* \$343.20.

Page 300.—*Ex. 84.* 29.49+ ft.; *85.* $41\frac{2}{3}\%$; *86.* \$200; *87.* 4.42+ ft.; *88.* \$408.33−; *89.* 343.36 sq. ft.; *90.* $\$4291\frac{2}{3}$; *91.* 127.06+ sq. yd.; *92.* $7\frac{1}{3}\%$; *93.* 14600 men; *94.* \$20.

Page 301.—*Ex. 95.* Diminished \$17.50, surplus \$61.75; *96.* 50%; *97.* 1st .274+ in., 2d .344+ in., 3d .491+ in., 4th 1.889+ in.; *98.* 195 ft.; *99.* 43; *100.* $\frac{1993}{5000000}$; *101.* 412+ balls; *102.* \$400; *103.* 10.52+ ft.; *104.* \$203; *105.* 58 ft.; *106.* $49\frac{1}{11}$ min.; *107.* \$220; *108.* $16\frac{8}{67}$ hr.

Page 302.—*Ex. 109.* 301.59+ cu. ft.; *110.* \$1389.74+; *111.* 38.48+ sq. in.; *112.* 101 shares, surplus \$12.81$\frac{1}{4}$; *113.* Circle .415 sq.rd.; *114.* 14$\frac{32}{47}$ hr.; *115.* 15$\frac{1}{5}$%; *116.* 10%; *117.* \$520.83$\frac{1}{3}$; *118.* 8 hr. 2$\frac{1}{7}$ min.

Page 303.—*Ex. 119.* \$358.498; *120.* \$60.89−; *121.* 33.92+%; *122.* \$2143.15+; *123.* 44%; *124.* 40+%; *125.* 3.19+ gal.; *126.* 50607; *127.* 40578; *128.* .261052+; *129.* 75.

Page 304.—*Ex. 130.* 21$\frac{17}{23}$%; *131.* 1$\frac{181777}{559790}$; *132.* 16.473+%; *133.* \$924.15+; *134.* 46 shares, surplus \$83; *135.* \$27000; *136.* 94.7+; *137.* 1$\frac{1}{2}$; *138.* \$603.39+; *139.* 1.6818−; *140.* 1.4678−; *141.* 2.375+ in.; *142.* 52$\frac{4}{11}$ da.; *143.* 1 hr. 5 min. 16.11+ sec.

Page 309.—*Ex. 3.* \$2025; *4.* \$1568; *5.* \$4887.50.

Page 310.—*Ex. 3.* \$480; *4.* \$5280; *5.* \$1200; *6.* \$1800; *7.* \$8000.

Page 312.—*Ex. 4.* \$2426.25; *5.* \$4845; *6.* £320; *7.* \$1007.75+; *8.* \$162.56−; *9.* 2176.65 francs; *10.* \$1440; *11.* \$2937.37$\frac{1}{2}$.

Page 313.—*Ex. 3.* \$101.25.

Page 314.—*Ex. 4.* \$4.32; *5.* \$4140; *6.* \$887.60.

Page 315.—*3.* \$1723.07+; *4.* \$3044.12−; *5.* \$9282.35+; *6.* \$11054.54+; *7.* \$13250; *8.* \$22666.67−.

Page 316.—*Ex. 3.* \$4801.64+; *4.* \$7451.75+; *5.* \$6494.22+; *6.* \$9995.02+; *7.* \$19304.34+.

Page 318.—*2.* \$1360; *3.* \$5089.84.

Page 319.—*Ex. 2.* \$1864.50; *3.* \$1671.60; *4.* \$2819.40.

Page 320.—*Ex. 2.* \$4152.96; *3.* \$2529.15.

Page 321.—*Ex. 2.* \$6699.71$\frac{1}{4}$; *3.* \$6871.50; *4.* \$5814.90.——*2.* 13.22+%.

Page 322.—*Ex. 3.* 6.44−%; *4.* 25.68+%; *5.* 21.1−%.——*2.* \$38.25, 8.63+%; *3.* 9.47+%; *4.* 8.23+%; *5.* 6.74−%.

Page 330.—*Ex. 1.* .7321$^{\text{cg.}}$; *2.* 456700$^{\text{dg.}}$; *3.* .86754$^{\text{Km.}}$; *4.* 4.7654$^{\text{Dm.}}$; *5.* 4.7375$^{\text{Hl.}}$; *6.* 647650$^{\text{cl.}}$; *7.* 67.56$^{\text{m.}}$; *8.* 647.6$^{\text{l.}}$; *9.* .93456$^{\text{Kg.}}$; *10.* 478654$^{\text{cg.}}$; *11.* 8.476543$^{\text{sq. Km.}}$; *12.* 4786953$^{\text{cu. cm.}}$; *13.* .095432$^{\text{cu. dm.}}$; *14.* 5.79$^{\text{l.}}$; *15.* .48567$^{\text{T.}}$; *16.* 100$^{\text{Dg.}}$; *17.* 1000$^{\text{Kg.}}$; *18.* .04516$^{\text{cl.}}$; *19.* 4287$^{\text{a.}}$; *20.* 1651$^{\text{Mg.}}$; *21.* .0184$^{\text{cl.}}$; *22.* 1.684$^{\text{Dl.}}$

Page 332.—*Ex. 6.* 283.459$^{\text{m.}}$; *7.* 328507.596$^{\text{l.}}$; *8.* 868.967$^{\text{g.}}$; *9.* 729.68$^{\text{sq. m.}}$; *10.* 1086.564$^{\text{cu. m.}}$; *11.* 1116.47$^{\text{l.}}$; *12.* 361.93$^{\text{m.}}$; *13.* 28.778$^{\text{g.}}$; *14.* 431.322$^{\text{cu. m.}}$; *15.* 756.019$^{\text{sq. m.}}$; *16.* .000056826$^{\text{Dl.}}$; *17.* 10371.84$^{\text{dm.}}$; *18.* .758976$^{\text{Kl.}}$; *19.* 855,414,000$^{\text{sq. mm.}}$; *20.* 306,720,000$^{\text{cu. cm.}}$; *21.* 963.9; *22.* 8110; *23.* 372; *24.* 21810; *25.* .002839$^{\text{Kg.}}$; *26.* 1877.8−$^{\text{cm.}}$

Page 333.—*Ex. 3.* 264.04+ ft.; *4.* 2237.53− yd.; *5.* 49.712 mi.; *6.* 29.26$^{\text{Dm.}}$; *7.* 80.465$^{\text{Hm.}}$; *8.* .6096$^{\text{Km.}}$; *9.* 14.82 acres; *10.* 503.516 sq. yd.; *11.* 2510611+ sq. rd.; *12.* 10.53486$^{\text{sq. Dm.}}$; *13.* 87.26+$^{\text{a.}}$; *14.* 1657.6$^{\text{sq. Km.}}$

Page 334.—*Ex. 15.* 4237.824 cu. ft.; *16.* 412.8048 cu. yd.; *17.* 7.248$^{\text{st.}}$; *18.* 37.12+$^{\text{st.}}$; *19.* 31.83+ liquid gal., 27.35+ dry gal.; *20.* 205.89+ bu.; *21.* 38.30+ liq. qt.; *22.* 23.85−$^{\text{Dl.}}$; *23.* 17.62$^{\text{Hl.}}$; *24.* 757.04$^{\text{cl.}}$; *25.* 771.6 gr.; *26.* 1.93+ lb.; *27.* 28356.44+ lb.; *28.* 5.59−$^{\text{Hg.}}$; *29.* 6.804$^{\text{Kg.}}$; *30.* 1829.818 cwt.——*1.* \$406.25; *2.* \$17.03; *3.* \$91.25; *4.* \$31.25; *5.* 4 hr.

Page 335.—*Ex. 6.* \$88.08$\frac{1}{3}$; *7.* \$248.64; *8.* \$16.19+; *9.* \$10.48; *10.* \$30705; *11.* \$5.47−; *12.* \$.154+; *13.* 88.9$^{\text{Kg.}}$; *14.* 48.279$^{\text{Km.}}$; *15.* 6070.5$^{\text{a.}}$; *16.* 25.4+$^{\text{T.}}$; *17.* 1187.3+$^{\text{Km.}}$; *18.* 386232$^{\text{Km.}}$; *19.* \$329.184+; *20.* 357.164+$^{\text{Kg.}}$

ADDITIONAL ANSWERS TO PROBLEMS IN BANK DISCOUNT.

The answers previously given for problems involving Bank Discount are calculated for the exact number of days named in the problems.

Below are given three other answers for each of these problems. The first is for *one* day more than the specified time, as in Pennsylvania, etc.; the second for *three* days more, as in several other states; and the third for *four* days more, as in Missouri.

Page 213.—*Ex. 2.* $2.06⅔, $2.20, $2.26⅔; *3.* $5.08⅓, $5.25, $5.33⅓; *4.* $1.425+, $1.45+, $1.47+; *5.* $3.29+, $3.64, $3.81+; *6.* $12.16⅔, $12.50, $12.66⅔; *7.* $16.818+, $17.188+, $17.37+; *8.* $1249.325, $1248.975, $1248.80; *9.* $20.22 2⁄9, $20.66⅔, $20.89−; *10.* $2351.60, $2350.80, $2350.40; *11.* $5005.265+, $5005.357+, $5005.403+; *12.* $15.19+, $16.41+, $17.02+; *13.* $1206.14−, $1206.096+, $1206.07+; *16.* $892.35, $892.05, $891.90.

Page 214.—*Ex. 2.* $2030.80+, $2031.488+, $2031.83+; *3.* $243.07+, $243.14−, $243.17+; *4.* $1212.325+, $1212.73+, $1212.94−; *5.* $2637.36+, $2580.64+, $2553.19+; *6.* $2829.235+, $2831.14+, $2832.09+; *7.* $1855.98+, $1856.627+, $1856.94+.

Page 215.—*Ex. 8.* $1530.96−, $1531.65+, $1532+; *9.* $731.09−, $731.33+, $731.46−; *10.* $1224.77−, $1225.32+, $1225.60+; *11.* $998.95+, $1000, $1000.52+; *12.* $1998.58+, $2000, $2000.71+.

Page 220.—*Ex. 6.* $69.19+, $69.17−, $69.16−; *7.* $1873.41+, $1874.04+, $1874.36+.

Page 221.—*Ex. 15.* $453.703+, $453.700+, $453.698+; *18.* $237.96⅔, $237.90, $237.86⅔; *19.* $599.46+, $599.368+, $599.35+.

Page 298.—*Ex. 61.* $443.175, $443.025, $442.95.

Page 302.—*Ex. 117.* $521.01+, $521.37+, $521.55+.

Page 303.—*Ex. 122.* $2142.55+, $2141.35−, $2140.74+.

Page 309.—*Ex. 5.* $4886⅔, $4885, $4884⅙.

Page 310.—*Ex. 5.* $1200.20+, $1200.61+, $1200.82−; *6.* $1800.25−, $1800.74+, $1800.99+; *7.* $8001.36+, $8004.09+, $8005.45+.

This book should be returned to the Library on or before the last date stamped below.

A fine of five cents a day is incurred by retaining it beyond the specified time.

Please return promptly.